1. **GRIFFITHS 6.5**

I will set $c=1$ in this problem.

\[ E = \sqrt{m_1^2 + p^2} + \sqrt{m_3^2 + p^2} \]

\[ E = m_1 \text{ by conservation of energy} \]

\[ m_1 = \sqrt{m_2^2 + p^2} + \sqrt{m_3^2 + p^2} \]

\[ m_1 = \sqrt{m_2^2 + p^2} - \sqrt{m_3^2 + p^2} \]

\[ p^2 + m_1^2 + m_2^2 - 2m_1m_2^2 + p^2 = m_3^2 + p^2 \]

\[ m_1^2 + m_2^2 - m_3^2 = 2m_1m_2^2 + p^2 \]

\[ m_1^4 + m_2^4 + m_3^4 + 2m_1^2m_2^2 = 2m_1^2m_2^2 - 2m_1^2m_3^2 - 2m_2^2m_3^2 = 4m_1^2m_2^2 + 4m_2^2p^2 \]

\[ p = \frac{1}{2m_1} \sqrt{m_1^4 + m_2^4 + m_3^4 - 2m_1^2m_2^2 - 2m_1^2m_3^2 - 2m_2^2m_3^2} \]

Actually, we had already calculated this in Homework 2, problem 7.

2. **Griffiths equation 6.32**

\[ M = \pm \frac{1}{8\pi \hbar m_4 c} |\mathbf{p}|^2 \]

Comparing $M(\pi^+ \to \mu^+ \nu_\mu)$ and $M(\pi^+ \to e^+ \nu_e)$, and assuming that $|M|$ is the same, I get

\[ R = \frac{M(\pi^+ \to \mu^+ \nu_\mu)}{M(\pi^+ \to e^+ \nu_e)} = \frac{|\mathbf{p}_1|}{|\mathbf{p}_2|} \]
Where $|\vec{p}_1|$ is the CM momentum in the $\pi^+\mu^-$ decay
$|\vec{p}_2|$ in the CM momentum in the $\pi^+e^+\nu_e$ decay
Using equation 6.31, with $m_\mu = 0, c = 1$

$$|\vec{p}_1| = \frac{1}{2m_\pi} \sqrt{m_\pi^4 + m_\mu^4 - 2m_\pi^2 m_\mu^2} = \frac{m_\pi^2 - m_\mu^2}{2m_\pi}$$

Similarly
$$|\vec{p}_2| = \frac{m_\pi^2 - m_e^2}{2m_\pi}$$

$$\Rightarrow R = \frac{m_\pi^2 - m_\mu^2}{m_\pi^2 - m_e^2} = \frac{(139.6 \text{ MeV})^2 - (105.7 \text{ MeV})^2}{(139.6 \text{ MeV})^2 - (0.5 \text{ MeV})^2} = \frac{8315.67}{19487.91} = 0.43$$

PDG: $\text{BR} (\pi^+ \rightarrow \mu^+\nu_\mu) = 99.99\%$
$\text{BR} (\pi^+ \rightarrow e^+\nu_e) = 1.2 \times 10^{-4}$

$$R = \frac{0.9999}{1.2 \times 10^{-4}} = 8.3 \times 10^3$$

\[\text{In natural units}\]

\[\text{BR}(K^+ \rightarrow \pi^+\pi^0) = 21.2\% = 0.212\]
\[\tau(K^+) = 1.24 \times 10^{-8} \text{ sec}\]
\[\Gamma(K^+ \rightarrow \pi^+\pi^0) = \frac{\text{BR}(K^+ \rightarrow \pi^+\pi^0)}{\tau(K^+)} = 1.77 \times 10^7 \text{ sec}^{-1}\]

To get this in units of energy, multiply by $h = 1.05 \times 10^{-34} \text{ J sec}$

$$\Gamma = 1.77 \times 10^7 \times 1.05 \times 10^{-34} \text{ J}$$

$$\Gamma = 1.86 \times 10^{-27} \text{ J} \approx 1.6 \times 10^{-19} \text{ J}$$

\[1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}\]
\[1 \text{ MeV} = 1.6 \times 10^{-13} \text{ J}\]
\[ I = 6.25 \times 10^{12} \text{ MeV} \]

We had
\[ \Gamma = 1.86 \times 10^{-27} \text{ J} = 1.86 \times 10^{-27} \times 6.25 \times 10^{12} \text{ MeV} = 11.6 \times 10^{-15} \text{ MeV} \]

\[ \Gamma = 1.2 \times 10^{-14} \text{ MeV} \]

4. From PDG: \( \Delta(1232) \Gamma \sim 120 \text{ MeV} \)
\( \alpha_s(1260) \Gamma \sim 400 \text{ MeV} \)

\[ \frac{\tau}{\Gamma} = \frac{1.05 \times 10^{-34} \text{ J sec}}{120 \text{ MeV}} = \frac{1.05 \times 10^{-24} \text{ J sec}}{120 \times 1.6 \times 10^{-13} \text{ J}} = 5.5 \times 10^{-24} \text{ sec} \]

\[ \frac{\tau}{\Gamma} = \frac{1.05 \times 10^{-34} \text{ J sec}}{400 \times 1.6 \times 10^{-13} \text{ J}} = 1.6 \times 10^{-24} \text{ sec} \]

5. **Griffiths 4.39**

First you have to establish whether they are made of matter or antimatter. You can do it as follows

Dear alien:
- Go to an accelerator.
- Study the decays of neutral particles.
- You will find two neutral particles with the same mass to one part in \(10^4\) but with lifetimes that differ by a factor of \(580\). We call the long-lived one \(K_0\), the short-lived one \(K_s\).
- Study the decay of \(K_0\).
- Look at the decay of the \(K_0\) into two charged particle and a neutral one that doesn't interact. These are \(K_0 \rightarrow \pi^- e^+ \nu\) or \(K_0 \rightarrow \pi^- e^- \bar{\nu}\).
- We call electron or positron the lightest of the two particles.
- The \(K_0\) prefers to decay into what we call \(K_L \rightarrow \pi^- e^+ \nu\).
- If the \(e^+\) is part of ordinary matter in your universe, then we say that your universe is made of antimatter.

Then you can define right vs left by saying that \(e^+\) in the weak interaction are right handed, \(e^-\) are left handed.

But then I am not sure that I know how to use this fact to tell the alien that my heart is on the left side.