Things I wish I'd known when I was a graduate student...that are related to particle physics (part 2)

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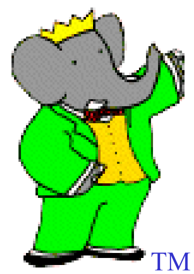
$$\begin{bmatrix} u \\ \ \end{bmatrix}\begin{bmatrix} c \\ s \end{bmatrix}\begin{bmatrix} \ \\ b \end{bmatrix}$$



TM

***B*A*B*A*R* Physics Analysis School**

Feb. 12 – 13, 2008

Outline

- **Lecture 1**
  - **Constants, natural units, and order-of-magnitude estimates**
  - **Overview of *B* physics**
  - **CP violation as an interference effect**
  - **3 kinds of CP violation**
  - **Direct CP violation**
- **Lecture 2**
  - **Thinking about symmetries**
  - **Meson oscillations**
  - **Time-dependent CP violation in *B* decays**
  - **Conclusions**

Thinking about Symmetries

Symmetries are fundamental to understanding the forces of nature.
<u>We characterize interactions by the symmetries they possess.</u>

In quantum mechanics, symmetries are nearly always represented by unitary transformations (U).

$$|\psi\rangle \to |\psi'\rangle = U|\psi\rangle$$

U modifies the state vector

...while preserving its norm.

$$\langle\psi'|\psi'\rangle = \langle U\psi|U\psi\rangle = \langle\psi|U^\dagger U|\psi\rangle = \langle\psi|\psi\rangle$$

$$[U,H]=0 \quad \Rightarrow \quad U \text{ is a symmetry of } H$$

If U is a symmetry, then

$$H|\psi\rangle = i\hbar\frac{d}{dt}|\psi\rangle \;\Rightarrow\; H\big(\underbrace{U|\psi\rangle}\big) = i\hbar\frac{d}{dt}\big(\underbrace{U|\psi\rangle}\big)$$

Solution to Schrodinger eq’n.

Also a solution to Sch. eq’n.

Continuous Symmetry Transformations

- Continuous symmetry transformations can be written as a function of a real parameter θ, which can be a vector of parameters.

$$U(\theta) = e^{-i\cdot\theta\cdot G} \quad (\text{ where } G^\dagger = G \text{ since } U \text{ is unitary})$$

$$\cong I - i\cdot\delta\theta\cdot G \quad \text{for small } \delta\theta$$

"Generator" of the transformation: a QM observable!

- Example: the translation operator is

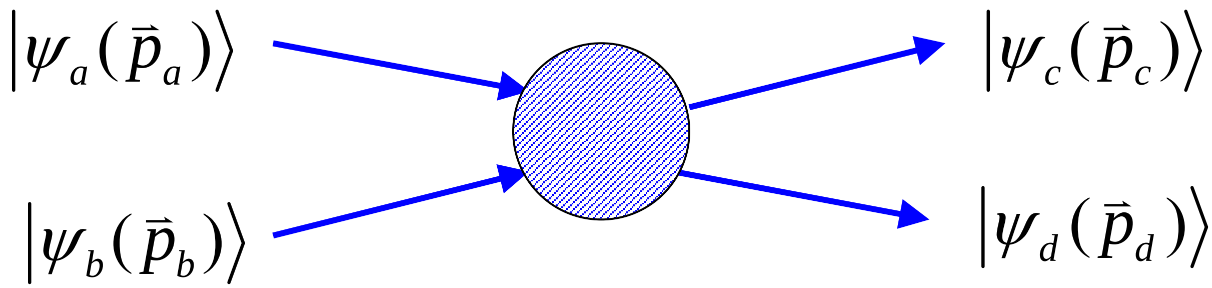
$$U(\vec{x}) = e^{-i\vec{P}\cdot\vec{x}/\hbar} = e^{-i(P_x x + P_y y + P_z z)/\hbar}$$

Suppose: $[H, U(\vec{x})] = 0$ for arb. $\vec{x}$

$\Rightarrow$ translational invariance along $\hat{x}$

$\Rightarrow [H, \vec{P}\cdot\hat{x}] = 0$ then momentum will be conserved (additively); see next slide

Conservation laws from continuous symmetry transformations



momentum eigenstates

$$\langle \psi_c,\psi_d | H | \psi_a,\psi_b \rangle = \langle \psi_c,\psi_d | U^\dagger U H U^\dagger U | \psi_a,\psi_b \rangle \qquad [U,H]=0$$

$$= \langle \psi_c,\psi_d | U^\dagger H U | \psi_a,\psi_b \rangle$$

$$= \langle \psi_c,\psi_d | e^{+i\vec{P}\cdot\vec{x}/\hbar} H e^{-i\vec{P}\cdot\vec{x}/\hbar} | \psi_a,\psi_b \rangle$$

$$= \langle \psi_c,\psi_d | H | \psi_a,\psi_b \rangle e^{+i[(\vec{p}_c+\vec{p}_d)-(\vec{p}_a+\vec{p}_b)]\cdot\vec{x}/\hbar}$$

Momentum is *additively* conserved!

$$\Rightarrow \begin{cases} \vec{p}_a + \vec{p}_b = \vec{p}_c + \vec{p}_d \\ \text{or} \quad \langle \psi_c,\psi_d | H | \psi_a,\psi_b \rangle = 0 \end{cases}$$

(or else transition is not allowed)

Discrete Symmetry Transformations

Discrete symmetry transformations cannot be written in terms of a continuous parameter. Such transformations must be performed in a single "jump."

$$C:\ a \to \bar{a}$$

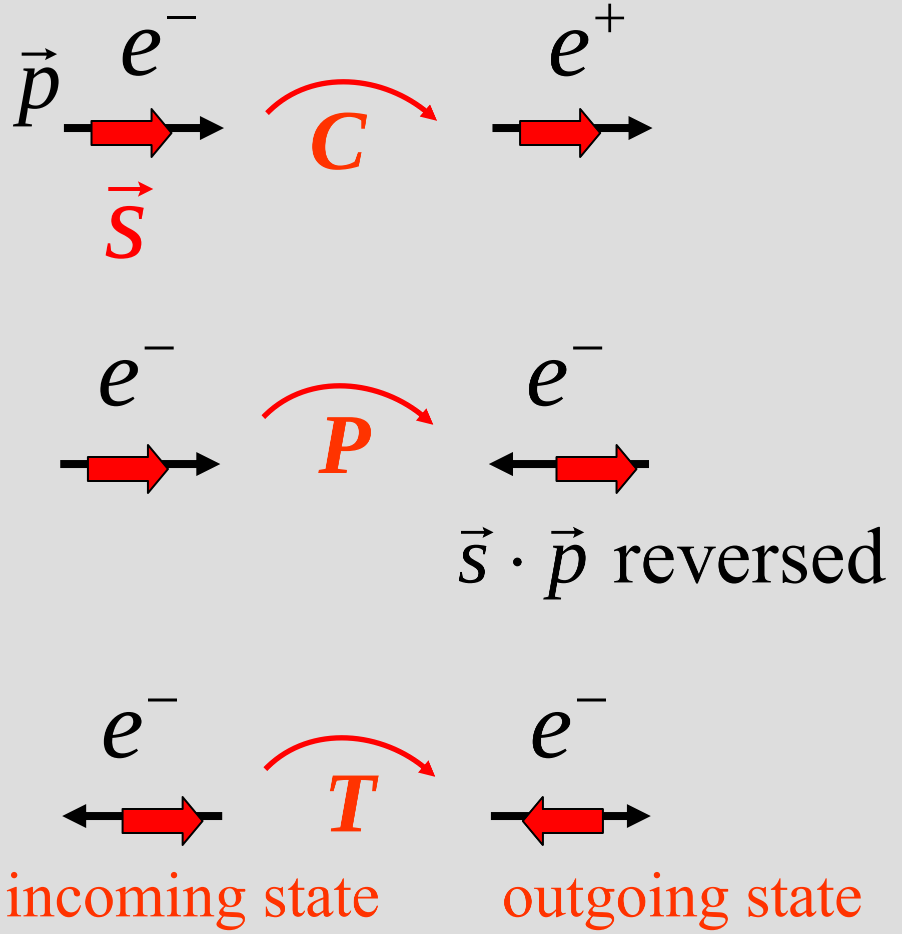
charge conjugation

$$P:\ \vec{r} \to -\vec{r}$$

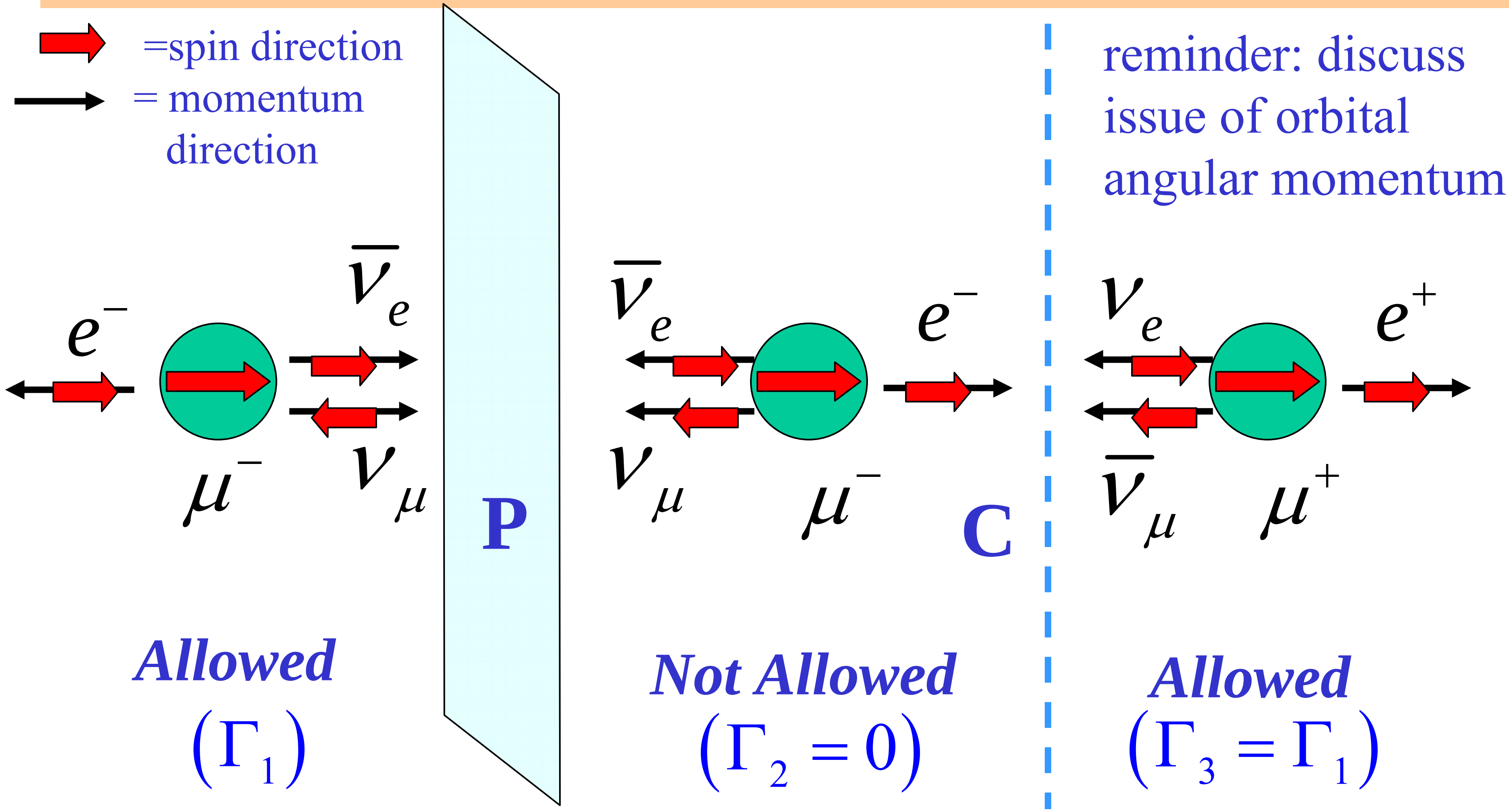
parity

$$T:\ t \to -t$$

motion reversal

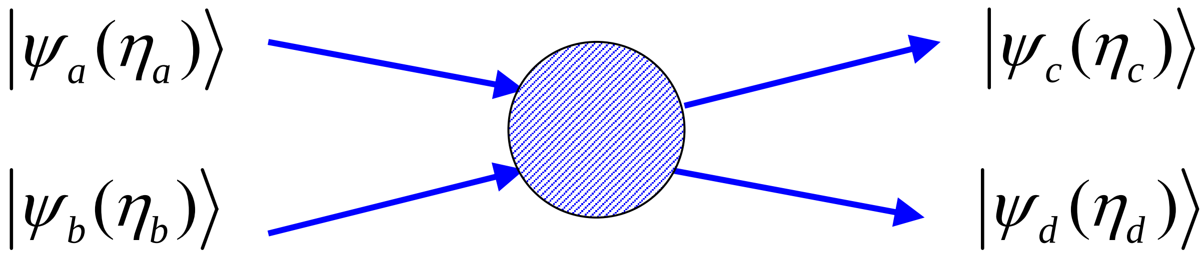


P and *C* violation in polarized muon decay



P and C are individually violated maximally in the weak interactions, but combined CP is a good symmetry even for most weak processes!

Conservation laws from discrete symmetry transformations



C eigenstates for simplicity

$$0 = \langle \psi_c, \psi_d | [H, C] | \psi_a, \psi_b \rangle = \langle \psi_c, \psi_d | H \eta_c \eta_d - \eta_a \eta_b H | \psi_a, \psi_b \rangle$$
$$= (\eta_c \eta_d - \eta_a \eta_b) \langle \psi_c, \psi_d | H | \psi_a, \psi_b \rangle$$

C is unitary & hermitian (discrete xf)

$$\Rightarrow \begin{cases} \eta_c \eta_d = \eta_a \eta_b \\ \text{or} \quad \langle \psi_c, \psi_d | H | \psi_a, \psi_b \rangle = 0 \end{cases}$$

The *C* eigenvalue is ***multiplicatively*** conserved!

(or else transition is not allowed)

Cosmology: Sakharov's three conditions

A. Sakharov (1967): How to generate an asymmetry between *N*(baryons) and *N*(anti-baryons) in the universe (assuming equal numbers initially)?

