1 Noise hit rate

1.1 Expected frequency of noise hits

With an ideal comparator the problem of estimating at which frequency comparator changes its state due to electronic noise, can be solved as follows [1]. Let's denote the noise process with $\mathbf{x}(t)$, by τ_i the time instances when $\mathbf{x}(t)$ crosses the threshold V_a and with $n_a(T)$ the number of points τ_i in an interval T. Writing $n_a(T)$ as

$$n_a(T) = \int_t^{t+T} \sum_i \delta(t - \tau_i) dt \tag{1}$$

then its expectation value is

$$\begin{aligned} E\{n_a(T)\} &= TE\{\sum_i \delta(t - \tau_i)\} & (2) \\ &= TE\{|\mathbf{x}'(t)|\delta[\mathbf{x}(t) - V_a]\} & (3) \end{aligned}$$

using the fundamental relation

$$E\{g(\mathbf{x}, \mathbf{y})\} = \int_\infty \int_\infty g(x, y) f(x, y)\ dxdy \tag{4}$$

where $f(x, y)$ is the joint of density of $\mathbf{x}(t)$ and $\mathbf{y}(t)$, we get:

$$\begin{aligned} E\{n_a(T)\} &= T \int \int |x'|\delta(x - V_a) f(x, x')\ dxdx' & (5) \\ &= T \int |x'| f(V_a, x')\ dx' & (6) \\ &= T f_x(a) \int |x'| f(x'|V_a)\ dx' & (7) \end{aligned}$$

where $f(x'|x)$ is the conditional density of $\mathbf{x}(t)$ and $\mathbf{x}'(t)$. Note that the conditional mean $E\{|\mathbf{x}'(t)||\mathbf{x}(t) = V_a\}$ is the average of the slopes $|\mathbf{x}'(t)|$ of all processes that cross threshold V_a at time t.

Application of equation 7 to a normal differentiable process with auto-correlation $R(\tau)$ and with zero mean, gives for the level crossing density λ_a [1]:

$$\lambda_a = \frac{1}{\pi}\sqrt{\frac{-R''(0)}{R(0)}} \cdot e^{-V_a^2/2R(0)}. \tag{8}$$

The frequency ν_a of noise hits is the half of the crossing density λ_a:

$$\nu_a = \frac{\lambda_a}{2} = \nu_0 e^{-V_a^2/2R(0)}. \tag{9}$$

where we have defined the noise hit rate at zero threshold ν_0

$$\nu_0 = \frac{1}{2\pi}\sqrt{\frac{-R''(0)}{R(0)}} \tag{10}$$

We used different methods to predict ν_a:

1. $R(\tau)$ is obtained by the noise measured at the shaper pad[1]; $R(0)$ and $R''(0)$ are calculated by approximating $R(\tau)$ near zero with:

$$R(\tau) = R(0) \cdot \left(1 - \frac{\tau^2}{2\sigma^2}\right) \tag{11}$$

2. noise hit rate is estimated directly, i.e. without using equation 9, by applying a comparator model to the noise measured at the shaper pad.

Figure 1 shows how $\sqrt{-R''(0)/R(0)}$ is estimated by the normalized autocorrelation functions of noise probed at the shaper pad for 100, 200, 300 and 400 ns peaking time. Figure 2 plots ν_a, obtained with method 1, as a function of threshold for the different peaking times. Results of method 2 are shown in figure 3.

1.2 Noise hit rate measurement

Frequency of noise hits at a given threshold is measured by probing comparator pad for relatively long time intervals and then counting the number of fires. Figure 4 shows the results of this measurement for a channel with C_D=33 pF.

1.3 Digital noise hit rate measurement

The firing efficiency f_a determined at the digital output for different thresholds V_a, is given by the probability of observing at least one low-high transition during the jitter time τ:

$$f_a = \mathcal{P}_a(n \geq 1, \tau) = 1 - \mathcal{P}_a(0, \tau) \tag{12}$$

If we assume a Poisson distribution for the number n of hits in a given time τ then

$$\mathcal{P}_a(n, \tau) = \frac{\mathrm{e}^{-\bar{n}_a} \cdot \bar{n}_a^n}{n!} \tag{13}$$

[1]If one knows the weight of the different kinds of input noise, then $R(\tau)$ can be determined by the knowledge of the δ response of the shaper. The results we obtained with this calculation are consistent with those of method 1.

