University of California at Santa Barbara
Physics Department
Physics 125 Homework 3
due 23 April 2003

• Problem 1
  Griffiths 3.22

• Problem 2
  Griffiths 3.23

• Problem 3
  Griffiths 3.24

• Problem 4
  Griffiths 4.5

• Problem 5
  Griffiths 4.10

• Problem 6
  Griffiths 4.11

• Problem 7
  Griffiths 4.23. Do parts (a), (b), (c). Skip part (d).

• Problem 8
  From the definition of a group, show that for a group $G$
  (a) the identity element is unique
  (b) every $a \in G$ has a unique inverse
  (c) $\forall a \in G, (a^{-1})^{-1} = a$
  (d) $\forall a, b \in G, (a \cdot b)^{-1} = b^{-1} \cdot a^{-1}$
• Problem 9

Consider an irreducible representation of a group in a space $L$. Let $A$ be a fixed operator in $L$, and let $O(a)$ be the operator in this vector space corresponding to the group element $a$.

(a) Show that if $A \cdot O(a) = O(a) \cdot A$ for all $a$, then $A = \lambda I$, where $\lambda$ is a number and $I$ is the identity operator (Schur’s first lemma).

(b) Use this result to show that the irreducible representations of an abelian$^1$ group are one-dimensional.

If you can’t do part (a), try to use the result of part (a) to do part (b). The key thing to remember is that the irreducible representations of an abelian group are one dimensional.

$^1$An abelian group is a group in which group multiplication is commutative, i.e., $\forall a, b \in G, a \cdot b = b \cdot a$. 