(a) $dss \rightarrow uds \bar{u}d$

The first of these two reaction ($\Xi^- \rightarrow \Lambda \pi^-$) has $\Delta S = 1$, the other one has $\Delta S = 2$. It is possible to have the first reaction through the weak interaction.

The second reaction ($\Xi^- \rightarrow n \pi^-$) cannot go through single $W$-exchange. Therefore we expect it to be highly suppressed.

PDG says $\Xi^- \rightarrow \Lambda \pi^- : BR = 99.9\%$
$\Xi^- \rightarrow n \pi^- : BR < 1.9 \times 10^{-5}$

(b) $D^0 \rightarrow K^- \pi^+$

proportional to $\left| V_{cs} V_{ud} \right|^2$

PDG: $V_{cs} \approx 0.974$
$V_{ud} \approx 0.975 \Rightarrow \left| V_{cs} V_{ud} \right|^2 \approx 0.90$
$D^0 \to \pi^- \pi^-$

Proportional to $|V_{cd} V_{ud}|^2$

PDG: $V_{cd} \approx 0.22$
$V_{ud} \approx 0.975$

$|V_{cd} V_{ud}|^2 \approx 4.3 \times 10^{-2}$

$D^0 \to K^+ \pi^-$

Proportional to $|V_{cd} V_{us}|^2$

PDG: $V_{cd} \approx 0.22$
$V_{us} \approx 0.22$

$|V_{cd} V_{us}|^2 \approx 2.3 \times 10^{-3}$

So expect

$\text{BR}(D^0 \to K^- \pi^+) : \text{BR}(D^0 \to \pi^+ \pi^-) : \text{BR}(D^0 \to K^+ \pi^-) \approx$

$0.90 : 4.3 \times 10^{-2} : 2.3 \times 10^{-3} =$

$1 : 4.8 \times 10^{-2} : 2.6 \times 10^{-3}$

PDG says

$\text{BR}(D^0 \to K^- \pi^+) : \text{BR}(D^0 \to \pi^+ \pi^-) : \text{BR}(D^0 \to K^+ \pi^-) \approx$

$3.8 \times 10^{-2} : 1.43 \times 10^{-3} : 1.48 \times 10^{-4} =$

$1 : 3.8 \times 10^{-2} : 3.9 \times 10^{-3}$

Pretty good agreement, considering that we've neglected a bunch of effects.

(c) The preferred transition is $b \to c$, so $B$-mesons decay to $D$-mesons.

$T$-mesons would decay to $B$-mesons with $t \to b$.

However, $T$-mesons don't exist! (This is another interesting story.)
(2) **GRIFFITHS 2.6**

![Diagram](image)

(3) **GRIFFITHS 3.4**

(a) \( L = \sqrt{t} = 0.998 \, \text{c} \, \tau = 0.998 \times 3 \times 10^8 \, \text{m} / \text{sec} = 2.2 \times 10^{-6} \, \text{sec} = 660 \, \text{m} \)

(b) \( L' = \gamma L \)

\[
\gamma = \frac{1}{\sqrt{1 - (0.998)^2}} = 15.8 \quad \Rightarrow \quad L' = 10.4 \, \text{km}
\]

(c) It makes it to the ground because of the \( \frac{1}{\gamma} \) length contraction.

(d) \( L'_{\tau} = \frac{1}{100} \quad L'_{\mu} = \frac{1}{100} \quad 10.4 \, \text{km} = 104 \, \text{m} \quad \Rightarrow \text{will not make it to the ground} \)

(4) **GRIFFITHS 3.7**

\[
I = (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2
\]

\[
x^0 = \gamma (x^0' + \beta x^1') \Rightarrow (x^0)^2 = \gamma^2 (x^0')^2 + \beta^2 (x^1')^2 + 2 \beta x^0' x^1'
\]

\[
x^1 = \gamma (x^1' + \beta x^0') \Rightarrow (x^1)^2 = \gamma^2 (x^1')^2 + \beta^2 (x^0')^2 + 2 \beta x^0' x^1'
\]

\[
x^2 = x^2' \quad \Rightarrow (x^2)^2 = (x^2')^2
\]

\[
x^3 = x^3' \quad \Rightarrow (x^3)^2 = (x^3')^2
\]

So

\[
I = \gamma^2 [(x^0')^2 + \beta^2 (x^1')^2 + 2 \beta x^0' x^1'] - \gamma^2 [(x^0')^2 + \beta^2 (x^1')^2 + 2 \beta x^0' x^1']
\]

\[
- (x^2')^2 - (x^3')^2
\]
\[ I = y^2(1 + \beta^2) \left( x^0 \right)^2 = y^2(1 - \beta^2) \left( x^1 \right)^2 - (x^2)^2 - (x^3)^2 \]

But \[ y^2(1 - \beta^2) = \frac{1 - \beta^2}{1 - \beta^2} = 1 \]

\[ I = \left( x^0 \right)^2 - \left( x^1 \right)^2 - (x^2)^2 - (x^3)^2 \]

5. Griffiths 3.13

Write the 4-vectors
(I will set \( c = 1 \))
(in this calculation)

\[ P_\pi = (\gamma m_\pi, \gamma m_\pi \beta_\pi, 0, 0) \]
\[ P_\nu = (E_\nu, 0, E_\nu, 0) \]
\[ P_\mu = (E_\mu, |P_\mu|^\cos \theta, -|P_\mu|^\sin \theta, 0) \]

Conservation laws:

\[ \begin{cases} \gamma m_\pi = E_\nu + E_\mu \quad \text{(4)} \\ \gamma m_\pi \beta_\pi = |P_\pi|^\cos \theta \quad \text{(2)} \\ E_\nu = |P_\mu|^\sin \theta \quad \text{(3)} \end{cases} \]