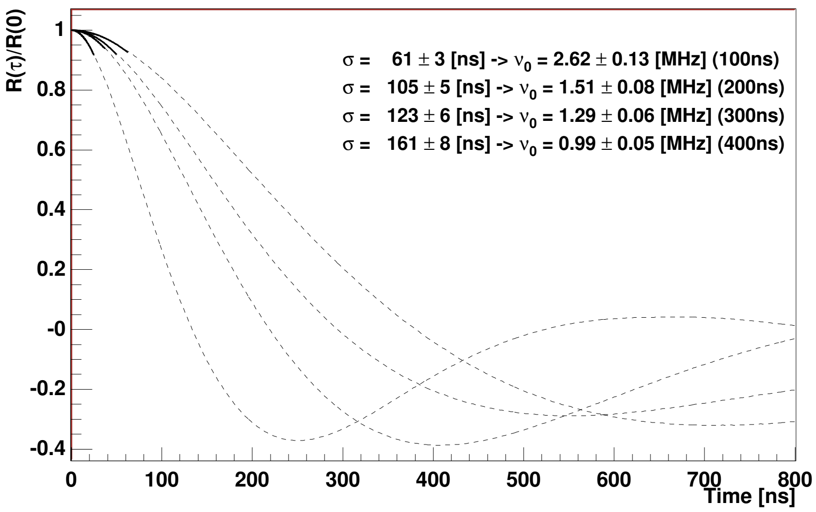

Figure 1: Normalized autocorrelation function of noise probed at the shaper pad for 100, 200, 300 and 400 ns peaking time. $\sqrt{-R''(0)/R(0)}$ is obtained by a parabolic fit of the autocorrelation function. C_D=33 pF.

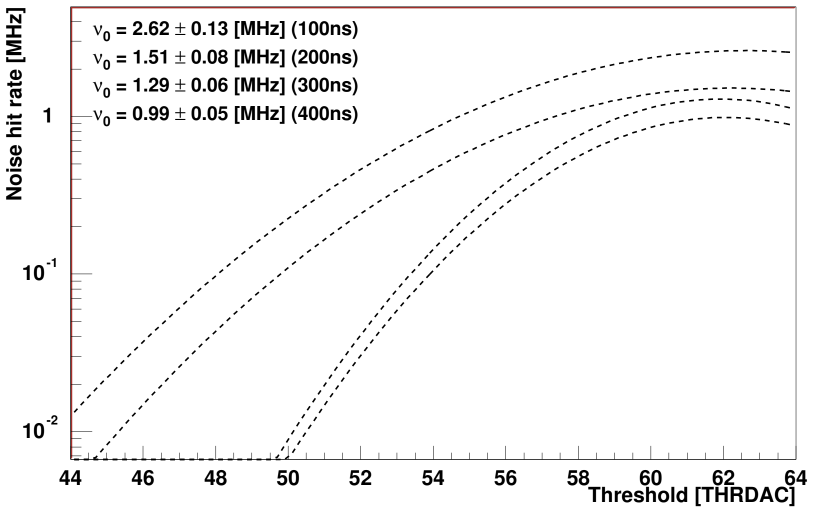

Figure 2: Expected hit rate frequency as a function of threshold for 100, 200, 300 and 400 ns peaking time. ν_0 is estimated by the autocorrelation function of the noise measured at the shaper pad. C_D=33 pF.

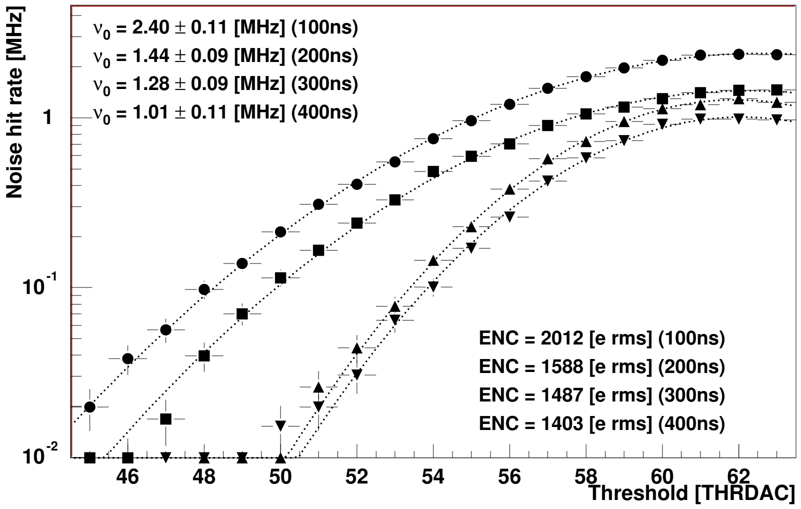

Figure 3: Expected hit rate frequency as a function of threshold for 100, 200, 300 and 400 ns peaking time. For each threshold hit frequency is estimated by applying a comparator model to the noise measured at the shaper pad. C_D=33 pF.

