(1) Let \( f(\theta_y) \, d\theta_y \) be the probability per unit length that the particle is scattered through an angle \( \theta_y \) in the (yz) plane.

\[
\langle \theta_y \rangle = \int_{-\infty}^{\infty} \theta_y f(\theta_y) \, d\theta_y = 0 \quad \text{because} \quad f(\theta_y) = f(-\theta_y)
\]

\[
\langle \theta_y^2 \rangle = \int_{-\infty}^{\infty} \theta_y^2 f(\theta_y) \, d\theta_y = \Theta_0^2 \quad \text{by definition}
\]

Consider the change in \( N \) going from \( z \rightarrow z + dz \)

\[
N(z+dz, y, \theta_y) = N(z, y-\theta_y \, dz, \theta_y) +
\]

\[
+ \int N(z, y, \theta_y-\theta_y') f(\theta_y') \, d\theta_y' \, dz +
\]

\[
- \int N(z, y, \theta_y) f(\theta_y') \, d\theta_y' \, dz
\]

where

- the 1st term is due to the drift, i.e. particles whose displacement changed because \( \theta_y \rightarrow 0 \) (in general),

- the 2nd term represents particles that had the "right" \( y \) but the "wrong" \( \theta_y \) and then scattered into the correct "right" \( \theta_y \) bin,

- the 3rd term represents particles that had
the "right" $y$ and $d y$, but, due to a further scatter ended up having the "wrong" $d y$

Now let us expand in Taylor series, keeping only leading non-zero terms. Assume that in 2nd and 3rd term contributions are important only when $d y$ is small. Then

$$
N(z, y, \theta_y) + \frac{\partial N}{\partial z} d z = N(z, y, \theta_y) - \frac{\partial N}{\partial y} \theta_y \ d z + \\
+ \int N(z, y, \theta_y) f(\theta_y) \ d \theta_y \ d z + \\
- \int \frac{\partial N}{\partial y} \theta_y \ f(\theta_y) \ d \theta_y \ d z + \\
\frac{1}{2} \int \frac{\partial^2 N}{\partial y^2} \theta_y^2 \ f(\theta_y) \ d \theta_y \ d z + \\
- \int N(z, y, \theta_y) f(\theta_y) \ d \theta_y \ d z
$$

$$
\frac{\partial N}{\partial z} = - \frac{\partial N}{\partial y} \theta_y - \frac{\partial N}{\partial y} \int \theta_y \ f(\theta_y) \ d \theta_y + \frac{1}{2} \frac{\partial^2 N}{\partial y^2} \int \theta_y^2 \ f(\theta_y) \ d \theta_y
$$

$$
= \langle \theta_y \rangle = 0
$$

$$
= \langle \theta_y^2 \rangle = \theta_o^2
$$

$$
\frac{\partial N}{\partial z} = - \theta_y \frac{\partial N}{\partial y} + \frac{1}{2} \theta_o^2 \frac{\partial^2 N}{\partial y^2}
$$
(11) Let \[ \frac{\partial N}{\partial z} = \frac{8 \sqrt{3} N_0}{\pi \theta_o^2} \exp[\ldots] \cdot \left\{ -\frac{4}{z^3} \theta_y^2 + \frac{3 \theta_y^2}{z^2} - \frac{8 \theta_y}{z^3} + \frac{3 \theta_y}{z^2} \right\} \]

\[ \frac{\partial N}{\partial y} = \frac{8 \sqrt{3} N_0}{\pi \theta_o^2} \exp[\ldots] \cdot \left\{ -\frac{4}{z^3} \theta_y^2 + \frac{3 \theta_y^2}{z^2} - \frac{8 \theta_y}{z^3} + \frac{3 \theta_y}{z^2} \right\} \]

\[ \frac{\partial N}{\partial \theta_y} = \frac{8 \sqrt{3} N_0}{\pi \theta_o^2} \exp[\ldots] \cdot \left\{ -\frac{4}{z^3} \theta_y^2 + \frac{3 \theta_y^2}{z^2} - \frac{8 \theta_y}{z^3} + \frac{3 \theta_y}{z^2} \right\} \]