Substituting (3) into (4), we can eliminate \( E_\nu \)

\[ \begin{cases} \gamma m_\pi = |P_\mu|^\sin \theta + E_\mu \\ \gamma m_\pi \beta_\pi = |P_\mu|^\cos \theta \end{cases} \]

\[ \begin{align*} \frac{(4)}{(5)} & \quad \tan \theta = \frac{\gamma m_\pi - E_\mu}{\gamma m_\pi \beta_\pi} \quad \text{(6)} \\
(4)^2 + (5)^2 & \quad |P_\mu|^2 = \gamma^2 m_\pi^2 + E_\mu^2 - 2\gamma m_\pi E_\mu + \gamma^2 m_\pi^2 \beta^2 \\
& \quad |P_\mu|^2 = \gamma^2 m_\pi^2 (1 + \beta^2) + m_\mu^2 + |P_\mu|^2 - 2\gamma m_\pi E_\mu \end{align*} \]
\[ E_\mu = \frac{\gamma^2 m^2_\mu (1 + \beta^2) + m^2_\mu}{2 \gamma m_\mu} = \frac{1}{2} \gamma \frac{m_\pi (1 + \beta^2) + m^2_\mu}{2 \gamma m_\pi} \]

Substitute into (6)
\[
\tan \theta = \gamma \frac{m_\pi - \frac{1}{2} \gamma m_\pi - \frac{1}{2} \gamma m_\pi \beta^2}{m_\pi - \frac{m^2_\mu}{2 \gamma m_\pi} \beta^2}
\]
\[
\tan \theta = \frac{1}{2} \gamma \frac{m_\pi (1 - \beta^2) + m^2_\mu}{2 \gamma m_\pi \beta^2}
\]
\[
\tan \theta = \frac{m_\pi}{\gamma} - \frac{m^2_\mu}{\gamma m_\pi} \beta^2
\]
\[
\tan \theta = \frac{(1 - \frac{m^2_\mu}{m^2_\pi})}{2 \gamma \beta^2}
\]

(6) \[ \text{GRIFFITHS 3.14} \]

\[ E \rightarrow \begin{array}{c}
A \\
B
\end{array} \]

At threshold, in the CM all the particles \( c_1, c_2, \ldots, c_n \) are at rest, i.e. there is no energy "wasted" as kinetic energy.

In LAB
\[ P_A : (E; \sqrt{E^2 - m^2_A}; 0; 0) \]
\[ P_B : (m_B; 0; 0; 0) \]
\[ P_A + P_B : (E + m_B; \sqrt{E^2 - m^2_A}; 0; 0) \]
\[ (P_A + P_B)^2 = (E + m_B)^2 - (E^2 - m^2_A) = \]
\[ (P_A + P_B)^2 = E^2 + m^2_B + 2E m_B - E^2 + m^2_A \]
\[ (P_A + P_B)^2 = m^2_A + m^2_B + 2E m_B \]
Now go to CM

\[
\begin{align*}
A &\xrightarrow{\text{B}} \quad & \text{AFTER} \\
\text{PREVIOUS} &\xrightarrow{\text{CM}} \quad & C_i\text{ all at rest}
\end{align*}
\]

\[
\begin{align*}
\vec{P}_A^{CM} &= (E_A; P_{CM}; 0; 0) \quad & \vec{P}_F^{CM} &= (\Sigma m_i; 0; 0; 0) \\
\vec{P}_B^{CM} &= (E_B; -P_{CM}; 0; 0) \quad & \vec{P}_F^{CM} &= (M; 0; 0; 0) \\
\vec{P}_A^{CM} + \vec{P}_B^{CM} &= (E_A + E_B; 0; 0; 0)
\end{align*}
\]

Now we have some useful relations:

(i) \( \vec{P}_A^{CM} + \vec{P}_B^{CM} = \vec{P}_F^{CM} \) From E-\( \vec{p} \) conservation

This gives \( E_A + E_B = M \)

(ii) \( (\vec{P}_A + \vec{P}_B)^2 = (\vec{P}_A^{CM} + \vec{P}_B^{CM})^2 \) From Lorentz invariance

\[
M_A^2 + M_B^2 + 2E_M = (E_A + E_B)^2 = M^2
\]

\[
\Rightarrow E = \frac{M^2 - M_A^2 - M_B^2}{2M_B}
\]
(a) In class we worked out the example of a particle at rest decaying into two other particles

\[ M \rightarrow m_1, m_2 \]

The momentum was

\[ P = \frac{\sqrt{[M^2 - (m_1 + m_2)^2][M^2 - (m_1 - m_2)^2]}}{2M} c \]

Here \( M = m_\pi \), \( m_1 = m_\mu \), \( m_2 = m_\nu \approx 0 \)

\[ P = \frac{\sqrt{(m_\pi^2 - m_\mu^2)(m_\pi^2 - m_\mu^2)}}{2m_\pi} c = \left( \frac{m_\pi^2 - m_\mu^2}{2m_\pi} \right) c \]

\[ L = v(\gamma t) = v\sqrt{\tau} \quad \text{But} \quad P = \gamma m_\mu v \]

So \( L = \frac{P \tau}{m_\mu} \)

\[ L = \frac{m_\pi^2 - m_\mu^2}{2m_\pi m_\mu} c \tau \sim 186 \text{m} \]

(b) Either this was a very improbable event, i.e., an event where the \( \mu \)-decayed faster than on average (which is possible) or the muon lost energy, (was slowed down) in going through matter. This would shorten its average decay length.