1. Baryon-number-violating process
2. Both C and CP violation (particle helicities not relevant to particle populations)
3. Departure from thermal equilibrium

$$\left(N_{\text{bar}} - N_{\text{anti-bar}}\right) \propto \sum_i \left[\Gamma(X \to Y_i) - \Gamma(\bar{X} \to \bar{Y}_i)\right] \cdot \Delta B_i$$

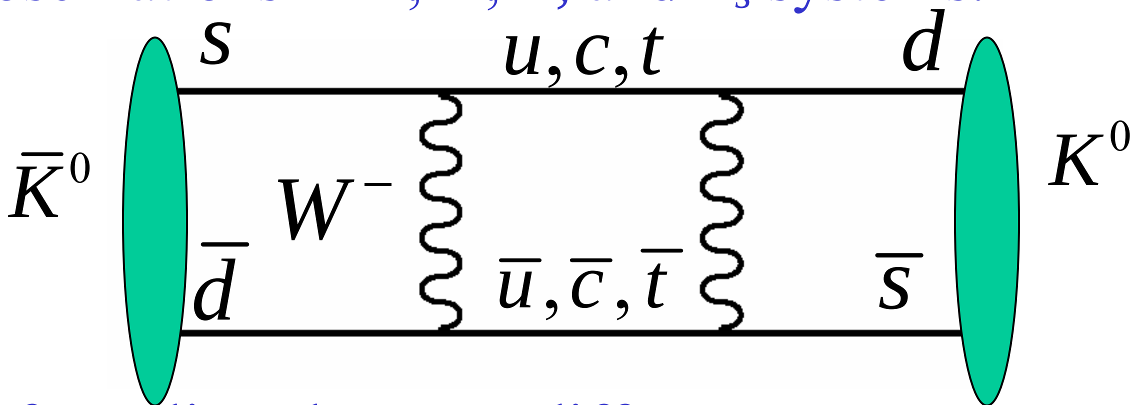
hypothetical heavy particle

We appear to owe our existence to some form of CP violation at work in the early universe.

A closer look at oscillations

A single, general formalism based on time-dependent perturbation theory describes meson oscillations in *K*, *D*, *B*, and B_s systems.

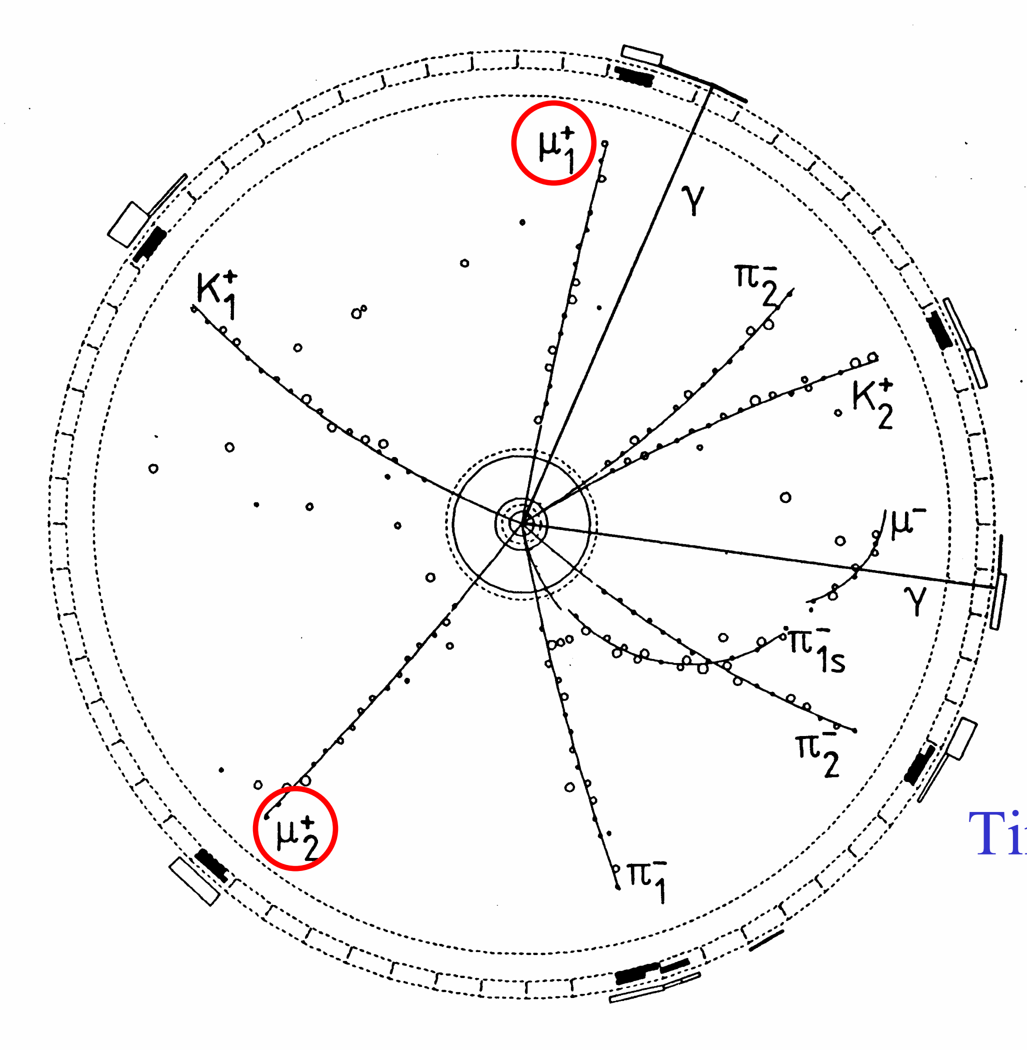
$$\left.\begin{array}{l} K^0\overline{K}^0 \\ D^0\overline{D}^0 \\ B^0\overline{B}^0 \\ B_s^0\overline{B}_s^0 \end{array}\right\} \boxed{P^0\overline{P}^0}$$



Common formalism, but very different parameter values → behavior in each case is very different!

Key point: since the weak interactions induce transitions between P^0 and $\overline{P}^0$, these flavor-eigenstate particles are not eigenstates of the total Hamiltonian, and <u>they do not have definite masses or lifetimes.</u> (They are superpositions of states P_L and P_H that do. Want to calculate Δm and $\Delta\Gamma$!)

Discovery of $B^0\bar{B}^0$ Oscillations



ARGUS experiment (1987)

$$\Upsilon(4S) \to B^0\bar{B}^0 \to B_1^0 B_2^0$$
$$B_1^0 \to D_1^{*-}\mu_1^+\nu_1,\ D_1^{*-} \to \bar{D}^0\pi_1^-$$
$$B_2^0 \to D_2^{*-}\mu_2^+\nu_2,\ D_1^{*-} \to D^-\pi^0$$

103 pb^{-1} ~ 110,000 B pairs

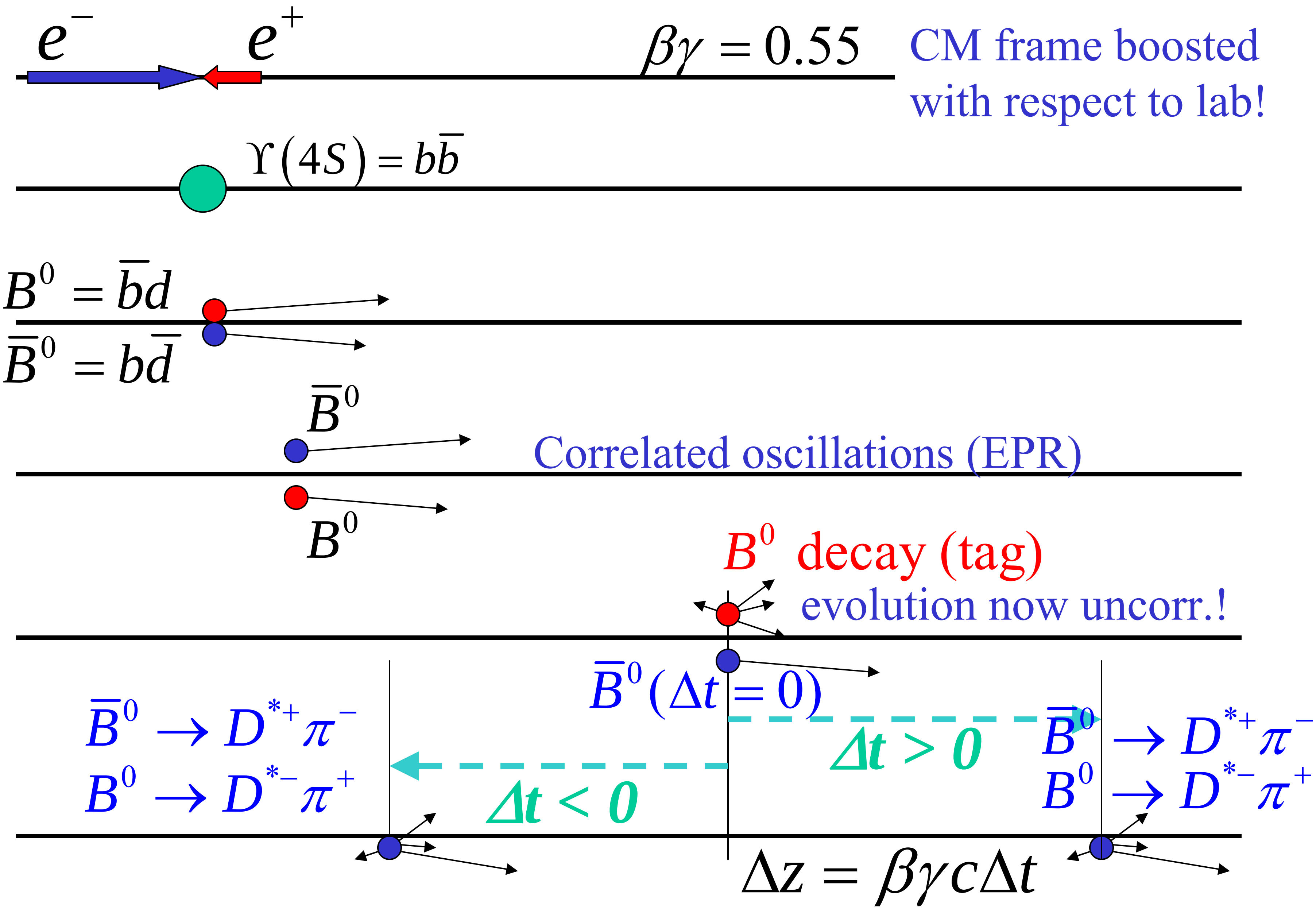
$$\chi_d = 0.17 \pm 0.05$$

ARGUS, PL B **192**, 245 (1987)

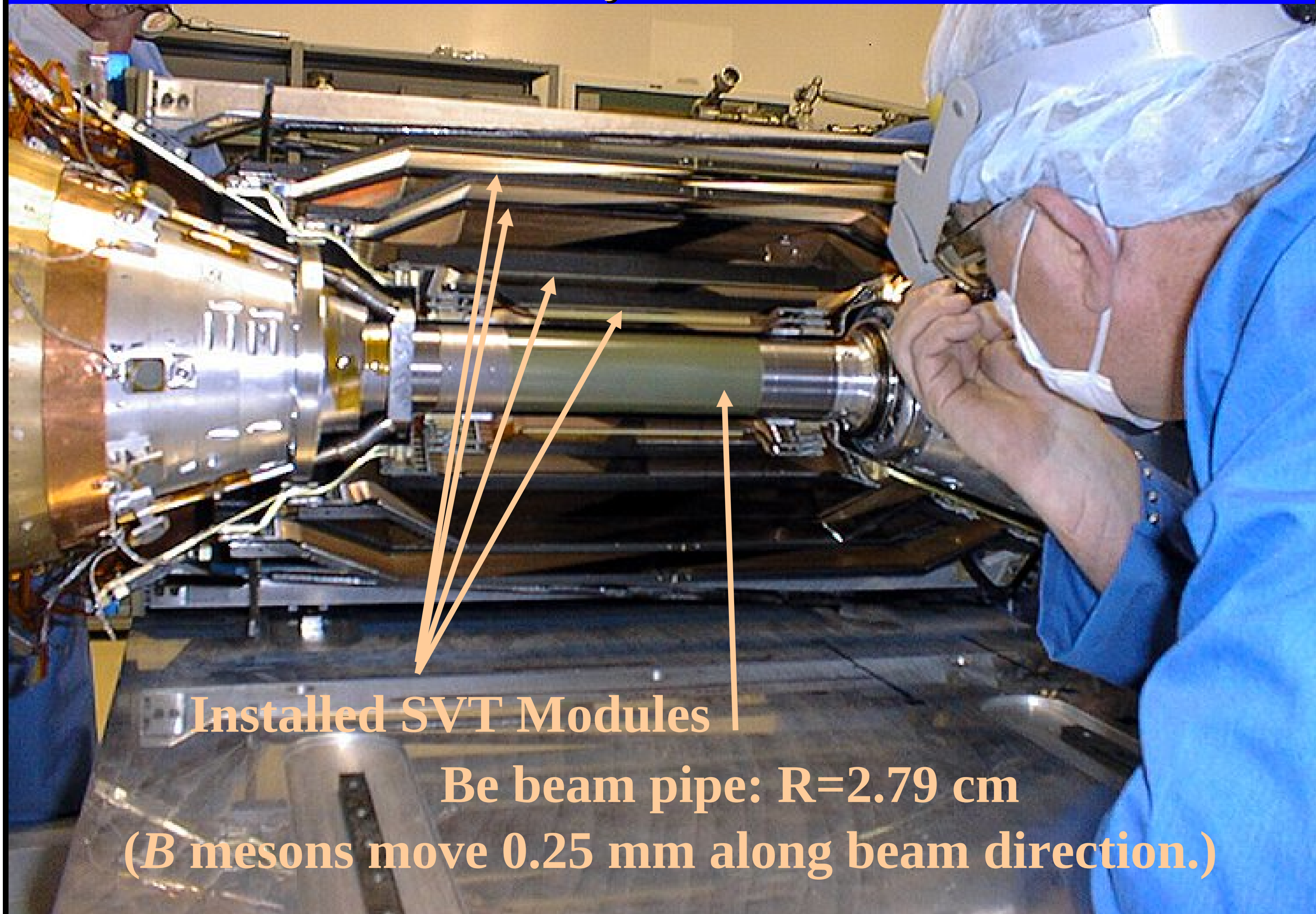
Time-integrated mixing rate: 21%

(fig. courtesy D. MacFarlane)

Time-dependent oscillation measurement



Innermost Detector Subsystem: Silicon Vertex Tracker



Measuring the $B^0\overline{B}^0$ oscillation frequency

$$\left(\frac{dN}{dt}\right)_{\text{nomix}} = \frac{1}{4\tau_B} \cdot e^{-\Gamma t} \cdot \left[1+\cos(\Delta m_d \cdot t)\right]$$

$$\left(\frac{dN}{dt}\right)_{\text{mix}} = \frac{1}{4\tau_B} \cdot e^{-\Gamma t} \cdot \left[1-\cos(\Delta m_d \cdot t)\right]$$

$$\Rightarrow \mathrm{A}_{\text{mix}} = \frac{\left(\frac{dN}{dt}\right)_{\text{nomix}} - \left(\frac{dN}{dt}\right)_{\text{mix}}}{\left(\frac{dN}{dt}\right)_{\text{nomix}} + \left(\frac{dN}{dt}\right)_{\text{mix}}} = \cos(\Delta m \cdot t)$$