\[ \frac{\partial N}{\partial \theta_y^2} = \frac{8 \sqrt{3} N_0}{\pi \theta_o^2} \exp[\ldots] \cdot \left\{ -\frac{4}{z^3} \theta_y^2 + \frac{3 \theta_y^2}{z^2} - \frac{8 \theta_y}{z^3} + \frac{3 \theta_y}{z^2} \right\} \]

\[ \frac{\partial N}{\partial \theta_y^3} = \frac{8 \sqrt{3} N_0}{\pi \theta_o^2} \exp[\ldots] \cdot \left\{ -\frac{4}{z^3} \theta_y^2 + \frac{3 \theta_y^2}{z^2} - \frac{8 \theta_y}{z^3} + \frac{3 \theta_y}{z^2} \right\} \]
\[
\frac{A_y^2}{2} \frac{\partial^2 N}{\partial A_y^2} = \frac{8\sqrt{3}N_0}{\pi A_0^2} \left[ -\frac{1}{Z^3} + \frac{4A_y^2}{A_0^2 Z^3} - \frac{12yA_y}{A_0^2 Z^5} - \frac{9y^2}{A_0^2 Z^6} \right] \exp[\ldots]
\]

We need to verify that \[1 = 2 + 3\]

The terms that multiply \(\frac{8\sqrt{3}N_0}{\pi A_0^2} \exp[\ldots]\) are:

\[1: \quad \frac{1}{Z^3} - \frac{A_y^2}{Z^4 A_0^2} - \frac{6yA_y}{Z^5 A_0^2} - \frac{9y^2}{Z^6 A_0^2}\]

\[2: \quad - \frac{3A_y^2}{Z^4 A_0^2} + \frac{6yA_y}{Z^5 A_0^2}\]

\[3: \quad - \frac{1}{Z^3} + \frac{4A_y^2}{Z^4 A_0^2} - \frac{12yA_y}{Z^5 A_0^2} - \frac{9y^2}{Z^6 A_0^2}\]

\[\text{YES!} \quad 1 = 2 + 3\]
(iii) \( N(z, \theta_y) = \frac{4\sqrt{3} N_0}{\pi \theta_0^2 z^2} \int_{-\infty}^{\infty} \exp \left[ -\frac{2\theta_y^2}{\theta_0^2 z^2} \right] \exp \left[ \frac{6y^4}{\theta_0^2 z^4} - \frac{6y^2}{\theta_0^2 z^2} \right] dy \)

CALL THIS A

Use \( \int_{-\infty}^{\infty} e^{-\left(ax^2 + bx\right)} dx = \frac{1}{2} \sqrt{\frac{\pi}{a}} e^{b^2/4a} \)

In our case \( a = \frac{6}{\theta_0^2 z^4}, \quad b = -\frac{6\theta_y^2}{\theta_0^2 z^2}, \quad \) so

\[ N(z, \theta_y) = A \frac{1}{2} \sqrt{\frac{\pi \theta_0^2 z^4}{6}} \exp \left( \frac{36\theta_y^2}{4 \theta_0^2 z^4} - \frac{1}{4} \theta_0^2 z^3 \right) \]

\[ N(z, \theta_y) = \frac{4\sqrt{3} N_0}{\pi \theta_0^2 z^2} \frac{1}{\sqrt{3} \sqrt{\theta_0}} \exp \left[ -\frac{2\theta_y^2}{\theta_0^2 z^2} + \frac{3\theta_y^2}{\theta_0^2 z^2} \right] \]

\[ N(z, \theta_y) \sim \frac{N_0}{\sqrt{2\pi} \theta_0 \sqrt{z}} \exp \left[ -\frac{\theta_y^2}{2\theta_0^2 z} \right] \]

Gaussian, normalized to 1, mean \( \mu \), rms \( \sigma \) is

\[ G(x; \mu, \sigma) = \frac{1}{\sqrt{2\pi} \sigma} e^{-\frac{(x-\mu)^2}{2\sigma^2}} \]

