Measuring the Coefficient of Quadratic Drag In Air

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Abstract:

An experiment designed to determine the quadratic drag coefficient in air acting on a spherical object. Carried out by using a simple pendulum and a motion detector to record the velocity of the pendulums counterweight. The experiment yielded $\gamma = (0.212 \ \pm 0.049)\frac{Ns^2}{m^4}$, which was $(15 \pm 11.1)\%$ less than the cited[1] quadratic drag coefficient $\gamma_{air} = 0.250\frac{Ns^2}{m^4}$.

[1] Taylor, John R.; pg. 46, *Classical Mechanics*, University Science Books, 2005

Introduction

When an object moves through a viscous medium, it experiences a retarding force. This retarding force is a nonconservative force known as the drag force. The drag force, primarily, has two components known as linear drag and quadratic drag. Objects that have a low cross-sectional area in the direction of motion (**v**) and a long, slender shape tend to be dominated by the linear drag force[2]. Conversely, objects that have a large cross-sectional area in its direction of motion tend to be dominated by the quadratic drag force. The quadratic drag force (QDF) retards the objects movement due the objects displacement of the viscous medium (air, in this case). The QDF is proportional to the objects velocity squared,

$$\boldsymbol{f}_{quadratic} = c\boldsymbol{v}^2, \quad \text{where} \quad c = \gamma D^2 \quad \text{(1) (for a spherical object)}$$

where D is the diameter of the object and γ is the coefficient of quadratic drag.

Incorporating the QDF into the equations of motion, we have:

$$m\dot{\boldsymbol{v}} = m\boldsymbol{a} - \gamma D^2 \boldsymbol{v}^2 \quad (2)$$

We can see this is a nonlinear differential equation. Equation 2 cannot be solved in terms of elementary functions when **v** is coupled in a two dimensional manner, v_x and v_y. However, when v_x and v_y we can solve the equation of motions in terms of elementary functions. In uncoupling dimensional dependence, it simplifies the system greatly. If we consider a real system, such as this experiment using a pendulum with a sphere attached as the counterweight, we can isolate our measurements to where the system is uncoupled and only consider the horizontal motion. In doing so, we find the equations of motion to follow

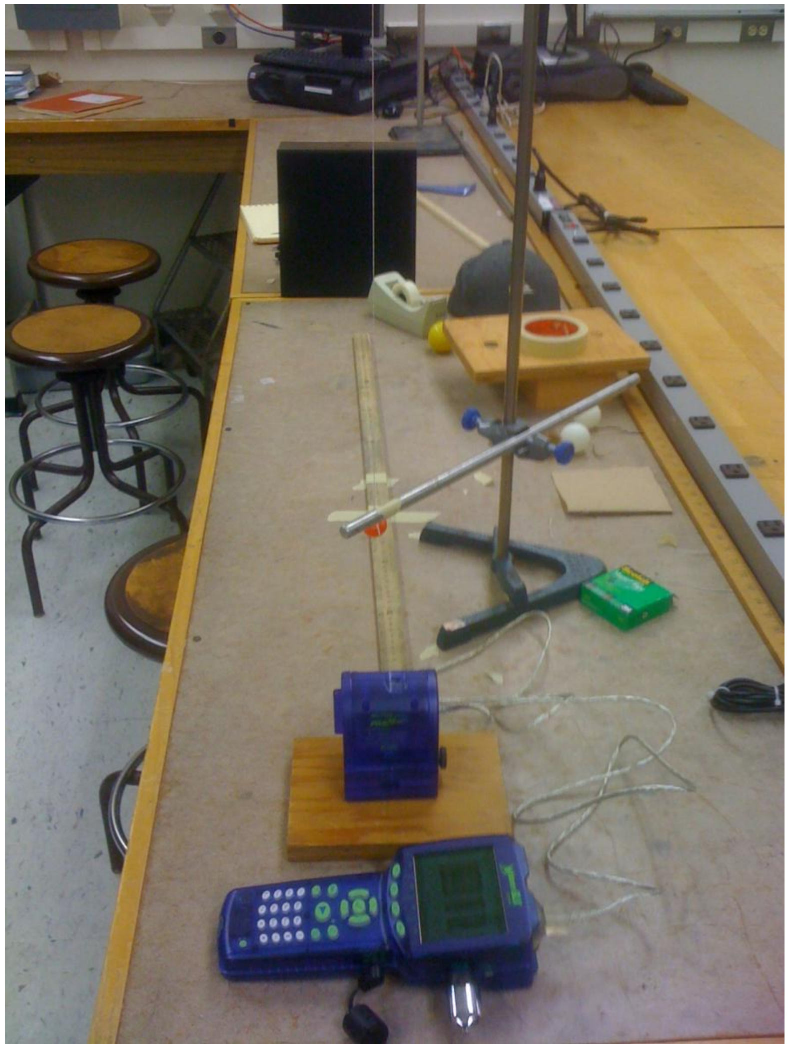
$$m\dot{v}_x = -\gamma D^2 {v_x}^2 \quad (3)$$

