Nevertheless it still makes sense to write down some rough numbers

<table>
<thead>
<tr>
<th>Interaction</th>
<th>Strength (arbitrary units)</th>
</tr>
</thead>
<tbody>
<tr>
<td>strong</td>
<td>( \sim 10 )</td>
</tr>
<tr>
<td>EM</td>
<td>( \sim 10^{-2} )</td>
</tr>
<tr>
<td>weak</td>
<td>( \sim 10^{-13} )</td>
</tr>
<tr>
<td>gravity</td>
<td>( \sim 10^{-42} )</td>
</tr>
</tbody>
</table>

Gravity is by far the weakest. It has little effect on the interaction of elementary particles (which are very light). Actually there are some new ideas about gravity, large extra dimensions, etc., where gravity might actually have effects (mini-black holes) could be produced in accelerator exps. - Very exciting.

We will leave gravity aside in this course.

The quantum mechanical, relativistic treatment of particle interactions is not through \( F \) but through exchange of virtual particles that mediate the interaction.

For example, consider the \( e^+e^- \) interaction.

Classically,

\[
\vec{F} = -\frac{e^2}{4\pi\varepsilon_0 r^2}\hat{\vec{r}}
\]
Instead

\[ \text{OUT}(t = \infty) \]
\[ e^- \]
\[ \text{IN}(t = 0) \]
\[ e^- \]

Example of a Feynman Diagram

We say that one electron emits a photon and the other electron absorbs it.

But is a real photon emitted, that is to say can I stick a detector in between and "catch" it?

No, this x is not real, it is virtual.

This picture here is a fantastic way of representing some truly awful (in my opinion) mathematics.

The genius of Feynman was the invention of these simple pictures that help tremendously in visualizing what is happening.

So what does this picture mean?

\[ (E_3 \vec{p}_3) \]
\[ q = 1 \]
\[ (E - E_3 \vec{p} - \vec{p}_3) \]
\[ q = 1 \]
\[ (E_2 + E_1 - E_3 \vec{p}_2 + \vec{p}_1 - \vec{p}_3) \]

In other words, the photon "transfers" energy and momentum (but also q and \( \phi \) and mom) from one particle to the other.
We say that the \( W \) is the \textbf{carrier} of the EM interaction. Note that \((E_i - E_f)^2 - (p_i - p_f)^2\) \(= M^2 \neq 0\) \(\Rightarrow\) a virtual particle can have any mass.

For the weak interaction, we have three carriers:

\[ W^+ \quad W^- \quad Z^0 \]

These are \( S = 1 \) particles (\( S = 1 \) particle is called a \textbf{vector} (\( S \) integer is called \textbf{boson})

\[ \Rightarrow \quad \textbf{VECTOR BOSONS} \]

\[ M_{W^+} = M_{W^-} = (80.423 \pm 0.039) \text{ GeV/c}^2 \]

\[ M_Z = (91.1876 \pm 0.0021) \text{ GeV/c}^2 \]

The difference between the weak and EM interaction can "mostly" be ascribed to the fact that \( M_Z = 0 \) and \( M_W \sim M_Z \sim 100 \text{ GeV/c}^2 \). If we do \( \text{experiments at } \epsilon > 100 \text{ GeV} \) we find that, for example, that EM and WEAK have about the same strength.

Example \( \beta^{-} \)-decay  \( n \rightarrow p + e^- + \bar{\nu}_e \)
What about the strong interaction?

Can think of the strong interaction as a generalization of EM but - EM has 1 kind of "charge" (electric charge)
- strong has 3 kinds of "charge"

We call these colors **Red**, **Green**, **Blue**

Also - anti-Red (R), G, B

Nord equivalent of -ve and +ve charge

The **carriers** of the strong interaction are called **gluons** - They are like photons but different

In one important respect - They carry color

\[
\begin{align*}
q(R) & \rightarrow q(BR) \\
q(B) & \rightarrow \text{other gluons}
\end{align*}
\]

There are 8 gluons

**QCD has some very weird properties**

Remember we broke up the constituents of matters into

- LEPTONS (L)
- QUARKS (Q)
- INTERACTION CARRIERS

Why do we make a distinction between L and Q?
(What is the distinction)
Because Q interact strongly \( \leftrightarrow \) couple "couple" to gluons \( \cdots \cdots \rightarrow g \) \( \rightarrow q \)

L do not \( \rightarrow g \) \( \rightarrow e \) \( \rightarrow e \)

\( e, \mu, \tau, \nu \)'s have no color.