Identify \( \mu = 0 \) \( \sigma = \theta_0 \sqrt{z} \)

and the normalization is correct, i.e. total number of particles is \( N_0 \)
(iv) Dropping the normalization now

\[ N(z, y) \, dy \sim \exp \left[ \frac{-6y^2}{\theta_0^2 z^3} \right] \int_{-\infty}^{\infty} \exp \left[ \frac{-2}{\theta_0^2} \left( \frac{\theta_0^2}{z} - \frac{3y \, dy}{z^2} \right) \right] \, dy \]

Again, we have

\[ \int_{-\infty}^{\infty} e^{-\left(ax^2+bx\right)} \, dx = \frac{1}{\sqrt{a}} \, e^{\frac{b^2}{4a}} \]

In this case \( a = \frac{2}{\theta_0^2 z} \), \( b = -\frac{6y}{\theta_0^2 z^2} \), so

\[ N(z, y) \, dy \sim \exp \left[ \frac{-6y^2}{\theta_0^2 z^3} \right] \, \exp \left[ \frac{\theta_0^2 y^2}{\theta_0^4 z^4} \frac{8}{8} \right] \]

\[ \boxed{N(z, y) \, dy \sim \exp \left[ \frac{-3y^2}{2\theta_0^2 z^3} \right]} \]

Gaussian \( \sim \exp \left[ \frac{-\left(y - \mu \right)^2}{2\sigma^2} \right] \Rightarrow \mu = 0 \)

\[ \sigma^2 = \frac{\theta_0^2 z^3}{3}, \quad \sigma = \frac{\theta_0 z^{3/2}}{\sqrt{3}} \]

(v) At \( z = t \), \( y = 0 \)

\[ N(\theta_0^2) \, d\theta_0 \sim \exp \left[ -\frac{2}{\theta_0^2} \left( \frac{\theta_0^2}{t} - \frac{3d \, d \theta_0}{t^2} \right) \right] \, d\theta_0 \, d\theta_0 \]

\[ \sim \exp \left[ -\left( \frac{2\theta_0^2}{t \theta_0^2} - \frac{6d \theta_0}{t^2 \theta_0^2} \right) \right] \, d\theta_0 \, d\theta_0 \]

\[ \sim \exp \left[ -\left( \frac{2\theta_0^2}{t \theta_0^2} - \frac{6d \theta_0}{t^2 \theta_0^2} + \frac{9d^2}{2t^3 \theta_0^4} - \frac{9d^2}{4t^3 \theta_0^4} \right) \right] \, d\theta_0 \]

(we are completing the square in the exponent)
\[ N(\theta_y) \, d\theta_y \sim \exp \left[ - \left( \frac{12 \theta_y}{\theta_0 \sqrt{E}} - \frac{3 d}{\sqrt{2} t^{3/2} \theta_0} \right)^2 \right] \]

\[ N(\theta_y) \, d\theta_y \sim \exp \left[ - \left( \frac{2 \theta_y t - 3 \frac{d}{\theta_0}}{\sqrt{2} t^{3/2} \theta_0} \right)^2 \right] \]

\[ N(\theta_y) \, d\theta_y \sim \exp - \frac{4 t^2 (\theta_y - \frac{3 d}{2 t \theta_0})^2}{2 t^3 \theta_0^2} \]

\[ N(\theta_y) \, d\theta_y \sim \exp - \frac{2 (\theta_y - \frac{3 d}{2 t \theta_0})}{t \theta_0^2} \]

This is Gaussian with

\[ \mu = \frac{3 d}{2 t \theta_0} \]

\[ 2\sigma^2 = \frac{t \theta_0^2}{2} \]

\[ \sigma = \frac{\theta_0 \sqrt{E}}{2} \]
(a) Bethe-Bloch, from PDG,

\[
\frac{dE}{dx} = - \frac{K \gamma^2}{A} \beta^2 \frac{\gamma}{\beta} \left[ \frac{1}{2} \log \frac{2m_e c^4 \beta^2 T_{max}}{I^2} - \frac{\gamma^2}{\gamma^2 + 1} \right]
\]

\[= - \frac{1}{2} \log \frac{2m_e c^2 \beta^2 T_{max}}{I^2} - \frac{\gamma^2}{\gamma^2 + 1} \]

\[T_{max} = \frac{2m_e c^2 \beta^2 \gamma^2}{1 + 2\gamma(m_e/m) + (m_e c^2)^2} \quad M = \text{mass of projectile} \]