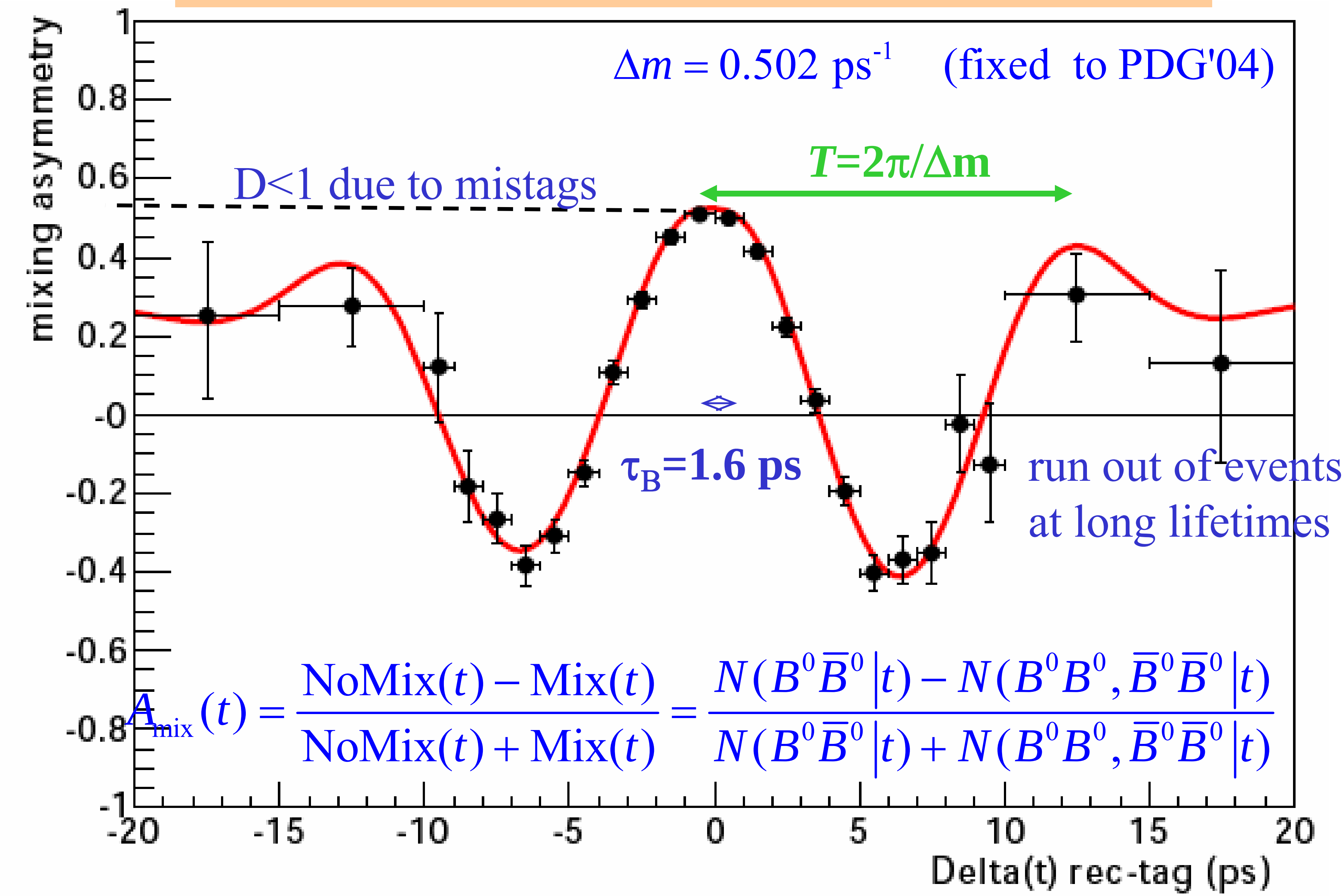
very simple!

amplitude=1

How do you actually do this measurement? Basic question: did the B oscillate or not? Need to know this as a function of time!

1. When it was produced, was the meson a B^0 or $\overline{B}^0$?
2. When it decayed, was the meson a B^0 or a $\overline{B}^0$?
3. What is the time difference between production and decay?

Mixing asymmetry vs. Δt



Does a mass really have units of s^{-1} ?

$$\mathrm{A}_{\mathrm{mix}} = \cos(\Delta m \cdot t)$$

1. Put in c^2

$$(\Delta m)c^2 \cdot t \sim ET$$

2. Divide by $\hbar \sim ET$ since phase must be dimensionless

$$\frac{(\Delta m)c^2 \cdot t}{\hbar} \sim \text{dimensionless!}$$

$$\frac{(\Delta m)c^2}{\hbar} = 0.5\ \mathrm{ps}^{-1} \qquad B^0 \overline{B}^0$$

$$(\Delta m)c^2 = (0.5 \cdot 10^{12}\ \mathrm{s}^{-1}) \cdot (66 \cdot 10^{6}\ \mathrm{eV} \cdot 10^{-23}\ \mathrm{s}) \approx 3 \cdot 10^{-4}\ \mathrm{eV}$$

Explains why we don't worry about B_{H} and B_{L} in most analyses!

Common formalism for $P^0\bar{P}^0$ oscillations

$$H = H_0 + H_w$$

Weak interactions (perturbation) induce
$$\left\{\begin{array}{l} P^0 \leftrightarrow \bar{P}^0 \\ P^0 \to f \\ \bar{P}^0 \to \bar{f} \end{array}\right.$$

Strong and EM interactions (create bound states)

$$|\Psi(t)\rangle = a(t)\left|P^0\right\rangle + b(t)\left|\bar{P}^0\right\rangle + \sum_f c_f(t)\left|f\right\rangle$$

We can **recast** the formalism in terms of a 2-dimensional vector space in which we only include P^0 and $\bar{P}^0$.

$$\left|P^0\right\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \qquad \left|\bar{P}^0\right\rangle = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

← important!

see my Les Houches lectures

$$\boldsymbol{H}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix} = i\frac{\partial}{\partial t}\begin{pmatrix} a(t) \\ b(t) \end{pmatrix}$$

$$\boldsymbol{H}^\dagger \neq \boldsymbol{H}$$

Solving the eigenvector problem in time-dependent perturbation theory

Eigenvectors of unperturbed Hamiltonian; notation

$$H_0 \left| P^0 \right\rangle = m_0 \left| P^0 \right\rangle$$

$$H_0 \left| \overline{P}^0 \right\rangle = m_0 \left| \overline{P}^0 \right\rangle$$

$$H_0 \left| f \right\rangle = E_f \left| f \right\rangle$$

Unperturbed energies of these states depend on quark masses, strong interations, and EM interactions that bind quarks into mesons.

$$CP \left| P^0 \right\rangle = e^{i\theta_{CP}} \left| \overline{P}^0 \right\rangle$$

$$CP \left| \overline{P}^0 \right\rangle = e^{-i\theta_{CP}} \left| P^0 \right\rangle$$

$$(CP)^2 \left| P^0 \right\rangle = \left| P^0 \right\rangle$$

Work in arbitrary phase convention; keep unphysical phases explicit. physical results must not depend on them!

$$H_{11} = \left\langle P^0 \right| H \left| P^0 \right\rangle$$

$$H_{12} = \left\langle P^0 \right| H \left| \overline{P}^0 \right\rangle$$

$$H_{21} = \left\langle \overline{P}^0 \right| H \left| P^0 \right\rangle$$

$$H_{11} = \left\langle P^0 \right| H \left| P^0 \right\rangle$$

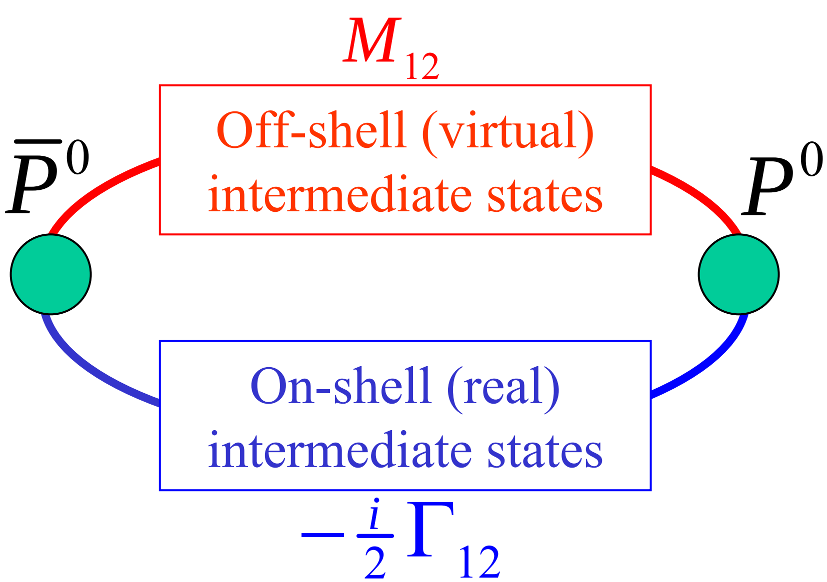
$$H = H_0 + H_w$$

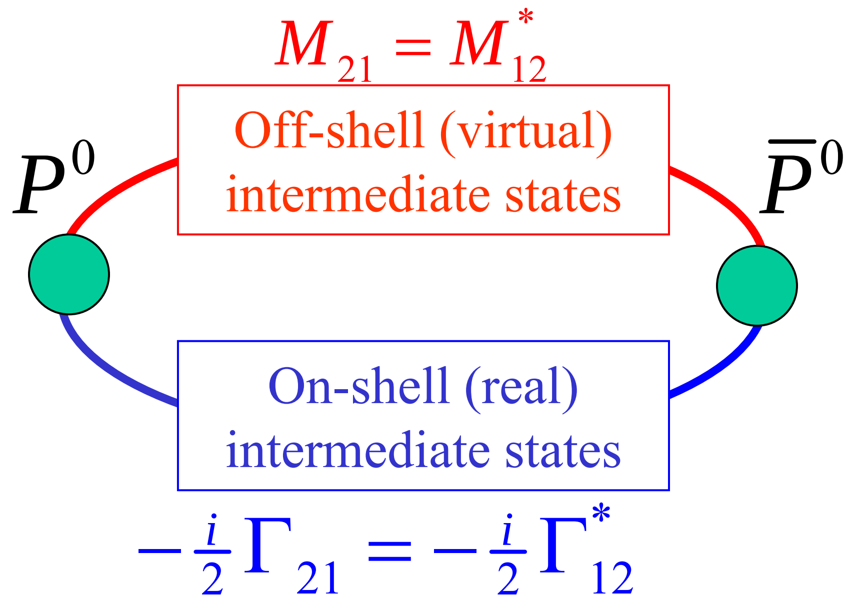
The two classes of transitions in mixing

Get specific form of $\boldsymbol{H}$ in terms of matrix elements of H_w.

$$H = \begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix} = \underbrace{\begin{pmatrix} M & M_{12} \\ M^*_{12} & M \end{pmatrix}}_{\text{mass matrix}} - \underbrace{\frac{i}{2}}_{\text{crucial factor!}} \underbrace{\begin{pmatrix} \Gamma & \Gamma_{12} \\ \Gamma^*_{12} & \Gamma \end{pmatrix}}_{\text{decay matrix}}$$

$$H_{21} \neq H^*_{12}$$





M_{ij}, Γ_{ij}, and the story of the factor $-i$

Results from time-dependent perturbation theory

$$M_{ij} = m_0\delta_{ij} + \underbrace{\langle i|H_w|j\rangle}_{\text{0 in SM}} + P\sum_f \frac{\langle i|H_w|f\rangle\langle f|H_w|j\rangle}{m_0 - E_f}$$

$$\Gamma_{ij} = 2\pi\sum_f \langle i|H_w|f\rangle\langle f|H_w|j\rangle\ \delta(m_0 - E_f)$$

Real/virtual separation comes from $i\varepsilon$

odd func of m_0-E_f

$$\lim_{\varepsilon\to 0^+}\frac{1}{(m_0 - E_f) + i\varepsilon} = \lim_{\varepsilon\to 0^+}\left[\frac{m_0 - E_f}{(m_0 - E_f)^2 + \varepsilon^2} - i\frac{\varepsilon}{(m_0 - E_f)^2 + \varepsilon^2}\right]$$

$$= P\left(\frac{1}{m_0 - E_f}\right) - i\pi\delta(m_0 - E_f)$$

even func

Solution to the eigenvalue problem

$$\boldsymbol{H} = H_{11}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} + \begin{pmatrix} 0 & H_{12} \\ H_{21} & 0 \end{pmatrix} = H_{11}\boldsymbol{I} + \boldsymbol{K}$$

H, ***K*** have same eigenvectors.

$$\begin{pmatrix} 0 & H_{12} \\ H_{21} & 0 \end{pmatrix}\begin{pmatrix} p \\ q \end{pmatrix} = \mu\begin{pmatrix} p \\ q \end{pmatrix} \quad \Rightarrow \mu^2 = H_{12}H_{21}$$

easy!

$$\Rightarrow \alpha \equiv \frac{q}{p} = \left(\frac{H_{21}}{H_{12}}\right)^{1/2} = \left(\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}\right)^{1/2}$$

To get eigenvalues of ***H***, just add H_{11} to μ

$$\mu_\pm = H_{11} \pm (H_{12}H_{21})^{1/2}$$
$$= M_\pm - \tfrac{i}{2}\Gamma_\pm$$

$$M_\pm = M \pm \mathrm{Re}(H_{12}H_{21})^{1/2}$$
$$\Gamma_\pm = \Gamma \mp \mathrm{Im}(H_{12}H_{21})^{1/2}$$

$$\Delta M = -2\,\mathrm{Re}(H_{12}H_{21})^{1/2} \qquad \Delta\Gamma = 4\,\mathrm{Im}(H_{12}H_{21})^{1/2}$$

Time evolution of the mass eigenstates

The ratio α=q/p determines the eigenstates of ***H*** in terms of superpositions of the original flavor-eigenstates:

$$\left|P_+^0\right\rangle = \frac{1}{\sqrt{1+|\alpha|^2}}\left(\left|P^0\right\rangle + \alpha\left|\bar{P}^0\right\rangle\right)$$

$$\left|P_-^0\right\rangle = \frac{1}{\sqrt{1+|\alpha|^2}}\left(\left|P^0\right\rangle - \alpha\left|\bar{P}^0\right\rangle\right)$$