which states the only acting force on the system at that specific moment is the QDF, the force of interest.

Materials and Methods

[2] Taylor, John R.; pg. 46, *Classical Mechanics*, University Science Books, 2005

Figure 1- Experiment Setup



The purpose of this experiment was to measure this coefficient of quadratic drag in air, γ. This was accomplished by setting up a simple pendulum system as depicted in Figure 1. The pivot of the pendulum was simply the string which held the counterweight, tired around a metal bar fixed to the stand of the pendulum. The counterweight was a rubber bouncy ball, with a measured mass of (0.02464 ± 0.0001) kg and diameter $(0.03705 \ \pm .0001)$ meters, which was taped to the string of the pendulum. The pendulum was set to have a near constant release angle of 12°, so using a small angle approximation in this system would be valid. This was done by measuring the length of the string and it's horizontal maximum value, such that $\frac{x}{L} = \theta \approx \sin\theta = 12°$. A protruding metal bar was attached to the stand so that a fixed height was established, and a mark on the bar where the string should be touching for a constant release position. Also, attached to the counterweight was a small string "tail" that was used to pull the counterweight back, minimizing shake during release. Below the path of the pendulums arc was a rule that was taped to the table top, to visually ensure there was little to no sway. If there appeared to be large amounts of sway of the pendulums arc in a specific run, it was noted and that data was not used in the final analysis. At one end of the arc, a sonic motion detector was stationed, recording position and velocity simultaneously (PASCO Xplorer GLX). The data recorded by the motion detector was used to fit a decaying exponential function to the damping envelope of the velocity. This gave an equation for the exponentially decaying amplitude of the counterweights velocity. A derivative of this velocity gave the acceleration at the bottom point of the pendulum, where all of the energy was in the form of kinetic energy and the force due to gravity need not be considered since all motion at this point was horizontal. Deriving both a(t) and v(t) allows one to solve for γ as a function of time

$$\gamma(t) \ = -\frac{m\dot{v}_x(t)}{D^2{v_x}^2(t)} \ (4).$$

The data was fitted using PASCO's DataStudio. The author used a built in function found in DataStudio to obtain the points of the amplitude peaks. The function found the largest value within a specified period over the whole data set. Once this was data was recovered, it was fit using DataStudio's decaying exponential equation fit. The parameters for all significant runs were averaged and described by