Express everything in terms of momentum, or \(p_M \)

\[\gamma \beta = \frac{p_M}{m} \quad \gamma \]

\[\frac{\beta^2}{1 - \beta^2} = \gamma^2 \Rightarrow \beta^2 = \frac{\gamma^2}{\gamma^2 + 1} \]

\[\gamma^2 = \frac{1}{1 - \beta^2} = \frac{\gamma^2}{\beta^2} = \gamma^2 + 1 \Rightarrow \gamma = \sqrt{\gamma^2 + 1} \]

\[\frac{dE}{dx} = - \frac{K \gamma^2}{A} \frac{\gamma^2 + 1}{\gamma^2} \left[ \frac{1}{2} \log \frac{2m_e c^2 \gamma^2 T_{max}}{I^2} - \frac{\gamma^2}{\gamma^2 + 1} \right] \]

We are concerned with \(T, \mu, K, p \).

For these particles \(M \gg m_e \Rightarrow T_{max} \sim 2m_e c^2 \beta^2 \gamma^2 \).

So

\[\frac{dE}{dx} = - \frac{K \gamma^2}{A} \frac{\gamma^2 + 1}{\gamma^2} \left[ \frac{1}{2} \log \frac{2m_e c^2 \gamma^4}{I^2} - \frac{\gamma^2}{\gamma^2 + 1} \right] \]

Using \(I = 16 \text{eV} \), plot \(\frac{dE}{dx} \) in arbitrary units.
Ignoring the bend
\[ t = \frac{R}{\beta c} \]
\[ t = \frac{\gamma^2 + 1}{\gamma} \frac{R}{c} \]
with \( R = 1.5 \text{ m} \) and \( c = 3 \times 10^8 \text{ m/s} \).

Including the bend, the path-length is not \( R \).

The radius of curvature \( p \) is given by \( p = \frac{p}{0.3 B} \)

where \( p \) is in GeV/c, \( p \) is in m, \( B \) is in Tesla.

\[ L = p \alpha \]
\[ R^2 = p^2 + p^2 - 2p^2 \cos \alpha \]
\[ R^2 = 2p^2(\cos^2 \alpha) \]
\[ R^2 = 2p^2 \left(1 - \sin^2 \frac{\alpha}{2}\right) \]
\[ \sin \frac{\alpha}{2} = \frac{R}{2p} \]
\[ \alpha = 2 \sin^{-1} \frac{R}{2p} \]

So \( L = 2p \sin^{-1} \frac{R}{2p} \), and \( t = \frac{\gamma^2 + 1}{\gamma} \frac{L}{c} \)

\[ t = 2 \frac{\gamma^2 + 1}{\gamma} \frac{p}{c} \sin^{-1} \frac{R}{2p} \]

with \( p = \frac{p}{0.3 B} \).
(a) Probability \( P(x) \, dx = \frac{dx}{D} \quad -\frac{D}{2} < x < \frac{D}{2} \)

\[ P(x) = 0 \quad x > \frac{D}{2} \]

Note: this is properly normalized \( \int_{-\infty}^{\infty} P(x) \, dx = 1 \)

\( P(x) \, dx \) here is the probability that the charged particle
goes through the detector between \( x \) and \( x + dx \) given
that the channel hit was centered at \( x = 0 \)

\[ \sigma^2 = \int_{-\infty}^{\infty} P(x) \, (x - \langle x \rangle)^2 \, dx \quad \text{but} \quad \langle x \rangle = 0 \]

\[ \sigma^2 = \int_{-\frac{D}{2}}^{\frac{D}{2}} \frac{x^2}{D} \, dx = \frac{1}{3D} \left[ \left( \frac{D}{2} \right)^3 + \left( \frac{D}{2} \right)^3 \right] = \frac{D^2}{12} \]

\[ \sigma = \frac{D}{\sqrt{12}} \]