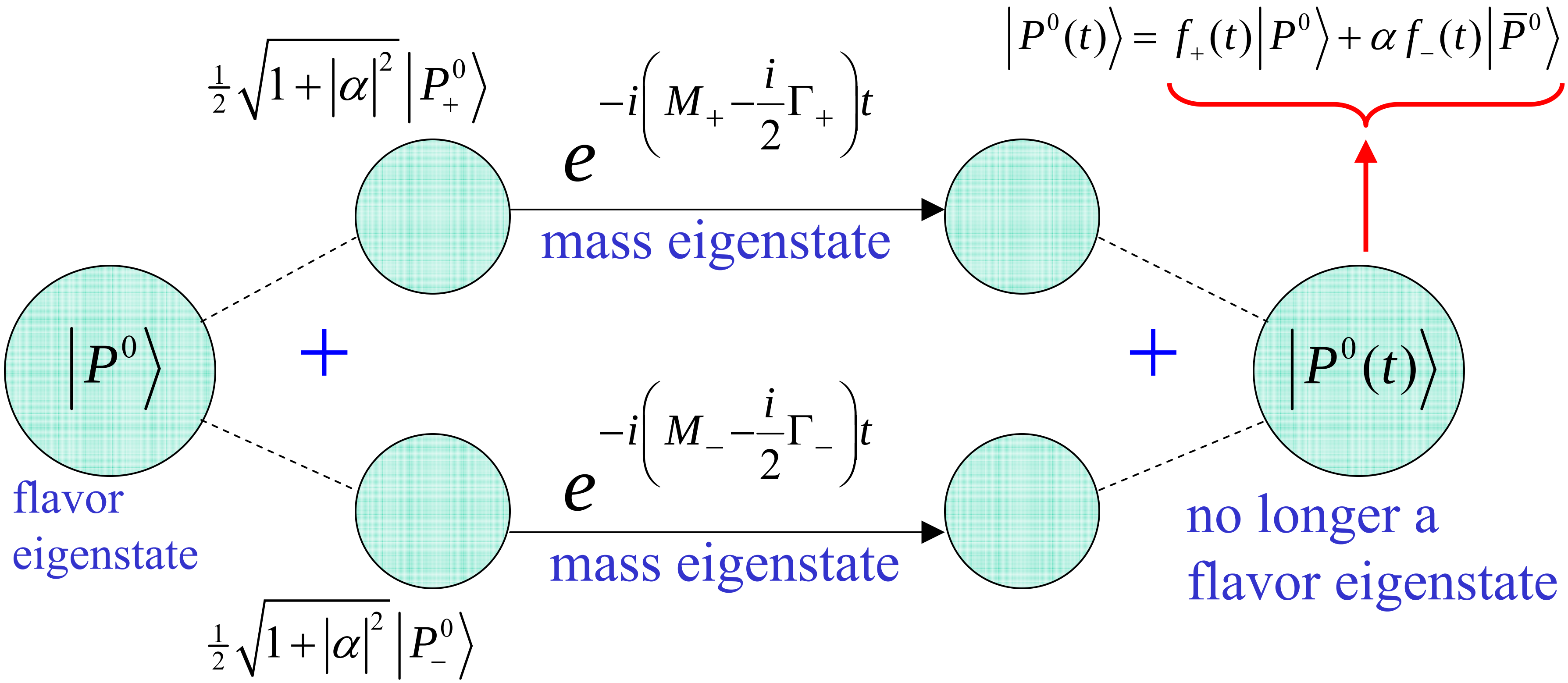
$$\alpha \equiv \frac{q}{p} = \left(\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}\right)^{1/2}$$

Since these are the eigenstates of ***H***, their time dependence is simple!

$$\left|P_+^0(t)\right\rangle = \frac{1}{\sqrt{1+|\alpha|^2}} e^{-i(M_+ - \frac{i}{2}\Gamma_+)t}\left(\left|P^0\right\rangle + \alpha\left|\bar{P}^0\right\rangle\right)$$

$$\left|P_-^0(t)\right\rangle = \frac{1}{\sqrt{1+|\alpha|^2}} \underbrace{e^{-i(M_- - \frac{i}{2}\Gamma_-)t}}_{\text{exponential time dependence}}\left(\left|P^0\right\rangle - \alpha\left|\bar{P}^0\right\rangle\right)$$

Time evolution of states that are initially flavor eigenstates



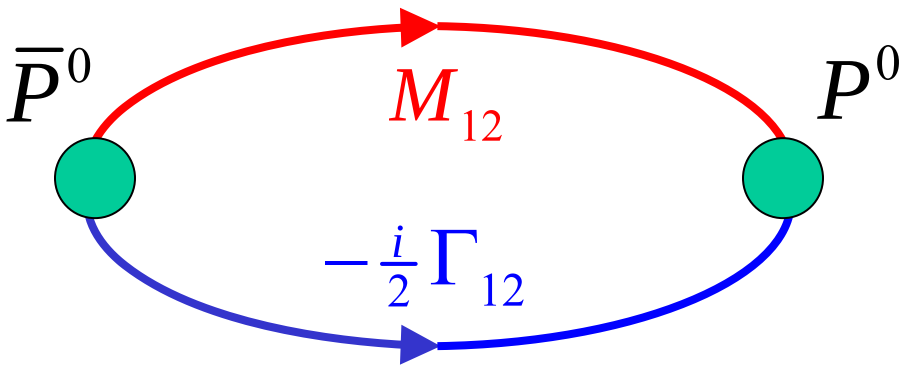
$$\left|P^0(t)\right\rangle = f_+(t)\left|P^0\right\rangle + \alpha f_-(t)\left|\bar{P}^0\right\rangle$$

$$\left|\bar{P}^0(t)\right\rangle = f_+(t)\left|\bar{P}^0\right\rangle + \frac{1}{\alpha} f_-(t)\left|P^0\right\rangle$$

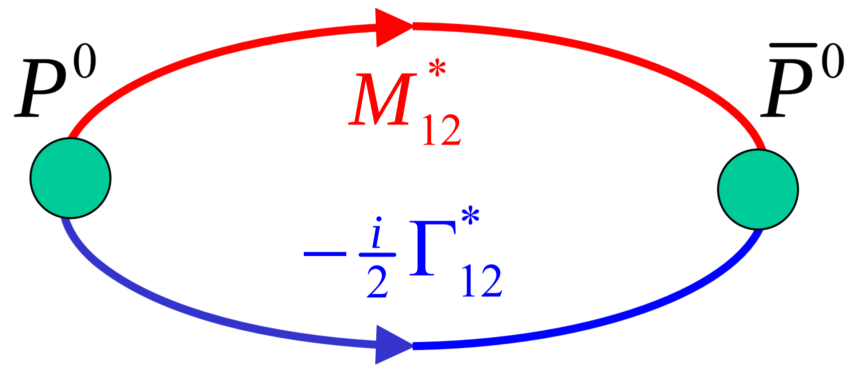
$$f_+(t) = \frac{1}{2}\left(e^{-iM_+t}e^{-\Gamma_+t/2} + e^{-iM_-t}e^{-\Gamma_-t/2}\right)$$

$$f_-(t) = \frac{1}{2}\left(e^{-iM_+t}e^{-\Gamma_+t/2} - e^{-iM_-t}e^{-\Gamma_-t/2}\right)$$

CP Violation in Oscillations



$\mathrm{Amp}(\bar{P}^0 \to P^0)$



$\mathrm{Amp}(P^0 \to \bar{P}^0)$

M_{12} = transition amplitude via intermediate states that are virtual (off-shell)

Γ_{12} = transition amplitude via intermediate states are are real (on-shell: both P^0 and $\bar{P}^0$ can decay into these!)

- The “-i” is a CP conserving phase factor. It doesn’t change sign!
- M_{12} and Γ_{12} behave like CP-violating phase factors, as long as they are not relatively real.

17 December 1998

PHYSICS LETTERS B

Physics Letters B 444 (1998) 43–51

First direct observation of time-reversal non-invariance in the neutral-kaon system

CPLEAR Collaboration

Abstract

We report on the first observation of time-reversal symmetry violation through a comparison of the probabilities of $\overline{\mathrm{K}}^0$ transforming into K^0 and K^0 into $\overline{\mathrm{K}}^0$ as a function of the neutral-kaon eigentime t. The comparison is based on the analysis of the neutral-kaon semileptonic decays recorded in the CPLEAR experiment. There, the strangeness of the neutral kaon at time $t=0$ was tagged by the kaon charge in the reaction $\mathrm{p}\overline{\mathrm{p}} \to \mathrm{K}^{\pm}\pi^{\mp}\mathrm{K}^0(\overline{\mathrm{K}}^0)$ at rest, whereas the strangeness of the kaon at the decay time $t=\tau$ was tagged by the lepton charge in the final state. An average decay-rate asymmetry

$$\left\langle \frac{R\left(\overline{\mathrm{K}}^0_{t=0} \to \mathrm{e}^+\pi^-\nu_{t=\tau}\right) - R\left(\mathrm{K}^0_{t=0} \to \mathrm{e}^-\pi^+\overline{\nu}_{t=\tau}\right)}{R\left(\overline{\mathrm{K}}^0_{t=0} \to \mathrm{e}^+\pi^-\nu_{t=\tau}\right) + R\left(\mathrm{K}^0_{t=0} \to \mathrm{e}^-\pi^+\overline{\nu}_{t=\tau}\right)} \right\rangle = \left(6.6 \pm 1.3_{\mathrm{stat}} \pm 1.0_{\mathrm{syst}}\right) \times 10^{-3}$$

was measured over the interval $1\,\tau_{\mathrm{S}} < \tau < 20\,\tau_{\mathrm{S}}$, thus leading to evidence for time-reversal non-invariance.

First Observation of T-violation (CP LEAR)

1. Introduction

Since weak interactions do not conserve strangeness, a K^0 meson can transform into a $\overline{K}^0$ in the course of time, and vice-versa, a $\overline{K}^0$ can transform into a K^0. Time-reversal (T) invariance, or microscopic reversibility, would require all details of the second process to be deducible from the first; in particular, the probability ($\mathscr{P}$) that a $K^0(t=0)$ is observed as a $\overline{K}^0$ at time τ should be equal to the probability that a $\overline{K}^0(t=0)$ is observed as a K^0 at the same time τ [1]. Any difference between these two probabilities is a signal for T violation and can be measured through the time-reversal asymmetry

$$\frac{\mathscr{P}(\overline{K}^0 \to K^0) - \mathscr{P}(K^0 \to \overline{K}^0)}{\mathscr{P}(\overline{K}^0 \to K^0) + \mathscr{P}(K^0 \to \overline{K}^0)} . \tag{1}$$

Experimentally this requires knowledge of the strangeness of the neutral kaon at two different times of its life.

A measurement of this asymmetry has become possible with the CPLEAR experiment, which produced K^0s and $\overline{K}^0$s through the strong interactions

$$p\bar{p} \to \begin{cases} K^-\pi^+K^0 \\ K^+\pi^-\overline{K}^0 , \end{cases}$$

enabling the initial strangeness of the neutral kaon to be tagged by the charge of the accompanying charged kaon. To tag the strangeness of the kaon at the moment of its decay we use semileptonic decays: positive lepton charge is associated to a K^0 and negative lepton charge to a $\overline{K}^0$. We measure, as a function of time, the decay-rate asymmetry

$$\frac{R\left(\overline{K}^0_{t=0} \to e^+\pi^-\nu_{t=\tau}\right) - R\left(K^0_{t=0} \to e^-\pi^+\overline{\nu}_{t=\tau}\right)}{R\left(\overline{K}^0_{t=0} \to e^+\pi^-\nu_{t=\tau}\right) + R\left(K^0_{t=0} \to e^-\pi^+\overline{\nu}_{t=\tau}\right)} . \tag{2}$$

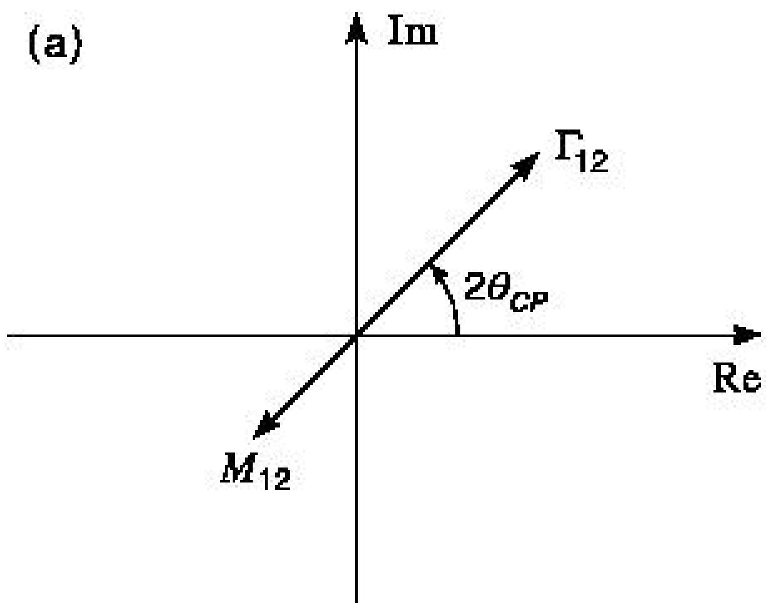
In the limit of CPT symmetry in the semileptonic decay process and of the validity of the $\Delta S = \Delta Q$ rule, this asymmetry is identical with the time-reversal asymmetry given in (1).

CP Violation in Oscillations: Visualization

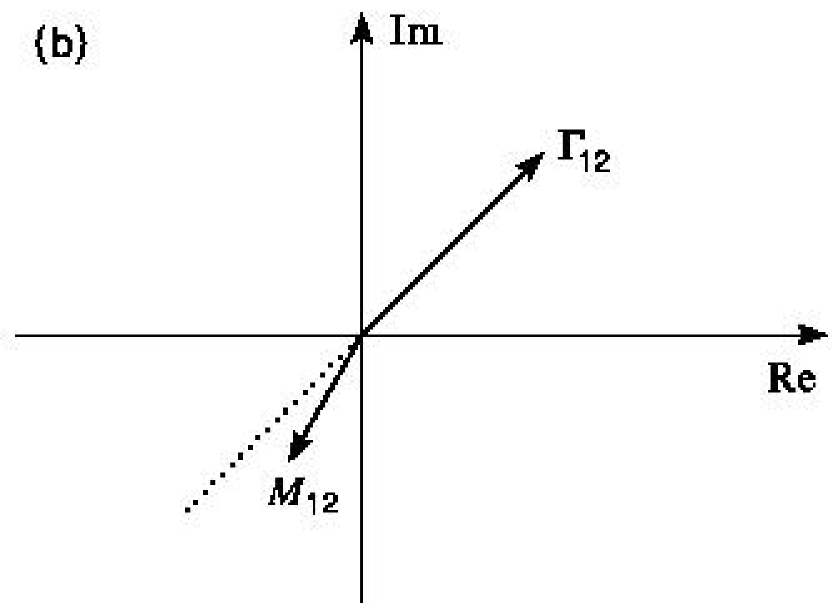
Equivalent statement of condition for CP violation in mixing:
M_{12} and Γ_{12} must not be collinear and both must be nonzero.

$$\mathrm{Im}(M_{12}\Gamma^{*}_{12}) = |M_{12}||\Gamma_{12}|\sin(\theta_{M_{12}} - \theta_{\Gamma_{12}}) \neq 0$$

no CP violation in mixing



CP violation in mixing



Overall rotation is merely a non-physical phase convention.

Oscillations in the $K^0\overline{K}^0$ System

Most striking feature of $K^0\overline{K}^0$ system: huge lifetime splitting between mass eigenstates. (This is quite different from the $B^0\overline{B}^0$ system, where the mass splitting is very small!)

$$\frac{\tau\left(K_S^0\right)}{\tau\left(K_L^0\right)} \simeq \frac{52\text{ ns}}{0.09\text{ ns}} \simeq \frac{15.5\text{ m}}{2.7\text{ cm}}$$

$$\simeq 580$$

Major experimental implication: a neutral K beam evolves over distance into a nearly pure $K_L{}^0$ beam.

$$\Delta\Gamma = \Gamma\left(K_L^0\right) - \Gamma\left(K_S^0\right) \simeq -\Gamma\left(K_S^0\right) \simeq -10^{-10}\text{ s}^{-1}$$

$$\Delta M = M\left(K_L^0\right) - M\left(K_S^0\right) = (0.5304 \pm 0.0014) \times 10^{10}\text{ s}^{-1}$$

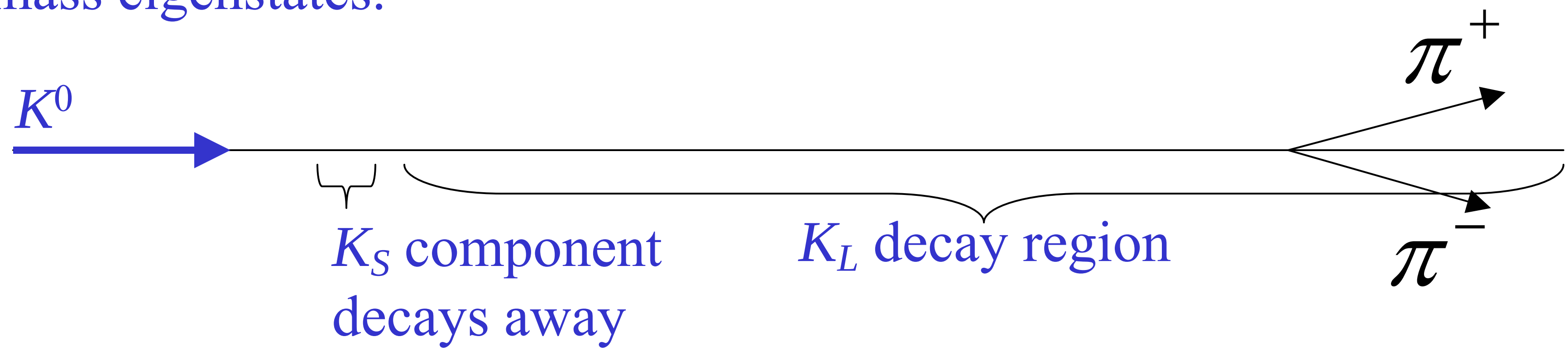
$$\simeq 3.5 \times 10^{-6}\text{ eV}$$

$$\Delta\Gamma \approx -2\Delta M$$

The mass and lifetime splittings are comparable!