$$v_x(t) = Ae^{-Ct} + B \quad (5)$$

$$\dot{v_x}(t) = -ACe^{-Ct} \quad (6)$$

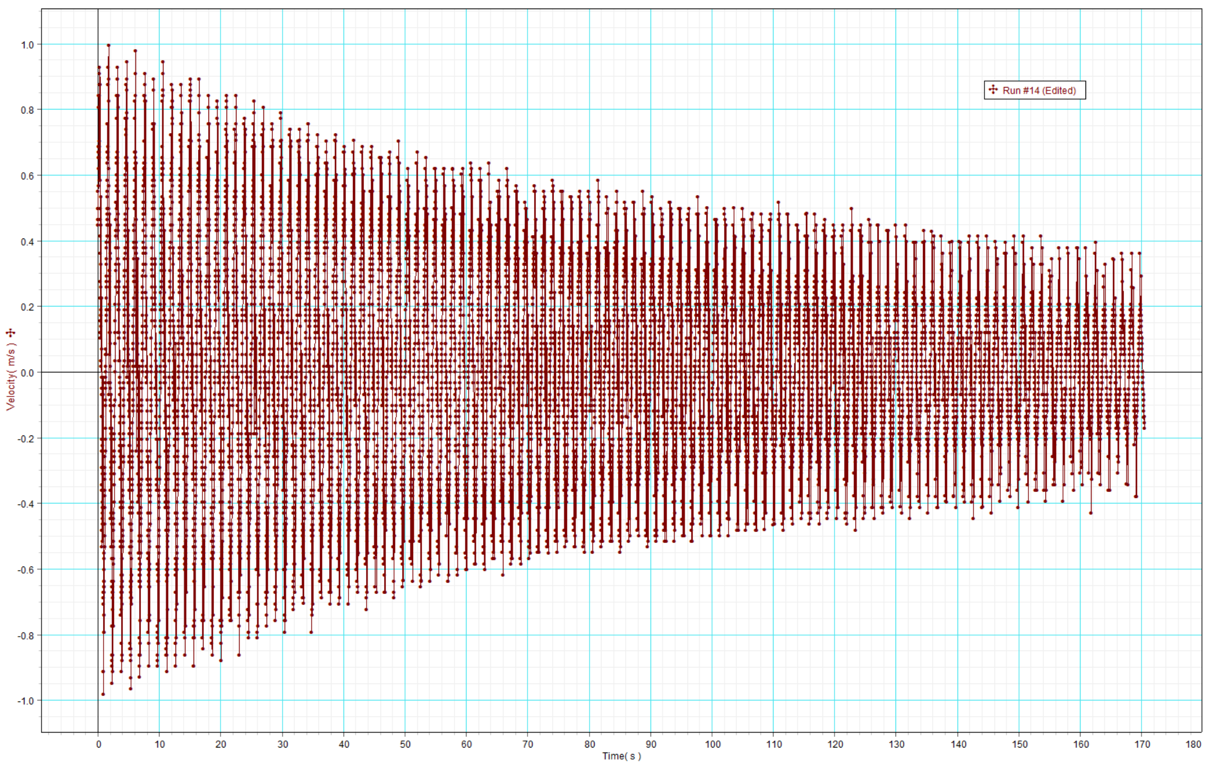


Figure 2 - Velocity vs Time data

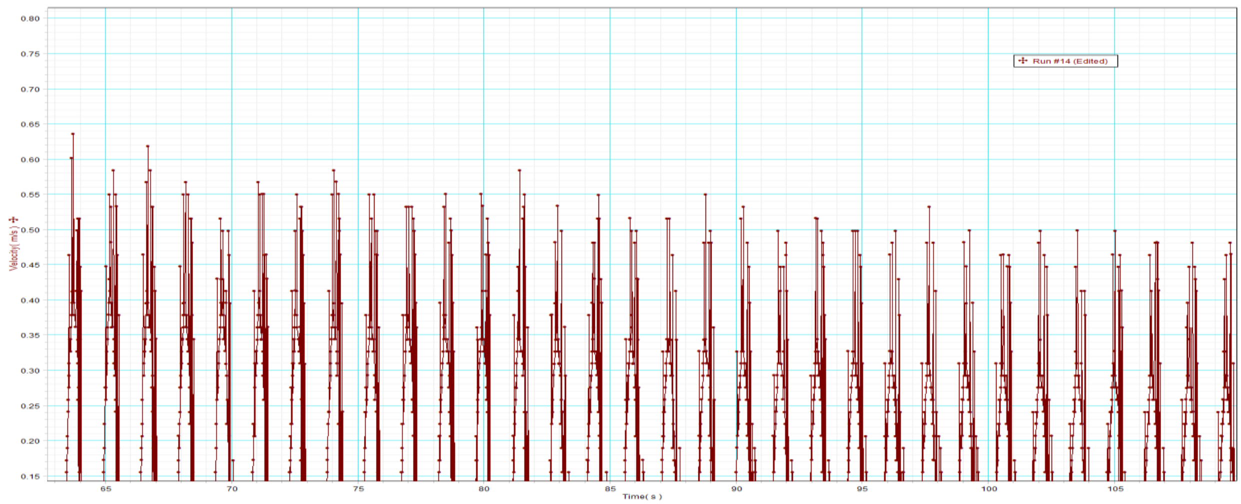


Figure 3 - Close-up of Velocity vs Time data

Analysis

The experiment yielded $\gamma(90s) = (0.212\ \pm 0.049)\frac{Ns^2}{m^4}$ at 20.5°C and 996 hPa (only one measurement for temperature and pressure over a matter of hours). The standard value for $\gamma_{air}\ = 0.250\frac{Ns^2}{m^4}$, at STP[3]. This gives a relative percentage error of $-(15 \pm 11.1)\%$. The author believes the large error on γ can be attributed to the algorithm used to obtain the amplitude peaks of the velocity. The algorithm used was y=(10,10,1.52,x) where 1.52 is the period, T, found to exist in the system. However, this peak to peak period T varies within a single data set of velocity by about $\pm\ 0.06$. Coupled with the noise found in the velocity data (see Figure 4), the peak detecting algorithm may have miscalculated some of the actual peak values, obtaining slightly larger or lower values based on where the period began and ended during the calculation. This changing period most likely was attributed to small side-to-side swaying on the pendulums arc of motion. Another large error was likely due to the lack of involvement of the friction of the pivot (string on metal), effectively making the damping coefficient greater.

[3] Taylor, John R.; pg. 46, *Classical Mechanics*, University Science Books, 2005