(b)

\[ z(R) = A + BR \]

\[ \begin{cases} z_1 = A + BR_1 \\ z_2 = A + BR_2 \end{cases} \]

\[ \begin{cases} A = (z_1 - z_2) / (R_2 - R_1) \\ B = (z_1 - z_2) / (R_2 - R_1) \end{cases} \]

\[ A = \frac{R_2}{R_2 - R_1} \frac{z_1}{R_2 - R_1} - \frac{R_1}{R_2 - R_1} \frac{z_2}{R_2 - R_1} \]
\[ z(R=0) = A = \frac{R_2}{R_2-R_1} z_2 - \frac{R_1}{R_2-R_1} z_z \]

Let \( \sigma^2 = \text{Resolution in } z(R=0) \)

\[ \sigma^2 = \left( \frac{R_2}{R_2-R_1} \right)^2 \sigma^2(z_2) + \left( \frac{R_1}{R_2-R_1} \right)^2 \sigma^2(z_z) \]

But \( \sigma^2(z_2) = \sigma^2(z_z) = \frac{D^2}{12} \)

Therefore

\[ \sigma^2 = \frac{D^2}{12} \frac{R_1^2 + R_2^2}{(R_2-R_1)^2} \]

\[ \sigma = \frac{D}{\sqrt{12}} \frac{\sqrt{R_1^2 + R_2^2}}{R_2-R_1} \]

Or, writing \( \tau = R_2/R_1 \),

\[ \sigma = \frac{D}{\sqrt{12}} \frac{\sqrt{1+\tau^2}}{\tau-1} \]
Let \( z = 0 \) origin point of particle
- \( z_f \): coordinate at beam pipe
- \( z_1 \): coordinate at first layer
- \( z_2 \): coordinate at 2nd layer
- \( \theta_0 \): scattering angle at beam pipe
- \( \theta_1 \): scattering angle at 1st layer

\[
Z_0 = R_0 \tan \alpha \\
Z_1 = Z_0 + (R_1 - R_0) \tan (\alpha - \theta_0) \approx Z_0 + (R_1 - R_0) (\tan \alpha - \theta_0) \quad \text{small} \theta_0 \\
Z_1 \approx R_0 \tan \alpha + R_1 \tan \alpha - R_0 \tan \alpha - (R_1 - R_0) \theta_0 \\
Z_1 \approx R_1 \tan \alpha - (R_1 - R_0) \theta_0 \quad \text{(1)}
\]

Similarly
\[
Z_2 \approx R_2 \tan \alpha - (R_2 - R_0) \theta_0 - (R_2 - R_1) \theta_1 \quad \text{(2)}
\]

From part (b)
\[
Z(0) = \frac{R_2}{R_2 - R_1} z_1 - \frac{R_1}{R_2 - R_1} z_2 \quad \text{(4)} \quad (3)
\]

\[
\sigma^2(z_1) = \frac{D_{\text{measurement error}}}{12} + \frac{(R_1 - R_0) \sigma_{E_0}^2}{\text{MS contribution}}
\]
\[ \sigma^2 (z_2) = \frac{D}{12} + (R_2 - R_0)^2 \sigma_\theta^2 + (R_2 - R_1)^2 \sigma_\theta_1^2 \]

Note that in order to get the uncertainty in \( z(0) \) I cannot do as in part (b) and write

\[ \sigma^2 (z_{\infty}) = \left( \frac{R_2}{R_2 - R_1} \right)^2 \sigma^2 (z_1) + \left( \frac{R_1}{R_2 - R_1} \right)^2 \sigma^2 (z_2) \]

This is because now \( z_1 \) and \( z_2 \) are correlated. What I can do is write down

\[ \sigma^2 (z_{\infty}) = \sigma^2 (\text{MEASUREMENT}) + \sigma^2 (\text{MS}) \quad (4) \]

where \( \sigma^2 (\text{MEASUREMENT}) = \frac{D^2}{12 \left( 1 + \frac{r^2}{r^2 - 1} \right)} \) as in (b)

Then to calculate \( \sigma^2 (\text{MS}) \), let me write down the deviation from \( z_{\infty} \) due to multiple scattering

