Physics 115A Seventh Problem Set

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1. $\Omega$ is a Hermitian operator; show that:

$$\langle \Delta \Omega \rangle^2 = \langle \Omega^2 \rangle - \langle \Omega \rangle^2.$$  

This is not quite the same as Equations 4.2.7 on page 128 of your text.

2. Consider the operator $S_\theta$, where $\theta$ is a real number:

$$S_\theta \doteq \frac{\hbar}{2} \begin{bmatrix} \cos \theta & \sin \theta \\ \sin \theta & -\cos \theta \end{bmatrix}.$$  

(a) Use the characteristic equation to find the eigenvalues of $S_\theta$.

(b) Show that the eigenvectors are represented by

$$| + \hbar/2 \rangle \doteq \begin{bmatrix} \cos(\theta/2) \\ \sin(\theta/2) \end{bmatrix}, \quad | - \hbar/2 \rangle \doteq \begin{bmatrix} -\sin(\theta/2) \\ \cos(\theta/2) \end{bmatrix}.$$  

It might help to remember that $\cos \theta = \cos^2(\theta/2) - \sin^2(\theta/2)$ and $\sin \theta = 2\sin(\theta/2)\cos(\theta/2)$.

(c) Consider the initial state:

$$|\psi\rangle \doteq \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

i. Evaluate $\langle S_\theta \rangle_\psi$, as a function of $\theta$.

ii. Evaluate $\langle S^2_\theta \rangle_\psi$, as a function of $\theta$.

iii. Evaluate $\langle \Delta S_\theta \rangle_\psi$, as a function of $\theta$.

3. Consider the two operators represented by:

$$\Omega \doteq \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \quad \Lambda \doteq \begin{bmatrix} \sqrt{2b} & b & 0 \\ b & \sqrt{2b} & b \\ 0 & b & \sqrt{2b} \end{bmatrix}$$

(a) Find an orthonormal basis that is an eigenbasis simultaneously of $\Omega$ and $\Lambda$; use the original representation to describe the eigenbasis.
(b) Do $\Omega$ and $\Lambda$ form a complete set of commuting observables?
(c) An initial state is described by:

$$|\psi\rangle = \begin{bmatrix} \sqrt{1/3} \\ 1/3 \\ 1/3 \\ 1/3 \end{bmatrix}.$$  

First $\Omega$ is measured, then $\Lambda$ is measured. Enumerate the sets of eigenvalues that result, and the final states (in the original basis) that correspond to each set of eigenvalues.