CP Violation in mixing: observation of $K_L \rightarrow \pi^+ \pi^-$

Exploit the large lifetime difference between the two neutral K mass eigenstates.



Demonstrates that K_L^0 decays into both CP=-1 (usually) and CP=+1 final states → K_L^0 is not a CP eigenstate.

$$\eta_{+-} \equiv \frac{A\left(K_L^0 \rightarrow \pi^+\pi^-\right)}{A\left(K_S^0 \rightarrow \pi^+\pi^-\right)} \qquad \eta_{00} \equiv \frac{A\left(K_L^0 \rightarrow \pi^0\pi^0\right)}{A\left(K_S^0 \rightarrow \pi^0\pi^0\right)}$$

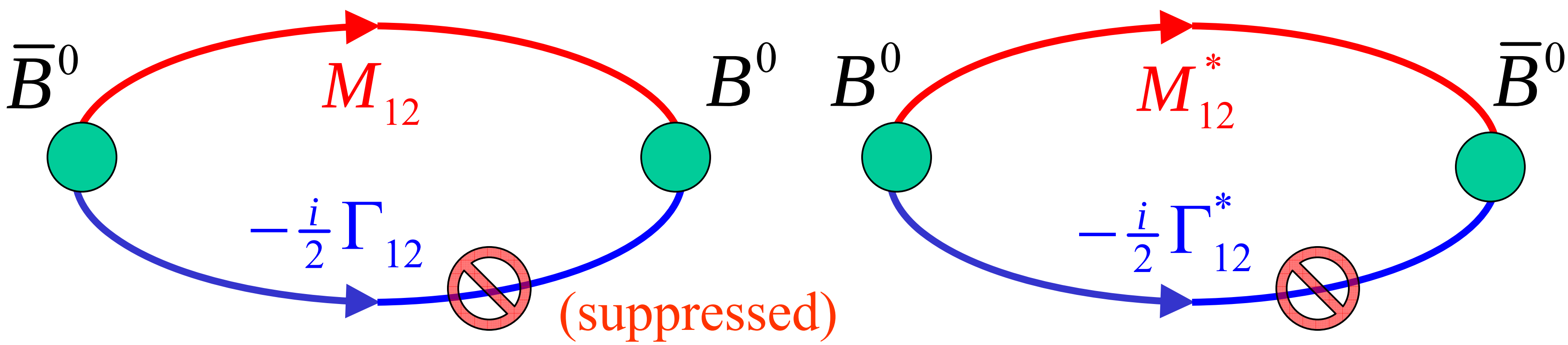
both are 2×10^{-3}

Key point: K_L beam is "self-tagging." (Tagging = method in which we identify a particle P_1 by studying a particle P_2 that is produced in association with particle P_1.)

Phenomenology of $B^0\overline{B}^0$ Oscillations

Oscillations in the $B\overline{B}$ and $K\overline{K}$ systems have very different parameters! This is due to different CKM factors and different intermediate states in the mixing diagrams.

$\mathrm{Amp}(\overline{B}^0 \to B^0)$ $\mathrm{Amp}(B^0 \to \overline{B}^0)$



M_{12}: Dominated by $t\overline{t}$ intermediate states; can be calculated reasonably well using input from lattice QCD

Γ_{12}: Small! Few on-shell intermediate states that both B and $\overline{B}$ can reach. (These are the states both can actually decay into.)

Γ_{12} is small in $B^0\overline{B}^0$ oscillations

- In the neutral B-meson system, the common modes that both B^0 and $\overline{B}^0$ can decay into have small branching fractions, since

$$b \to c \qquad \text{and} \qquad \overline{b} \to \overline{c}$$

- These decays usually lead to different final states. There are some exceptions, but the branching fractions are small. Examples:

$$(B^0, \overline{B}^0) \to c\overline{c}d\overline{d}$$ Cabibbo suppressed

$$(B^0, \overline{B}^0) \to u\overline{u}d\overline{d}$$ $b \to u$ is CKM suppressed

SM predicts

$$\left|\frac{\Gamma_{12}}{M_{12}}\right| = O(m_b^2 / m_t^2) \ll 1$$

Expect CP violation in $B^0\overline{B}^0$ mixing to be $O(10^{-3})$. not yet observed

Implications of a small Γ_{12} in $B^0\bar{B}^0$ oscillations

$$\alpha = \frac{q}{p} = \frac{H_{21}}{H_{12}} = \left(\frac{M_{12}^* - \cancel{\frac{i}{2}\Gamma_{12}^*}}{M_{12} - \cancel{\frac{i}{2}\Gamma_{12}}} \right)^{1/2} = \left(\frac{M_{12}^*}{M_{12}} \right)^{1/2} \qquad \Rightarrow |\alpha| = 1$$

$$\Gamma_1 = \Gamma_2 = \Gamma$$

No CP violation in mixing since don't have second amplitude with non-zero CP-conserving relative phase.

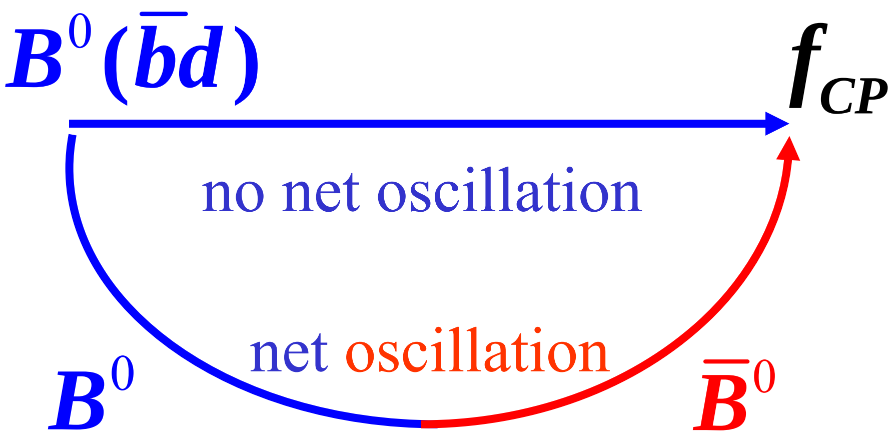
$$\mathrm{Prob}\left(P^0 \text{ at } t \,|\, P^0 \text{ at } t=0\right) = \frac{1}{2} e^{-\Gamma t}\left[1 + \cos(\Delta M t)\right]$$

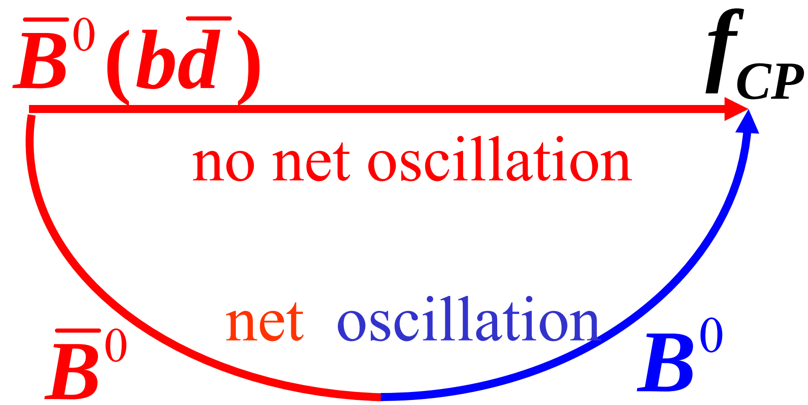
$$\left.\begin{array}{l} \mathrm{Prob}\left(\bar{P}^0 \text{ at } t \,|\, P^0 \text{ at } t=0\right) = \frac{1}{2} e^{-\Gamma t}\left[1 - \cos(\Delta M t)\right] \\ \mathrm{Prob}\left(P^0 \text{ at } t \,|\, \bar{P}^0 \text{ at } t=0\right) = \frac{1}{2} e^{-\Gamma t}\left[1 - \cos(\Delta M t)\right] \end{array}\right\} \text{used to have } |\alpha|^2,\ 1/|\alpha|^2$$

$$\mathrm{Prob}\left(\bar{P}^0 \text{ at } t \,|\, \bar{P}^0 \text{ at } t=0\right) = \frac{1}{2} e^{-\Gamma t}\left[1 + \cos(\Delta M t)\right]$$

Time-dependent CP asymmetries from the interference between mixing and decay amplitudes

By modifying the mixing measurement, we can observe whole new class of CP-violating phenomena: pick final states that both B^0 and $\bar{B}^0$ can decay into. (Often a CP eigenstate, but doesn't have to be.)