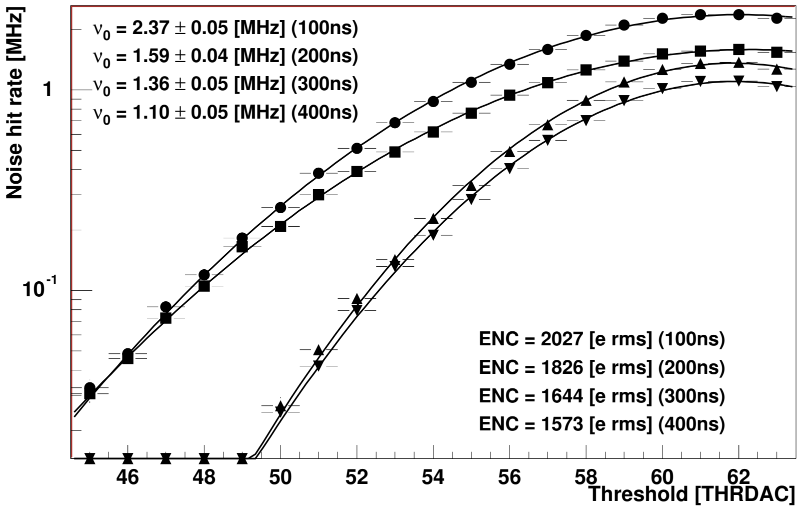

Figure 4: Measured noise hit rate as a function of threshold at 100, 200, 300 and 400 ns peaking time. C_D=33 pF.

where the mean number of hits $\bar{n}_a$ is related to the noise hit frequency by $\bar{n}_a = \tau \cdot \nu_a$. Hence:

$$f_a = 1 - \mathrm{e}^{-\tau \cdot \nu_a} \tag{14}$$

$$\nu_a = \frac{1}{\tau} \cdot \ln \left(\frac{1}{1 - f_a} \right) \tag{15}$$

This way of determining ν_a can be more accurate than the one of section 1.2, because there is no need to interfere with the system through an external probe. Four observations can be done to this approach:

1. if $\tau \nu_a \ll 1$ then $f_a \approx \tau \cdot \nu_a$ and f_a is Gaussian like ν_a;

2. hits which are shorter than half clock can be lost during the pipe line sampling; at 100 ns the fraction of hits at zero threshold which is less than 30 ns, is 0.5% (see figure 5.a);

3. previous discussion is based on the assumption of a Poisson distribution for n; this hypothesis is verified for values of $\tau < \sigma$ (where σ is the width of the noise autocorrelation function); figure 5.b plots $\mathcal{P}_a(0, \tau)$ from equation 13 versus $\mathcal{P}_a(0, \tau)$ measured at the comparator for τ=134 ns (i.e. jitter=2 at the nominal sampling speed of 15 MHz);

4. since the error on the measure of ν_a is

$$\delta \nu_a = \frac{\partial \nu_a}{\partial f_a} \delta f_a = \frac{1}{\tau} \left(\frac{1}{1 - f_a} \right) \delta f_a \tag{16}$$

   one needs to have $f_a < 1$ in order to determine ν_a; this request can be accomplished by choosing small values of τ.

References

[1] Athanasios Papoulis, *Probability, random variables and stochastic processes*, 3rd ed., McGraw Hill (1991)

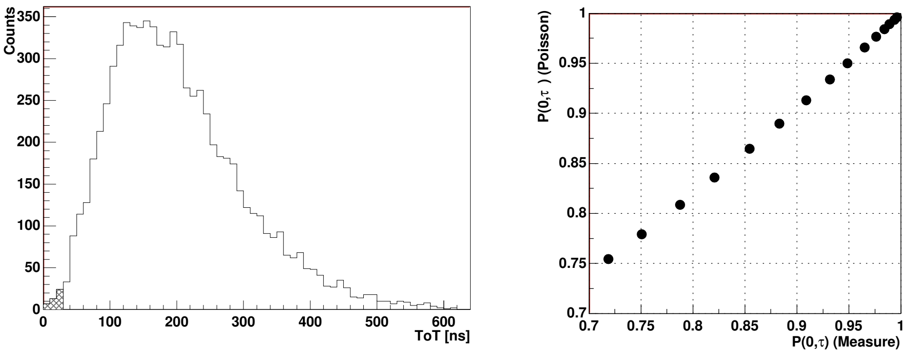

Figure 5: ToT distribution at 100 ns peaking time and zero threshold (shaded area is for ToT $<$ 30 ns; C_D=33 pF) (a). Comparison of $\mathcal{P}_a(0,\tau)$ from equation 13 with $\mathcal{P}_a(0,\tau)$ measured at the comparator (τ=134 ns; C_D=33 pF) (b).