Deviation (from equations (4), (2), (3))

\[ \delta z_0 = \frac{R_2}{R_2 - R_1} \left[ -(R_2 - R_0) \theta_0 - (R_2 - R_1) \theta_1 \right] \]

\[ \delta z_\theta = \frac{-R_1 R_2 \theta_0 + R_0 R_2 \theta_2 + R_1 R_2 \theta_0 - R_1 R_0 \theta_0 + R_1 R_2 \theta_1 - R_1^2 \theta_1}{R_2 - R_1} \]

\[ \delta z_\phi = \frac{R_0 (R_2 - R_1) \theta_0 + R_1 (R_2 - R_1) \theta_1}{R_2 - R_1} = R_0 \theta_0 + R_1 \theta_1 \]

\[ \sigma^2_{\text{MS}} = \sigma_\theta^2 + \sigma_\phi^2 \]

Therefore, equation (4)

\[ \sigma^2 = \frac{D^2}{12} \left( 1 + \frac{r^2}{r^2 - 1} \right) + \frac{\sigma_\theta^2 + \sigma_\phi^2}{r^2 - 1} \]
(d) \( R_0 = 2 \text{ cm} \quad R_1 = 3 \text{ cm} \quad R_2 = 4 \text{ cm} \quad r = \frac{9}{3} \quad D = 50 \mu \text{m} \)

\[
\sigma = \frac{13.6 \text{ MeV}}{\beta \gamma} \sqrt{N} \quad \text{where } N = \text{number of radiation lengths}
\]

This equation is from page 198 of the 2002 PDG, neglecting the small log term.

Beam pipe is 1.2 mm thickness Be - Be is good because it is low Z, so long X
Silicon is 300 \( \mu \text{m} \) thick
\[
\begin{align*}
X(\text{Be}) &= 35.28 \text{ cm} & \text{PDG, page 84} \\
X(\text{Si}) &= 9.36 \text{ cm} & \text{PDG, page 84}
\end{align*}
\]

\[
\begin{align*}
N(\text{Be}) &= 1.2 \text{ mm} / 35.28 \text{ cm} = 3.4 \times 10^{-3} \\
N(\text{Si}) &= 0.3 \text{ mm} / 93.6 \text{ mm} = 3.2 \times 10^{-3}
\end{align*}
\]

With no multiple scattering,

\[
\sigma^2 = \frac{D}{\sqrt{12}} \cdot \frac{1 + 16/9}{(\beta \gamma - 1)} = \frac{D}{\sqrt{12}} \cdot \frac{5/3}{71/9} = \frac{15}{7} \frac{D}{\sqrt{12}} = \frac{15 \times 50}{7} \mu \text{m}
\]

\[
\sigma^2 = 31 \mu \text{m}
\]

With multiple scattering,

\[
\sigma^2 = (31 \mu \text{m})^2 + R_0^2 \sigma_{\text{Be}}^2 + R_1^2 \sigma_{\text{Si}}^2
\]

Plugging the numbers in,

\[
\sigma^2 = (31 \mu \text{m})^2 + 4 \text{ cm}^2 \left( \frac{13.6 \text{ MeV}}{\beta \gamma} \right)^2 \times 3.4 \times 10^{-3} + 9 \text{ cm}^2 \left( \frac{13.6 \text{ MeV}}{\beta \gamma} \right)^2 \times 3.2 \times 10^{-3}
\]

\[
\sigma^2 = (31 \mu \text{m})^2 + \left( \frac{28 \mu \text{m MeV}}{\beta \gamma} \right)^2
\]
In Fe, \[ \frac{dE}{dx}_{\text{ion}} \approx 1.45 \text{ MeV cm}^2 \text{ g}^{-1} \]

\[ P = 2.8 \times 10^8 \text{ cm} \Rightarrow \frac{dE}{dx}_{\text{ion}} = \frac{11.4 \text{ MeV}}{\text{cm}} \]

Kinetic Energy \[ T = E - m = \sqrt{p^2 c^2 - m^2} \]