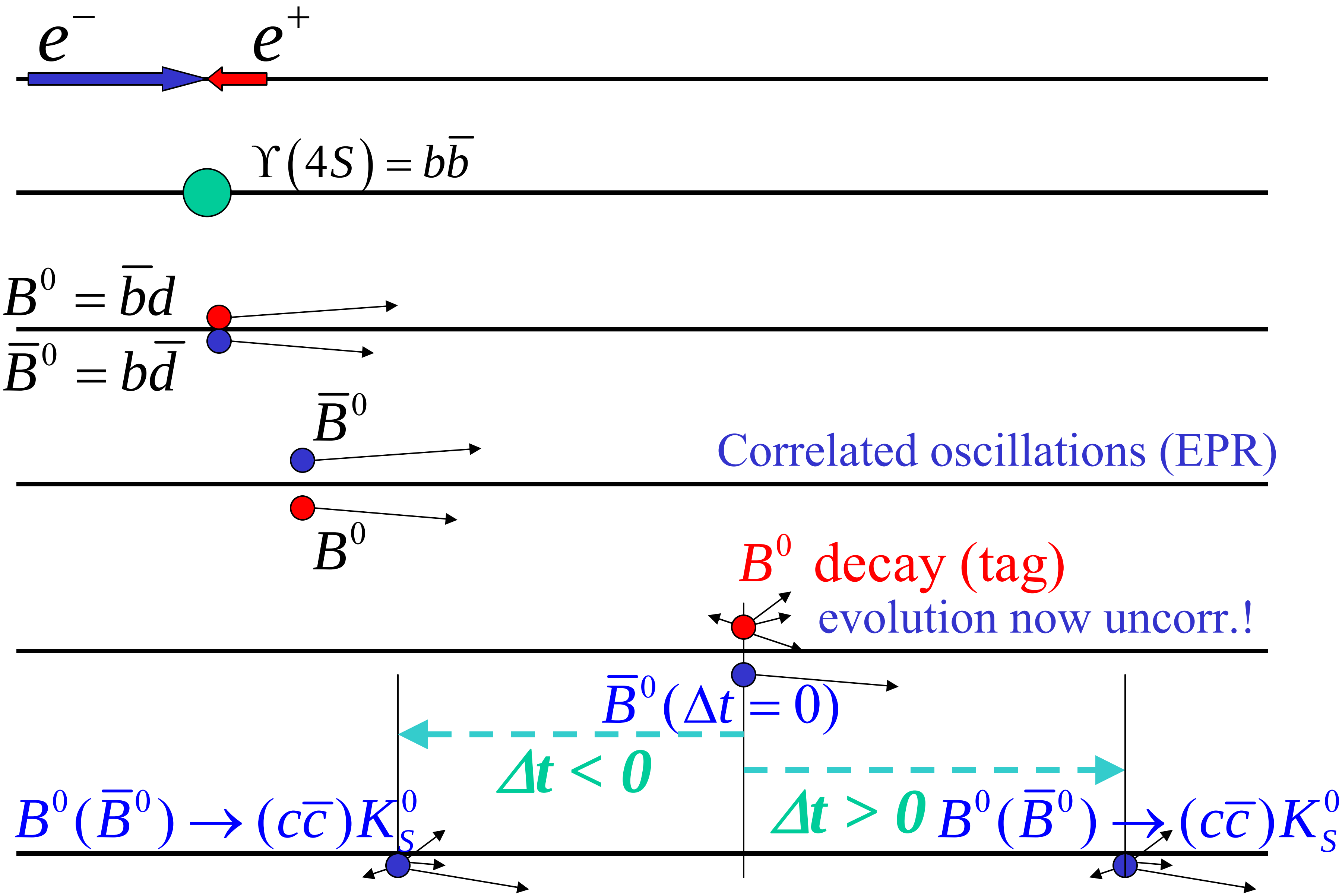




$$\Gamma(B^0_{phys}(t) \to f_{CP})$$

$$\Gamma(\bar{B}^0_{phys}(t) \to f_{CP})$$

Time-dependent CP asymmetry measurement



Ingredients of the CP Asymmetry Measurement

1. Determine initial state: "tag" using other B

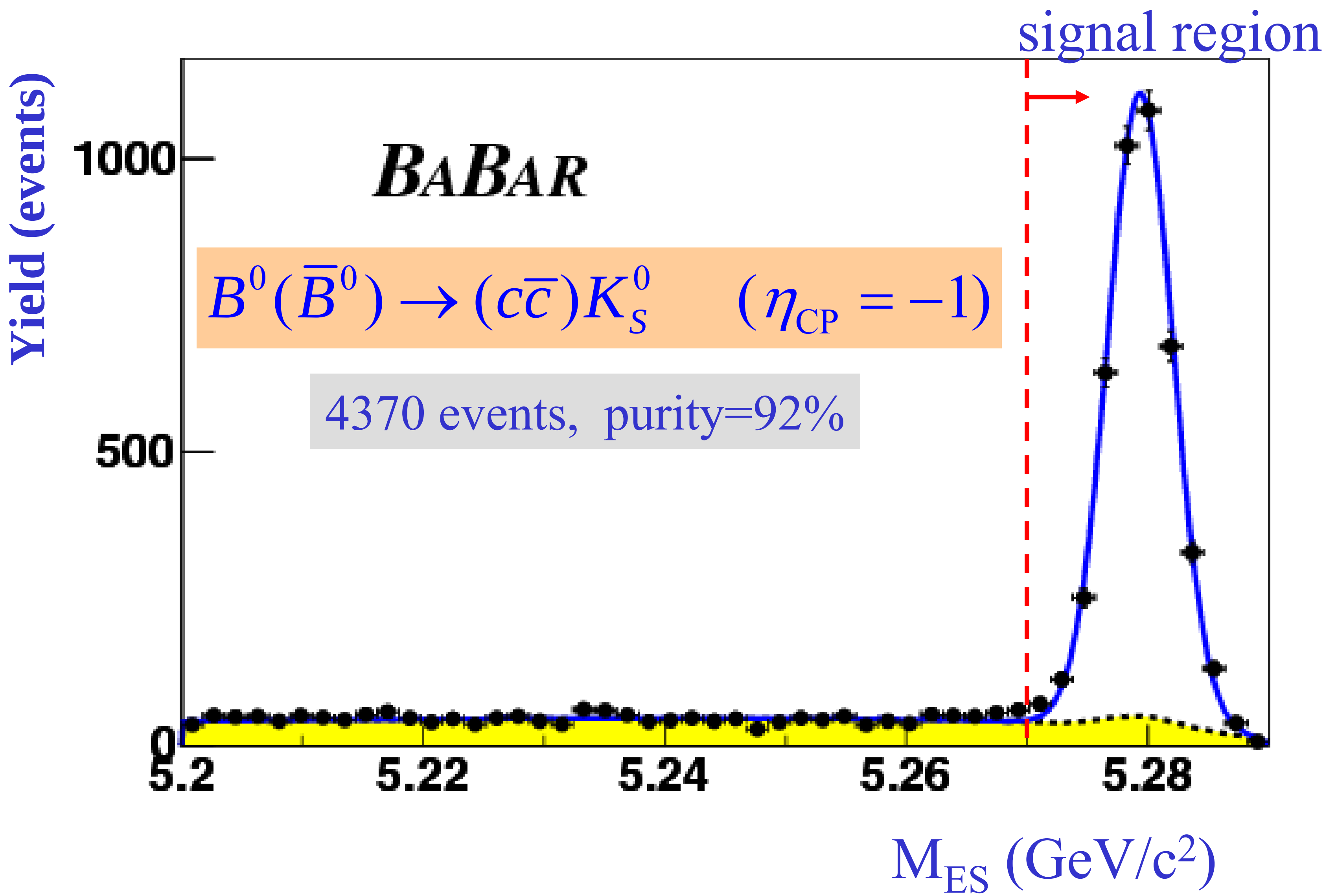
$$A_{CP}(\Delta t) \equiv \frac{\Gamma\left(\bar{B}^0(\Delta t) \to f_{CP}\right) - \Gamma\left(B^0(\Delta t) \to f_{CP}\right)}{\Gamma\left(\bar{B}^0(\Delta t) \to f_{CP}\right) + \Gamma\left(B^0(\Delta t) \to f_{CP}\right)}$$

2. Reconstruct the final state system.

3. Measure Δt dependence

Different final states f_{CP} provide access to different CKM elements and hence different CP-violating phases.
Reason for time dependence: one of the amplitudes is due to mixing.

Data: tagged signal events for $B \rightarrow J/\psi\ K_s$ and other $\eta_{CP} = -1$ sin2β modes



Computing the CP asymmetry for final states common to B^0 and $\overline{B}^0$

Time evolution of tagged states

$$\left|B^0(t)\right\rangle = \quad e^{-\frac{\Gamma}{2}t} e^{-iMt}\left(\cos\frac{\Delta M\cdot t}{2}\left|B^0\right\rangle \ - i\cdot\alpha\cdot\sin\frac{\Delta M\cdot t}{2}\left|\overline{B}^0\right\rangle\right)$$

$$\left|\overline{B}^0(t)\right\rangle = \quad e^{-\frac{\Gamma}{2}t} e^{-iMt}\left(\cos\frac{\Delta M\cdot t}{2}\left|\overline{B}^0\right\rangle - i\cdot\frac{1}{\alpha}\cdot\sin\frac{\Delta M\cdot t}{2}\left|B^0\right\rangle\right)$$

We have used $\Delta\Gamma/\Gamma \ll 1$ and set $\Gamma \cong \Gamma_1 \cong \Gamma_2 \qquad M = \frac{1}{2}(M_+ + M_-)$

Goal: calculate $\begin{cases} \left\langle f_{CP}\middle|H\middle|B^0(t)\right\rangle \\ \left\langle f_{CP}\middle|H\middle|\overline{B}^0(t)\right\rangle \end{cases}$ $\qquad f_{CP}$ = CP eigenstate

$\underbrace{\langle f_{CP}|H|}_{}$ Simply project above eq'ns onto this!

Decay amplitudes for $B^0(t) \rightarrow f_{CP}$ vs. $\bar{B}^0(t) \rightarrow f_{CP}$

$$\langle f_{CP} | H | B^0(t) \rangle = e^{-\frac{\Gamma}{2}t} e^{-iMt} \langle f_{CP} | H | B^0 \rangle \left(\cos\frac{\Delta M \cdot t}{2} - i \cdot \alpha \cdot \frac{\langle f_{CP} | H | \bar{B}^0 \rangle}{\langle f_{CP} | H | B^0 \rangle} \sin\frac{\Delta M \cdot t}{2} \right)$$

$$\langle f_{CP} | H | \bar{B}^0(t) \rangle = e^{-\frac{\Gamma}{2}t} e^{-iMt} \langle f_{CP} | H | \bar{B}^0 \rangle \left(\cos\frac{\Delta M \cdot t}{2} - i \cdot \frac{1}{\alpha} \cdot \frac{\langle f_{CP} | H | B^0 \rangle}{\langle f_{CP} | H | \bar{B}^0 \rangle} \sin\frac{\Delta M \cdot t}{2} \right)$$

blue: mixing green: decay

The key quantity in these CP asymmetries is:

$$\lambda \equiv \alpha \cdot \frac{\langle f_{CP} | H | \bar{B}^0 \rangle}{\langle f_{CP} | H | B^0 \rangle} = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}} \cdot \frac{\langle f_{CP} | H | \bar{B}^0 \rangle}{\langle f_{CP} | H | B^0 \rangle}$$

Calculation of the time-dependent CP asymmetry

$$A_{f_{CP}}(t) = \frac{\left|\left\langle f_{CP}\left|H\right|\overline{B}^0(t)\right\rangle\right|^2 - \left|\left\langle f_{CP}\left|H\right|B^0(t)\right\rangle\right|^2}{\left|\left\langle f_{CP}\left|H\right|\overline{B}^0(t)\right\rangle\right|^2 + \left|\left\langle f_{CP}\left|H\right|B^0(t)\right\rangle\right|^2}$$

$$= \frac{\Gamma\left(\overline{B}^0(t) \to f_{CP}\right) - \Gamma\left(B^0(t) \to f_{CP}\right)}{\Gamma\left(\overline{B}^0(t) \to f_{CP}\right) + \Gamma\left(B^0(t) \to f_{CP}\right)}$$

$$A_{f_{CP}}(t) = S \cdot \sin(\Delta m \cdot t) \text{ - } C \cdot \cos(\Delta m \cdot t)$$

$$S = \frac{2 \cdot \mathrm{Im}(\lambda)}{1 + |\lambda|^2} \qquad C = \frac{1 - |\lambda|^2}{1 + |\lambda|^2}$$

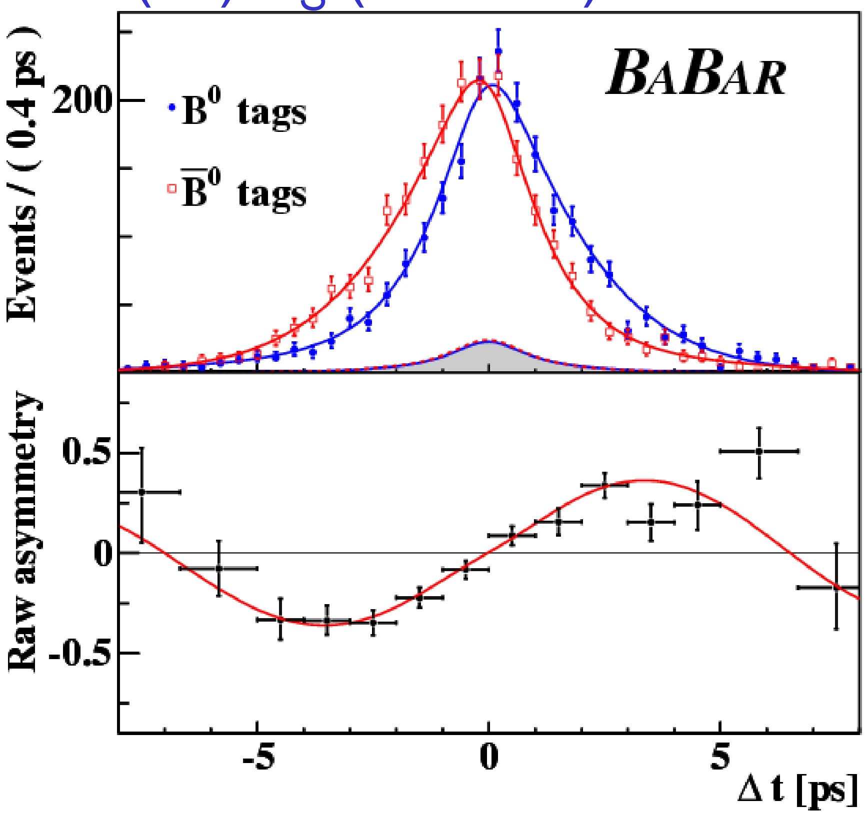
1 decay amplitude:

$$|\lambda| = 1 \quad \Rightarrow \quad S = \mathrm{Im}\left(\lambda\right), \;\; C = 0$$

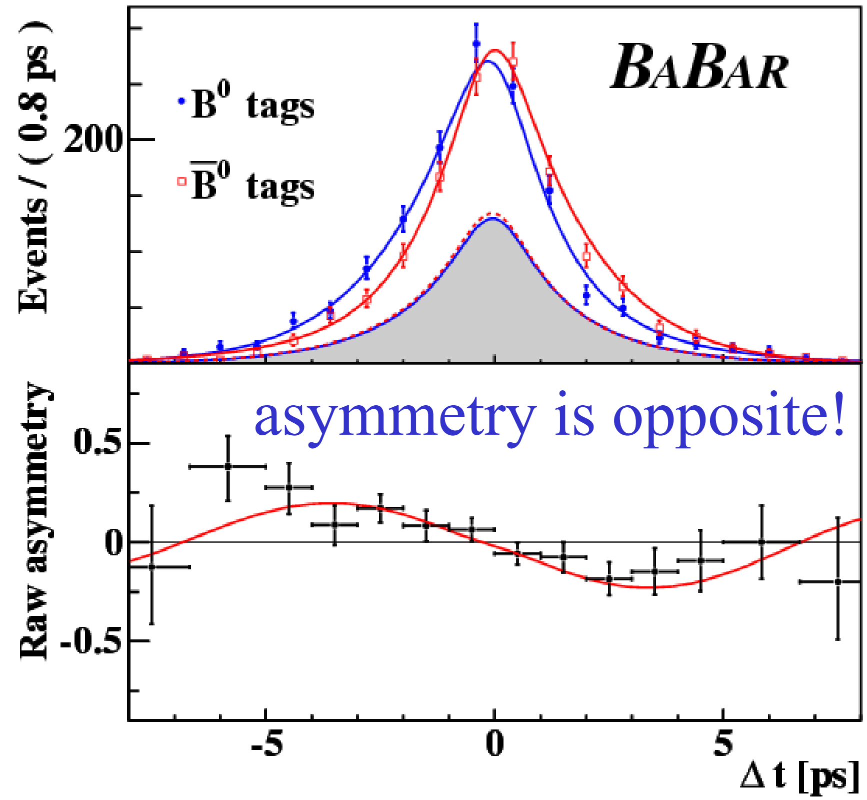
$$A_{f_{CP}}(t) = \mathrm{Im}(\lambda) \cdot \sin(\Delta m \cdot t)$$

Results on sin2β from charmonium modes

$(c\bar{c})\ K_S$ (CP odd) modes



$J/\psi\ K_L$ (CP even) mode



$\sin 2\beta = 0.722 \pm 0.040$ (stat) $\pm\ 0.023$ (sys)

$|\lambda| = 0.950$ +/- 0.031 (stat) +/- 0.013 (sys)

227 M $B\bar{B}$ events

(raw asymmetry shown above must be corrected for the dilution)

Calculating λ

$$\lambda = \sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}} \cdot \frac{\langle f_{CP} | H | \overline{B}^0 \rangle}{\langle f_{CP} | H | B^0 \rangle} = \frac{q}{p} \cdot \frac{\overline{A}_f}{A_f}$$

Factor from mixing

$$\alpha \cong \sqrt{\frac{M_{12}^*}{M_{12}}}$$

$$\simeq \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \cdot e^{i\theta_{CP}}$$

$$\simeq e^{i(\theta_{CP} + 2\phi_M)}$$

Factor from decay

assuming 1 decay amplitude:

$$\langle f_{CP} | H | B^0 \rangle = |a| e^{i(\delta + \phi_D)}$$

$$\langle f_{CP} | H | \overline{B}^0 \rangle = \eta_{CP}(f) e^{-i\theta_{CP}} |a| e^{i(\delta - \phi_D)}$$

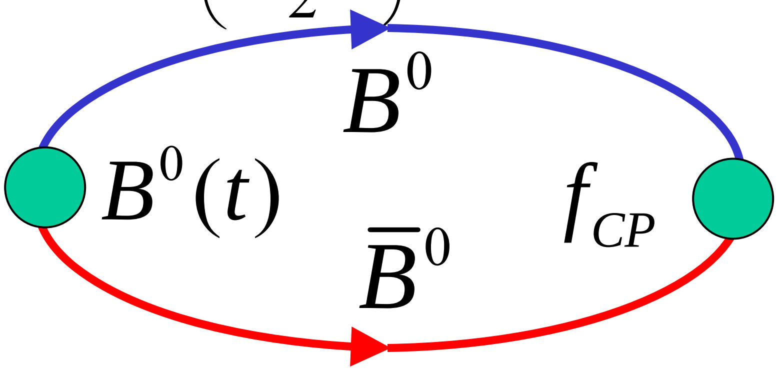
$$\frac{\langle f_{CP} | H | \overline{B}^0 \rangle}{\langle f_{CP} | H | B^0 \rangle} = \eta_{CP}(f) e^{-i(\theta_{CP} + 2\phi_D)}$$

$$\Rightarrow \quad \lambda = \eta_{CP}(f) e^{2i(\phi_M - \phi_D)}$$

(the unphysical phase, the strong phase, and |a| ALL cancel)

How the magic works

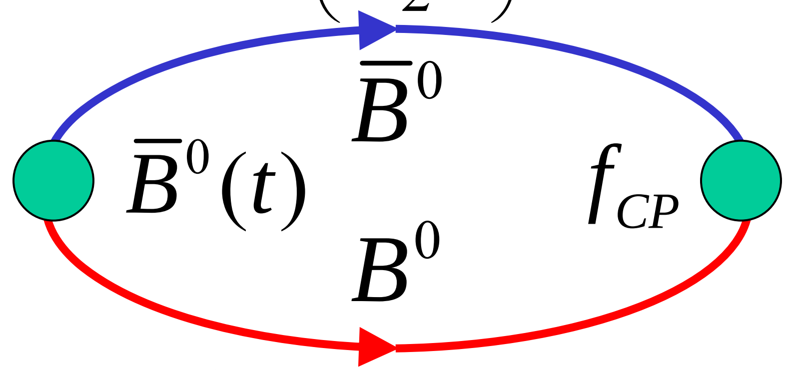
$$\cos\left(\frac{\Delta m\cdot t}{2}\right)\cdot|a|e^{i(\delta+\phi_D)}$$



$$-i\cdot\eta_{CP}(f)\sin\left(\frac{\Delta m\cdot t}{2}\right)\cdot|a|e^{i(\delta-\phi_D)}e^{2i\phi_M}$$

$$\mathrm{Amp}(B^0\to f_{CP})$$

$$\eta_{CP}(f)\cdot\cos\left(\frac{\Delta m\cdot t}{2}\right)\cdot|a|e^{i(\delta-\phi_D)}$$



$$-i\cdot\sin\left(\frac{\Delta m\cdot t}{2}\right)\cdot|a|e^{i(\delta+\phi_D)}e^{-2i\phi_M}$$

$$\mathrm{Amp}(\bar{B}^0(t)\to f_{CP})$$

In each case, the two interfering amplitudes have the same CP conserving phase from strong interactions, so it is irrelevant.

$$A(t)=\mathrm{Im}(\lambda)\cdot\sin\left(\Delta m\cdot t\right)$$

Conclusions

- **Measurements in the B system are extraordinarily powerful probes of CP violation and the CKM matrix.**
- **From our observations so far, the CKM framework for quark couplings to the W-boson is confirmed, although there is still room for non-SM effects.**

Jim Cronin's Nobel Prize lecture:

"...the effect is telling us that at some tiny level there is a fundamental asymmetry between matter and antimatter, and it is telling us that at some tiny level interactions will show an asymmetry under the reversal of time. We know that improvements in detector technology and quality of accelerators will permit even more sensitive experiments in coming decades. We are hopeful then, that at some epoch, perhaps distant, this cryptic message from nature will be deciphered."