Both L and Q interact EM (except \( \nu \) because \( Q_\nu = 0 \))

Everybody interacts weekly

QCD has some very weird properties

Classical EM potential

\[ u(r) \sim -\frac{1}{4\pi\epsilon_0} \frac{e^2}{r} \]

\( r \rightarrow \infty \) no interaction

\( r \rightarrow 0 \) big interaction

A QCD potential \( \sim \) between two quarks is approximately of the form

\[ u(r) \sim -\frac{4}{3} \frac{x_s}{r} + kr \]

\( \text{(Don't worry about } x_s \text{ and } k \text{ for now)} \)
It takes an infinite amount of energy to separate quarks. As a result QCD tells us that free quarks do not exist. (Although people look for them – I posted a recent nice paper on the website.)

This means that quark only exist in bound states (A bound state must be color-neutral (colorless) just like an atom is electrically neutral.) With three quarks, colors, can take 3-quarks and make a neutral bound state

\[ q_1(r) \ q_2(b) \ q_3(g) \text{ in neutral} \]

Bound states of 3-quarks are called BARYONS

What is \( J \)?
\[ J = \left( \frac{1}{2} \oplus \frac{1}{2} \oplus \frac{1}{2} \right) \oplus L \]

(Spin of a baryon)
\[ J = \left( \frac{3}{2} \oplus \frac{1}{2} \right) \oplus L \]

\[ J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}, \frac{7}{2}, \ldots \text{ A huge spectrum of states} \]

The lowest lying states have \( L = 0 \) \( J = \frac{1}{2} \)

\[ uud \rightarrow p \]
\[ udd \rightarrow n \]
\[ uus \rightarrow \Xi^+ \]
\[ u\bar{s}s \rightarrow \Xi^0 \]
\[ s\bar{s}s \rightarrow \Xi^- \]
\[ u\bar{d}s \rightarrow \Omega \] etc (see Griffiths)
Can also have a neutral $q\bar{q}$ state

$q_1(R)$  $q_2(\bar{R})$

These bound states are called **mesons**

$J = (\frac{1}{2} + \frac{1}{2}) \oplus L$

$J = (0 \text{ or } 1) \oplus L$

Lowest lying states have $J = 0$

- $u\bar{s} : K^+$  $u\bar{d} : \pi^+$
- $d\bar{s} : K^0$  $u\bar{u} - d\bar{d} : \pi^0$
- $s\bar{d} : K^-$  $u\bar{u} : \pi^-$
- $s\bar{s} : K^0$

Order out of chaos.

There are also "excited" states

Take a $\pi^+ u\bar{d}$, $J=0$ - Give it $J=1$, get a $p^+$ meson. Heavier.
Interaction vertices

**EM**

At each vertex, conserve charge. Also does not change particle species, e.g.

\[ e \rightarrow e^\gamma \] OK

but \[ \mu \rightarrow e^\gamma \] not OK

**Strong**

Leptons do not participate.

\[ q_{(R)} \xrightarrow{g_{(RG)}} q_{(R)} \]

Conserve charge and also does not change particle species.

\[ u \rightarrow u^g \] OK

\[ d \rightarrow u^g \] Not OK (charge also not conserved)

\[ t \rightarrow u^g \] Not OK (charge now conserved but quark flavor change)
**WEAK**

More complicated \((u) \ (c) \ (t)\)

\((d) \ (s) \ (b)\)

\((e^+) \ (\ell^+) \ (\nu^+) \ (\bar{\nu}^+)\)

Interaction carriers: \(W^+ \ W^- \ Z\)

The vertex with a \(Z\) looks just like the vertex for a photon

\[\text{charge conserved}\]

Note \(\nu_x \rightarrow \nu_x \ "Z"\), same flavor again

Interactions with \(W^+\) more complicated

First leptons

\[\text{but not } \nu_x \rightarrow e^- \ W^-\]

That is to say the \(W\) couples only leptons and neutrinos in *same generation*

This result in "**conservation of lepton number**"
3 lepton numbers \( L_e \), \( L_\mu \), \( L_\tau \)

\( e^-, \nu_e \) \( L_e = 1 \)

\( e^+, \bar{\nu}_e \) \( L_e = -1 \)

L is additive

Same for \( \mu, \tau \).