In order to lose all the kinetic energy, we must have a thickness

\[ \Delta x \approx \frac{T}{\left( \frac{dE}{dx}_{\text{ion}} \right)} = \left( \frac{1.45 \text{ MeV cm}^2 \text{ g}^{-1}}{1.4 \times 10^3} \right) \frac{\text{cm}}{\text{cm}} \]

\[ P = 5 \text{ GeV} \Rightarrow \Delta x = \frac{5 - 0.102}{11.4 \times 10^3} \text{ cm} \]

\[ \Delta x = 4.56 \text{ cm} \]

\[ P = 10 \text{ GeV} \Rightarrow \Delta x = \frac{10 - 0.102}{11.4 \times 10^3} \text{ cm} \]

\[ \Delta x = 8.76 \text{ cm} \]

Also, \[ \lambda_{\gamma} = \frac{1.32}{11.4} \text{ cm} \Rightarrow \lambda_{\gamma} = \frac{1.32}{7.84} \text{ cm} \approx 17 \text{ cm} \]

For \( \Delta x = 8.76 \text{ cm} \), \[ \lambda_{\gamma} \approx 17 \text{ cm} \]

\[ \Delta x = 4.56 \text{ cm} \]

\[ \Delta x = 8.76 \text{ cm} \]
Let \( \lambda = \frac{b}{v} \) and \( \phi = \text{mean residence time of } T \) before decay.

For a \( T^0 \) produced at \( x \), the probability that it decays at \( y \) is

\[
p(y) = \frac{e^{-\lambda y}}{\lambda}
\]

Then suppose \( T^0 \rightarrow \delta \) (this is almost unit probability).

The probability that at least one of the \( \delta \)-converts

\[
p(\frac{\delta(x-y-x)}{x_0}) \quad \text{for } x_0 > \delta
\]

Then if \( T^0 \) are produced at a constant rate

\[\lambda dx \text{ uniformly in } x, \text{ the rate of } \delta \text{-converts from } T^0 \rightarrow \delta \text{ is}
\]

\[
R(t) = \int \int k dx \ e^{-\lambda t} \ \frac{d\delta}{x} = \left( e^{-\lambda t} - x \right)
\]

where I neglected the difference between 96.8% and 100%. 

\[ \]
\[ R(t) = \int_0^t dx \frac{e}{\lambda X_0} \int_0^x dy \left( t - x - y \right) e^{-\lambda y} \]

\[ R(t) = \frac{eR}{\lambda X_0} \int_0^t dx \left\{ \left( t - x \right) \left( e^{-\lambda x} - 1 \right) \right\} \]

For the second integral, use \( \int dx \ e^{-\lambda x} = \frac{e^{-\lambda x}}{-\lambda} \)

So, \( \int_0^t y e^{-\lambda y} e^{-\lambda x} = -\lambda \left[ e^{-\lambda x} \left( y + \lambda \right) \right]_0^t \)

\[ = -\lambda \left[ e^{-\lambda x} (t - x + \lambda) - \lambda \right] \]

\[ R(t) = \frac{eR}{\lambda X_0} \int_0^t dx \left\{ \lambda (t - x) \left( 1 - e^{-\lambda x} \right) + \lambda (t - x) e^{-\lambda x} \right\} \]

\[ R(t) = \frac{eR}{\lambda X_0} \int_0^t dx \left\{ \lambda e^{-\lambda x} (t - x) + \lambda e^{-\lambda x} \right\} \]

\[ R(t) = \frac{e^2 R}{X_0} \left[ e^2 - \frac{e^2}{2} - \lambda e^2 \right] + \frac{e^2 R}{X_0} \int_0^t dx e^{-\lambda x} e^{\lambda x} \]

\[ R(t) = \frac{R}{X_0} \left[ e^2 - 2 - 2 \lambda e \right] + \frac{e^2 R}{X_0} \int_0^t dx e^{\lambda x} \left( e^{\lambda x} - \lambda e^{-\lambda x} \right) \]
\[ R(t) = \frac{2k^2}{x_0} \left[ \frac{t^2}{2} - \lambda + \frac{\Delta^2}{t} \left( e^{-\frac{\lambda}{t}} - 1 \right) \right] \]

This neglects the contribution of $W^+ \to e^+ \nu$ decay.

They contribute at a rate $3k^2$.

So

\[ R(t) = 3k^2 + \frac{3}{x_0} \left[ \frac{t^2}{2} - \lambda + \frac{\Delta^2}{t} \left( 1 - e^{-\frac{\lambda}{t}} \right) \right] \]