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Backup Slides

Some consequences of symmetries

1. Conserved quantum numbers

$$0 = \langle \psi_b | [H, G] | \psi_a \rangle = \langle \psi_b | HG - GH | \psi_a \rangle$$
$$= (g_b - g_a) \langle \psi_b | H | \psi_a \rangle$$
$$\Rightarrow \begin{cases} g_b = g_a & \text{(quantum number conserved)} \\ \text{or} \quad \langle \psi_b | H | \psi_a \rangle = 0 & \text{(no transition)} \end{cases}$$

2. Relations between amplitudes

$$0 = \langle \phi | U^\dagger H U - H | \psi \rangle = \langle U\phi | H | U\psi \rangle - \langle \phi | H | \psi \rangle$$
$$\Rightarrow \langle U\phi | H | U\psi \rangle = \langle \phi | H | \psi \rangle$$

Same amplitudes for these transitions!

3. Existence of multiplets (states with same energies)

$$[H, U] = 0 \quad \Rightarrow \quad \langle U\psi | H | U\psi \rangle = \langle \psi | H | \psi \rangle$$

Testing for Violation of Symmetries

1. Non-conserved quantum numbers

$$B^0 \to \pi^+\pi^-$$

$$J^P = 0^- \to \underbrace{0^-0^-}$$ Violates parity (weak decay).

$$\eta_P = \eta_{\pi^+}\eta_{\pi^-}(-1)^\ell = +1 \qquad (\ell = 0)$$

2. Broken relationships between amplitudes

$$\Gamma(B^0 \to K^+\pi^-) \neq \Gamma(\overline{B}^0 \to K^-\pi^+)$$

Violates CP

3. Masses of particles in multiplet not the same

$$\begin{pmatrix} p \\ n \end{pmatrix} \qquad m_p = 938.27 \text{ MeV}/c^2 \qquad m_n = 939.57 \text{ MeV}/c^2$$

I-spin violation (quark masses, EM interaction)

Conjugate amplitudes when CP is conserved

What is the relation between an amplitude and its conjugate?

$$\left.\begin{aligned} CP\left|P\right\rangle &= e^{i\theta(P)}\left|\overline{P}\right\rangle \\ CP\left|\overline{P}\right\rangle &= e^{-i\theta(P)}\left|P\right\rangle \\ (CP)^2\left|P\right\rangle &= \left|P\right\rangle \end{aligned}\right\}$$

Often, people choose a specific phase convention. I like to keep the non-physical CP phase explicit.

$$A = \left\langle f \right| H \left| P \right\rangle = \left\langle f \right| (CP)^\dagger (CP) H (CP)^\dagger (CP) \left| P \right\rangle$$

$$= \left\langle \overline{f} \right| (CP) H (CP)^\dagger \left| \overline{P} \right\rangle e^{i[\theta(P)-\theta(f)]}$$

assume $[H,CP]=0$

$$= \left\langle \overline{f} \right| H \left| \overline{P} \right\rangle e^{i[\theta(P)-\theta(f)]}$$

$$= \overline{A} e^{i[\theta(P)-\theta(f)]} \qquad \Rightarrow \left|\frac{\overline{A}}{A}\right| = 1 \quad \text{if CP conserved}$$

Probabilities vs. time: master equations

$$\text{Prob}\left(P^0 \text{ at } t \mid P^0 \text{ at } t=0\right) = \frac{1}{4}\left[e^{-\Gamma_+ t} + e^{-\Gamma_- t} + 2e^{-\Gamma t}\cos(\Delta M t)\right]$$

$$\text{Prob}\left(\bar{P}^0 \text{ at } t \mid P^0 \text{ at } t=0\right) = |\alpha|^2 \frac{1}{4}\left[e^{-\Gamma_+ t} + e^{-\Gamma_- t} - 2e^{-\Gamma t}\cos(\Delta M t)\right]$$

$$\text{Prob}\left(P^0 \text{ at } t \mid \bar{P}^0 \text{ at } t=0\right) = \left|\frac{1}{\alpha}\right|^2 \frac{1}{4}\left[e^{-\Gamma_+ t} + e^{-\Gamma_- t} - 2e^{-\Gamma t}\cos(\Delta M t)\right]$$

$$\text{Prob}\left(\bar{P}^0 \text{ at } t \mid \bar{P}^0 \text{ at } t=0\right) = \frac{1}{4}\left[e^{-\Gamma_+ t} + e^{-\Gamma_- t} + 2e^{-\Gamma t}\cos(\Delta M t)\right]$$

Notes:

$$\Delta M \equiv M_- - M_+ \qquad \Gamma = \tfrac{1}{2}(\Gamma_+ + \Gamma_-)$$

To calculate, need 5 numbers: $M_+, M_-, \Gamma_+, \Gamma_-, |\alpha|$

$$|\alpha| \neq 1 \quad \Rightarrow \quad \text{Prob}\left(\bar{P}^0 \text{ at } t \mid P^0 \text{ at } t=0\right) \neq \text{Prob}\left(P^0 \text{ at } t \mid \bar{P}^0 \text{ at } t=0\right)$$

$$\text{CPT} \quad \Rightarrow \quad \text{Prob}\left(\bar{P}^0 \text{ at } t \mid \bar{P}^0 \text{ at } t=0\right) = \text{Prob}\left(P^0 \text{ at } t \mid P^0 \text{ at } t=0\right)$$

Apply the condition for CP violation in mixing

What are the implications of CP violation for the state vectors?

$$\langle P_-^0 | P_+^0 \rangle = \frac{|p|^2 - |q|^2}{|p|^2 + |q|^2} = \frac{1 - |\alpha|^2}{1 + |\alpha|^2} \neq 0$$

Mass eigenstates not orthogonal!
(***H*** not hermitian).

Since $[H, CP] \neq 0$, expect that mass eigenstates are not simultaneously CP eigenstates.

$$|P_{CP=+1}^0\rangle = \tfrac{1}{\sqrt{2}}\left(|P^0\rangle + e^{i\theta_{CP}}|\bar{P}^0\rangle\right)$$

$$|P_{CP=-1}^0\rangle = \tfrac{1}{\sqrt{2}}\left(|P^0\rangle - e^{i\theta_{CP}}|\bar{P}^0\rangle\right)$$

You can verify that these are

1. CP eigenstates
2. If CP is violated, they are not mass eigenstates.

Condition for CP Violation in Oscillations

CP conservation ([H,CP]=0) therefore implies

$$e^{2i\theta_{CP}} H_{12} = H_{21} \qquad \Rightarrow \qquad \frac{H_{21}}{H_{12}} = e^{i2\theta_{CP}}$$

Conversely, there will be observable CP violation in the oscillations if

$$\left|\frac{\mathrm{Amp}(P^0 \to \overline{P}^0)}{\mathrm{Amp}(\overline{P}^0 \to P^0)}\right| = \left|\frac{H_{21}}{H_{12}}\right| = \left|\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}\right| = |\alpha|^2 = \left|\frac{q}{p}\right|^2 \neq 1$$

Key point: for CP violation to occur in mixing, both M_{12} and Γ_{12} must be non-zero. CP violation in mixing will not occur due to interference of the amplitudes within M_{12} (or Γ_{12}). This is why the formalism is so simple!

CP violation and the $K_S K_L$ lifetime splitting

CP violation is a small effect in $K^0 K^0$ oscillations.

$$\left|K_S^0\right\rangle = \frac{1}{\sqrt{2(1+|\varepsilon|^2)}}\left[(1+\varepsilon)\left|K^0\right\rangle + (1-\varepsilon)\left|\bar{K}^0\right\rangle\right]$$

$$\left|K_L^0\right\rangle = \frac{1}{\sqrt{2(1+|\varepsilon|^2)}}\left[(1+\varepsilon)\left|K^0\right\rangle - (1-\varepsilon)\left|\bar{K}^0\right\rangle\right]$$

$$\varepsilon = \frac{1-\alpha}{1+\alpha} \qquad |\varepsilon| = (2.284 \pm 0.014)\times 10^{-3}$$

K_S^0	Mostly CP=+1 → can decay to $\pi^+\pi^-$, $\pi^0\pi^0$ → <u>faster decay rate</u>
K_L^0	Mostly CP=-1 → decays to $\pi^0\pi^0\pi^0$, $\pi^+\pi^-\pi^0$, $\pi e\nu$, $\pi\mu\nu$ → 3 body decays: <u>slower decay rate</u>

Experimental setup used for discovery of $K_L \rightarrow \pi^+\pi^-$

J.H. Christenson, J.W. Cronin, V.L. Fitch, and R.Turlay, Phys. Rev. Lett. 13, 138 (1964).

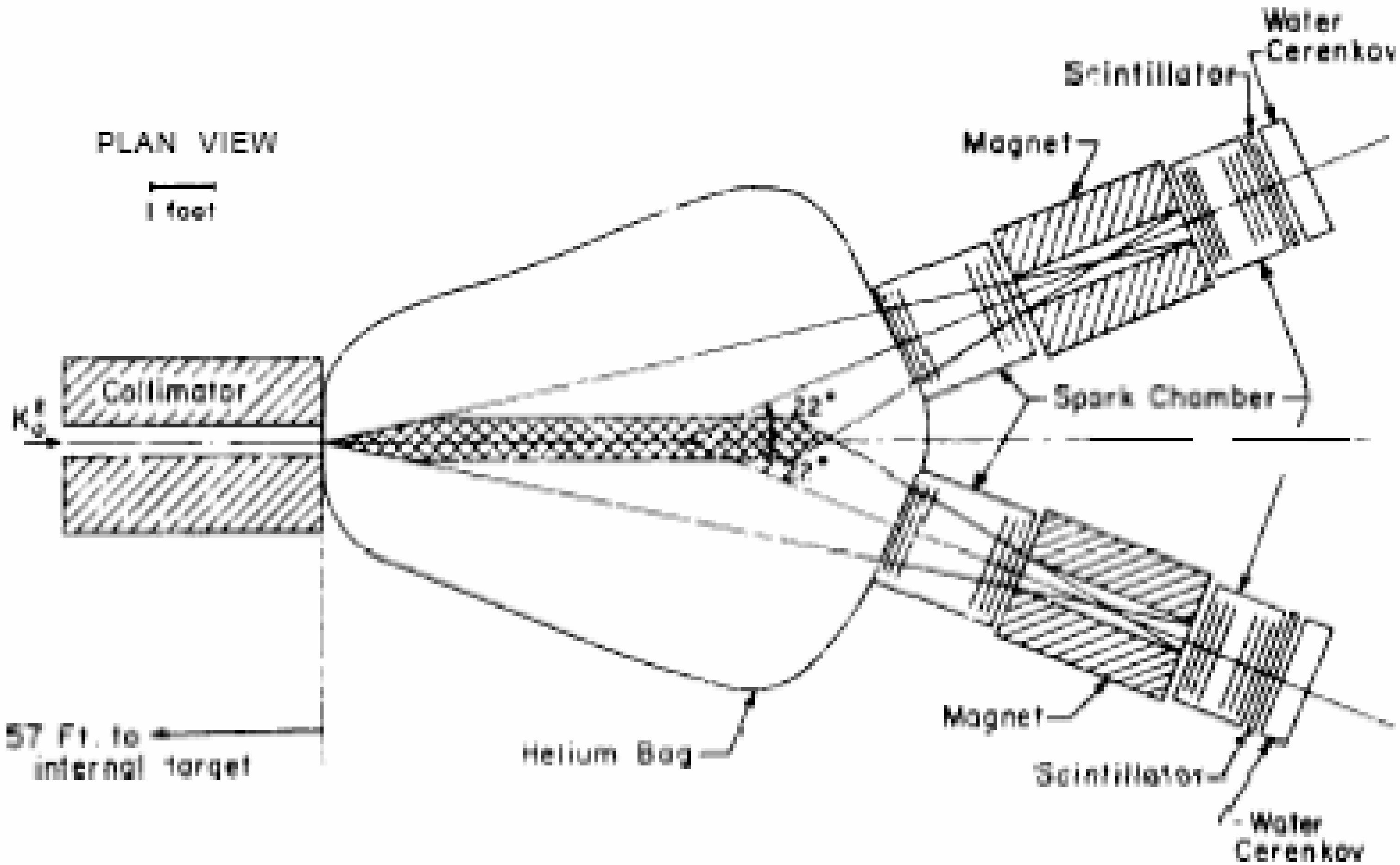


Fig. I. Plan view of the apparatus as located at the A. G. S.

