Fluid Flow

Steady: pattern does not change in time
velocities

"funnel"

higher velocities

specks of fluid

trajectories called "streamlines"
if not curly, "laminar flow"

"flow tubes" follow a pre-defined set of specks

Whole thing above is a flow tube.

Continuity Equation
Like "Play-Doh" what goes in must come out
really, \( \frac{\# \text{ atoms}}{\text{mass}} \)
In time \( dt \), compare volume in the flow tubes.

\[ v_1 \, A_1 \, dt + A_1 \]

\[ A_2 \]

\[ v_2 \, dt \]

\[ \text{mass: } p_1 \, A_1 \, v_1 \, dt = p_2 \, A_2 \, v_2 \, dt \]

\[ (p_1 \, v_1) \times A_1 = (p_2 \, v_2) \times A_2 \]

"continuity equation"

true when \( \vec{v} \perp \) to area

\[ \vec{p} \, \vec{v} = \vec{d} = \text{"flux"} \]

\[ \frac{dV}{dt} = V \cdot A = \text{volume change w/r to time} \]

\[ p_1 = p_2 \text{ for incompressible fluids} \]

"Kinematics"
Bernoulli's Principal

apply energy considerations to some fluid.

surprise result:

\[ \text{FASTER} = \text{LESS PRESSURE} \]

\[ \begin{align*}
\text{slug of liquid} & \quad \text{initial} \quad \text{ds}_2 \quad \text{dc} \quad \text{p}_2 \\
\text{final} & \quad \text{flow tube} \\
\text{dW} & = p_1 A_1 \, ds_1 - p_2 A_2 \, ds_2 \\
& = dU + dK \\
& \quad \text{kinetic energy} \\
& \quad \text{potential energy} \\
& \quad \text{assume incompressible} \\
\text{dU} & = - \left( p \cdot A_1 \, ds_1 \right) g y_1 + \left( p A_2 \, ds_2 \right) g y_2 \\
& \quad \text{mass loss from bottom} \\
& \quad \text{gain at the top} 
\end{align*} \]
\[ dK = -\frac{1}{2} (p \cdot A_1) ds_1 v_1^2 + \frac{1}{2} (pA_2 ds_2) v_2^2 \]

loss at bottom

\[ \text{Incompressibility:} \]
\[ \phi A_1 ds_1 = \phi A_2 ds_2 \]
\[ A_1 ds_1 = A_2 ds_2 \]
\[ \rightarrow \text{they all divide out!} \]

\[ p_1 - p_2 = \rho g (y_2 - y_1) + \frac{1}{2} \rho (v_2^2 - v_1^2) \]

\[ p_1 + \rho g y_1 + \frac{1}{2} \rho v_1^2 = p_2 + \rho g y_2 + \frac{1}{2} \rho v_2^2 \]
\[ = \text{constant (in flow tube)} \]

Examples:

\[ p_1 + \frac{1}{2} \rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2} \rho v_2^2 + \rho g y_2 \]
\[ V_1 = V_2 = 0 \]
\[ p_1 + p_2 \]
\[ \rho_1 + \rho_2 \]
\[ p_1 = p_2 + \rho g (y_2 - y_1) \]
\[ h = y_2 - y_1 \]
\[ p_1 = p_2 + \rho gh \]