At each vertex \( L \) is conserved

\[ \mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu \]

\( L_\mu = 1 \), \( L_e = 1 \), \( L_\mu = -1 \), \( L_\mu = 1 \)

Note \( L \) is conserved also in EM-STRONG

\( \Rightarrow \) conserved always!!

\( W^+ \) interaction

\( \begin{bmatrix} u \end{bmatrix}, \begin{bmatrix} c \end{bmatrix}, \begin{bmatrix} t \end{bmatrix} \)

\( \begin{bmatrix} \nu_e \end{bmatrix}, \begin{bmatrix} \nu_\mu \end{bmatrix}, \begin{bmatrix} \nu_\tau \end{bmatrix} \)

Leptons

Quarks

\( W^+ \) interaction

\( \begin{bmatrix} u \end{bmatrix}, \begin{bmatrix} c \end{bmatrix}, \begin{bmatrix} t \end{bmatrix} \)

\( \begin{bmatrix} \nu_e \end{bmatrix}, \begin{bmatrix} \nu_\mu \end{bmatrix}, \begin{bmatrix} \nu_\tau \end{bmatrix} \)

\( \begin{bmatrix} d \end{bmatrix}, \begin{bmatrix} s \end{bmatrix}, \begin{bmatrix} b \end{bmatrix} \)

\( \begin{bmatrix} u \end{bmatrix}, \begin{bmatrix} c \end{bmatrix}, \begin{bmatrix} t \end{bmatrix} \)

\( \begin{bmatrix} \nu_e \end{bmatrix}, \begin{bmatrix} \nu_\mu \end{bmatrix}, \begin{bmatrix} \nu_\tau \end{bmatrix} \)

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\( \begin{bmatrix} u \end{bmatrix}, \begin{bmatrix} c \end{bmatrix}, \begin{bmatrix} t \end{bmatrix} \)

\( \begin{bmatrix} \nu_e \end{bmatrix}, \begin{bmatrix} \nu_\mu \end{bmatrix}, \begin{bmatrix} \nu_\tau \end{bmatrix} \)

\( \begin{bmatrix} d \end{bmatrix}, \begin{bmatrix} s \end{bmatrix}, \begin{bmatrix} b \end{bmatrix} \)
Note \( d' \neq d \), \( s' \neq s \), \( b' \neq b \), but "almost"

\[
\begin{pmatrix}
    d' \\
    s' \\
    b'
\end{pmatrix} =
\begin{pmatrix}
    V_{ud} & V_{us} & V_{ub} \\
    V_{cd} & V_{cs} & V_{cb} \\
    V_{td} & V_{ts} & V_{tb}
\end{pmatrix}
\begin{pmatrix}
    d \\
    s \\
    b
\end{pmatrix}
\]

Cabibbo Kobayashi Maskawa
(CKM) matrix

CKM matrix almost diagonal

In magnitude

\[
\begin{pmatrix}
    0.974 & 0.976 & 0.029 \\
    \sim 0.975 & \sim 0.22 & \sim 0.003 \\
    \sim 0.22 & \sim 0.974 & \sim 0.04 \\
    \sim 0.008 & \sim 0.04 & \sim 0.99
\end{pmatrix}
\]

\[\text{see PDG for more details}\]

then the amplitude for \( q_1 \to q_2 W \) transition
is proportional to \( V_{q_1 q_2} \) and its rate to \( |V_{q_1 q_2}|^2 \)

\[\Rightarrow \text{Weak transition between quarks occur preferentially in the same generation}
\]

(\text{unless otherwise but not completely})
One more important conservation law: **Baryon Number.**

In all interactions the "number of quarks" is conserved:

\[
\begin{align*}
q & \rightarrow q + 2g + W^- \\
q & \rightarrow q + W^+ \\
q & \rightarrow q + g
\end{align*}
\]

Baryon number \( B = \frac{1}{3} \) for each \( q \), \( B = -\frac{1}{3} \) for each \( \bar{q} \)

Conserved - Defined like this so that \( \text{p (uud)} \) and \( \text{n (udd)} \) have \( B = \pm 1 \)

(Same theories beyond SM allow for B-violation.)