CP violation in $K^0\bar{K}^0$ oscillations: semileptonic decays

The K^0 has a slightly higher probability of decaying as a K_L than as a K_S.

$$\delta \equiv \frac{\Gamma\left(K_L^0 \to \pi^-\ell^+\nu\right) - \Gamma\left(K_L^0 \to \pi^+\ell^-\nu\right)}{\Gamma\left(K_L^0 \to \pi^-\ell^+\nu\right) + \Gamma\left(K_L^0 \to \pi^+\ell^-\nu\right)} = \frac{\left|\left\langle K^0 \middle| K_L^0\right\rangle\right|^2 - \left|\left\langle \bar{K}^0 \middle| K_L^0\right\rangle\right|^2}{\left|\left\langle K^0 \middle| K_L^0\right\rangle\right|^2 + \left|\left\langle \bar{K}^0 \middle| K_L^0\right\rangle\right|^2}$$

$$= \frac{1-|\alpha|^2}{1+|\alpha|^2} = \left\langle K_L^0 \middle| K_S^0\right\rangle = (3.27 \pm 0.12) \times 10^{-3}$$

$\bar{s}d$ $s\bar{d}$

δ gives direct measure of non-orthogonality of mass eigenstates.

$$|\alpha| \simeq 1 - \delta \simeq 0.9967$$

Time evolution of particles initially tagged as $K^0(\bar{K}^0)$

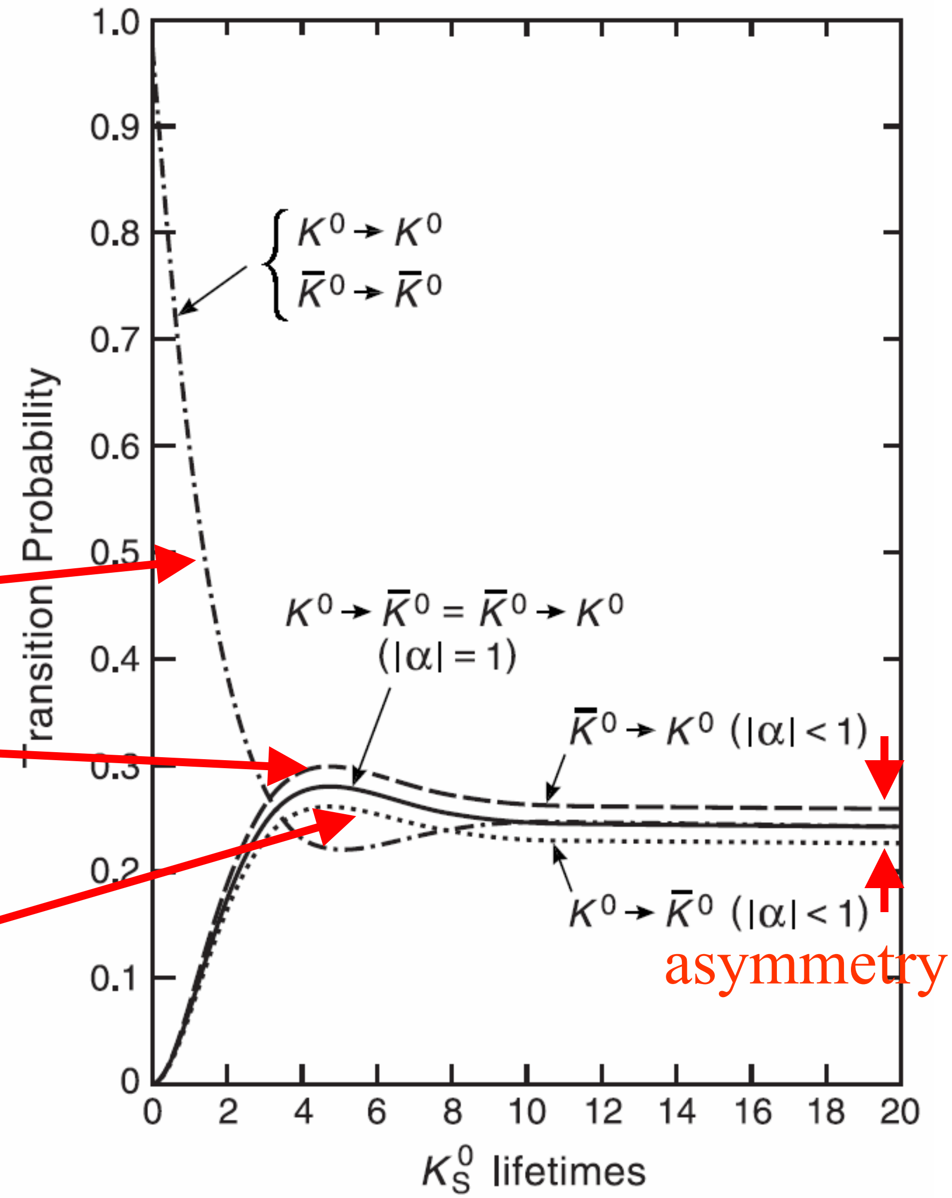
$$|\alpha| \simeq 1-\delta \simeq 0.9967$$

In fig., increase δ by 10X
→ |α|=0.967

$$\frac{1}{4}\left[e^{-\Gamma_+ t}+e^{-\Gamma_- t}+2e^{-\Gamma t}\cos(\Delta M t)\right]$$

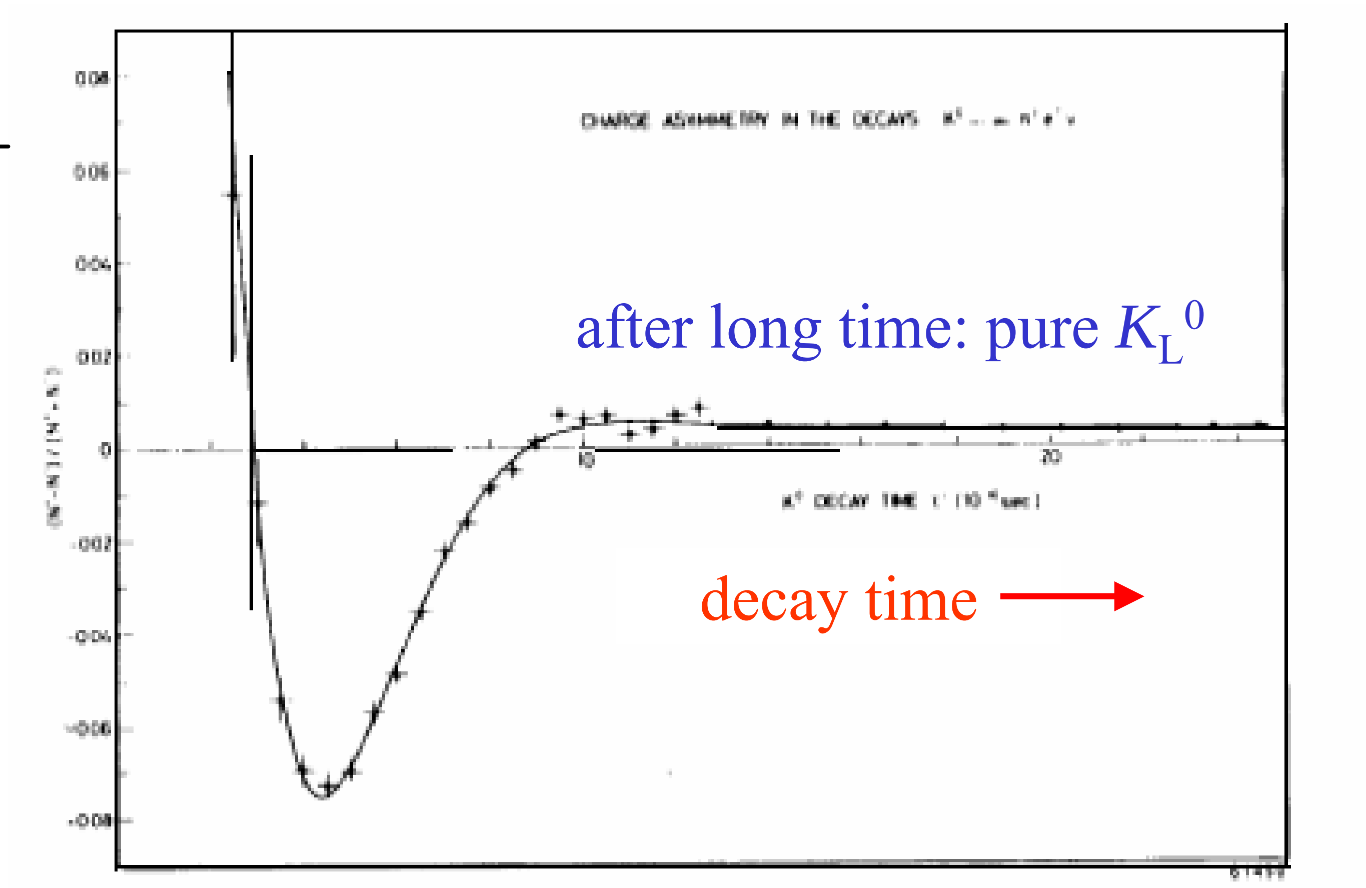
$$\frac{1}{4}|\alpha|^2\left[e^{-\Gamma_+ t}+e^{-\Gamma_- t}-2e^{-\Gamma t}\cos(\Delta M t)\right]$$

$$\frac{1}{4}\left|\frac{1}{\alpha}\right|^2\left[e^{-\Gamma_+ t}+e^{-\Gamma_- t}-2e^{-\Gamma t}\cos(\Delta M t)\right]$$



Measurement of the charge asymmetry in K^0 semileptonic decays

$$A = \frac{N^+ - N^-}{N^+ + N^-}$$



CP violation is a small effect in $K^0\bar{K}^0$ oscillations, but it is possible to observe it!

Direct CP violation in K decays

In the neutral K decays, does CP violation occur only between mixing amplitudes, or does it also occur between decay amplitudes?

Compare two CP violating amplitudes

$$\eta_{+-} \equiv \frac{A\left(K_L^0 \to \pi^+\pi^-\right)}{A\left(K_S^0 \to \pi^+\pi^-\right)} \qquad \eta_{00} \equiv \frac{A\left(K_L^0 \to \pi^0\pi^0\right)}{A\left(K_S^0 \to \pi^0\pi^0\right)}$$

$$\left.\begin{aligned} \eta_{+-} &\simeq \varepsilon + \varepsilon' \\ \eta_{00} &\simeq \varepsilon - 2\varepsilon' \end{aligned}\right\} \quad \begin{aligned} \varepsilon &\simeq \tfrac{2}{3}\eta_{+-} + \tfrac{1}{3}\eta_{00} \\ \varepsilon' &\simeq \tfrac{1}{3}(\eta_{+-} - \eta_{00}) \end{aligned}$$

CP violation from mixing only

CP violation from interference between direct decay amps

$$\mathrm{Re}\left(\frac{\varepsilon'}{\varepsilon}\right) = (1.67 \pm 0.26) \times 10^{-3}$$

(Direct CP violation due to tree-penguin interference.)

$B^0\bar{B}^0$ oscillation frequency in the SM

Inami-Lim function

$$\Delta m_d = \frac{G_F^2}{6\pi^2}\eta_B m_{B_d} f_{B_d}^2 B_d m_W^2 S(x_t)\left|V_{td}V_{tb}^*\right|^2$$

$$\eta_B = 0.551 \pm 0.007$$

$$x_t = (m_t^2 / m_W^2)$$

pert. QCD correction

B meson decay constant

"Bag" constant

$$\Delta m_s = \frac{G_F^2}{6\pi^2}\eta_B m_{B_s} \xi^2 f_{B_d}^2 B_d m_W^2 S(x_t)\left|V_{ts}V_{tb}^*\right|^2$$

$$\xi \equiv \frac{f_{B_s}\sqrt{B_s}}{f_{B_d}\sqrt{B_d}}$$

$$\xi = 1.16 \pm 0.05$$

$$\frac{\Delta m_s}{\Delta m_d} = \frac{m_{B_s}}{m_{B_d}}\xi^2\left|\frac{V_{ts}}{V_{td}}\right|^2 \gg 1$$

B_s oscillations are very fast.
Current limit: $\Delta m_s > 14.4\ \text{ps}^{-1}$.

Matrix form of the CP operator for 2-state system

$$[CP, H] = 0 \qquad (CP \text{ is conserved})$$

What does this statement imply for CP violation in oscillations?

$$CP\left|P^0\right\rangle = e^{i\theta_{CP}}\left|\bar{P}^0\right\rangle$$

$$CP\left|\bar{P}^0\right\rangle = e^{-i\theta_{CP}}\left|P^0\right\rangle$$

$$CP \rightarrow \begin{pmatrix} 0 & e^{-i\theta_{CP}} \\ e^{i\theta_{CP}} & 0 \end{pmatrix}$$

CP conservation is equivalent to $(CP)^{-1} H (CP) = H$

$$\begin{pmatrix} 0 & e^{-i\theta_{CP}} \\ e^{i\theta_{CP}} & 0 \end{pmatrix}\begin{pmatrix} 0 & H_{12} \\ H_{21} & 0 \end{pmatrix}\begin{pmatrix} 0 & e^{-i\theta_{CP}} \\ e^{i\theta_{CP}} & 0 \end{pmatrix} = \begin{pmatrix} 0 & e^{-2i\theta_{CP}} H_{21} \\ e^{2i\theta_{CP}} H_{12} & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 0 & H_{12} \\ H_{21} & 0 \end{pmatrix}$$

(Only need to look at off-diag components of **H**.)

$B^0\overline{B}^0$ coherent wave function at the Y(4S)

The B^0 and B^0 mesons are produced in a coherent quantum state.

$$\Upsilon(4S) \to \underbrace{B^0\overline{B}^0}$$

must be in a C= -1 state, since the Y(4S) decay is a strong interaction process and conserves C.

$$\left|\Psi(t_1,t_2)\right\rangle_{C=\pm1} = \frac{1}{\sqrt{2}}\left(\left|B^0(t_1);\vec{p}\right\rangle\left|\overline{B}^0(t_2);-\vec{p}\right\rangle \pm \left|\overline{B}^0(t_1);\vec{p}\right\rangle\left|B^0(t_2);-\vec{p}\right\rangle\right)$$

Major implications

1. The asymmetry between the time-integrated decay rates is zero! At the Y(4S), you must measure Δt to perform a useful CP asymmetry measurement.
2. The two neutral B mesons oscillate coherently until one of them decays. (example of Einstein-Podolsky-Rosen paradox)

Calculating λ for specific final states

$$B^0 \to \pi^+\pi^- \qquad (b \to u\bar{u}d)$$

$$\lambda = \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \cdot \frac{V_{ud}^* V_{ub}}{V_{ud} V_{ub}^*} \qquad \mathrm{Im}(\lambda) = \sin(2\alpha)$$

(assuming only tree diagram for illustration)

$$B^0 \to J/\psi K_S^0 \qquad \lambda = (-1) \cdot \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \cdot \frac{V_{cs}^* V_{cb}}{V_{cs} V_{cb}^*} \cdot \frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*} \qquad \mathrm{Im}(\lambda) = \sin(2\beta)$$

$$(b \to c\bar{c}s) \times (K^0 \to K_S^0)$$

$$B^0 \to J/\psi K_L^0 \qquad \lambda = (+1) \cdot \frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*} \cdot \frac{V_{cs}^* V_{cb}}{V_{cs} V_{cb}^*} \cdot \frac{V_{cd}^* V_{cs}}{V_{cd} V_{cs}^*} \qquad \mathrm{Im}(\lambda) = -\sin(2\beta)$$

$$(b \to c\bar{c}s) \times (K^0 \to K_L^0)$$

Angles of the unitarity triangle

Consider two complex numbers z_1 and z_2.

$$z_1 = |z_1| e^{i\theta_1} \qquad z_2 = |z_2| e^{i\theta_2} \quad \Rightarrow \quad \frac{z_2/|z_2|}{z_1/|z_1|} = e^{i(\theta_2-\theta_1)} \qquad \arg\left(\frac{z_2}{z_1}\right) = \theta_2 - \theta_1$$

$$\alpha = \arg\left(-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right)$$

$$\beta = \arg\left(-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right)$$

$$\gamma = \arg\left(\frac{-V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right)$$