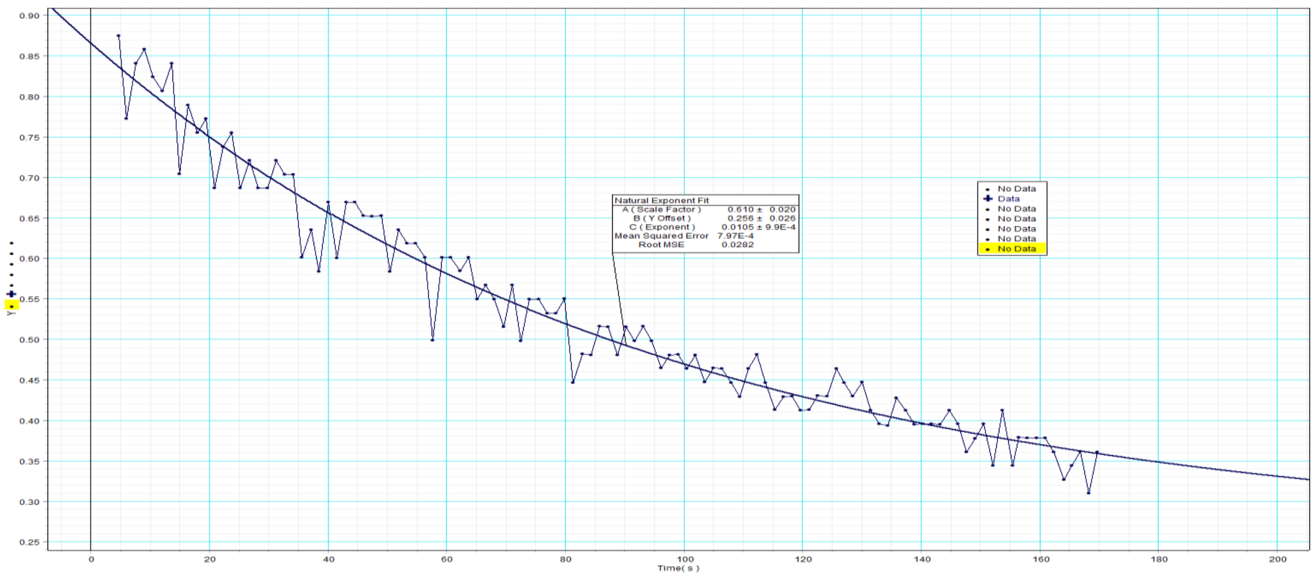
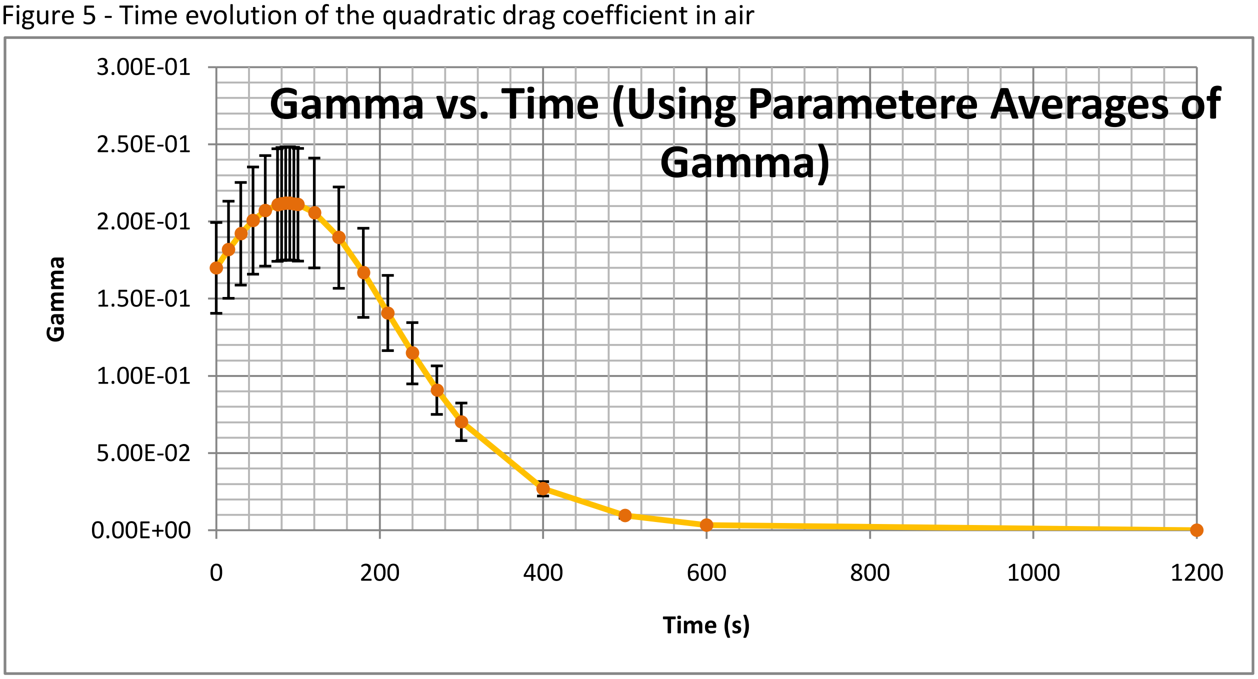


Figure 4- Decaying Exponential of V vs T envelope

Since the author chose to use the shown method using decaying exponentials to describe the velocity, $\gamma(t)$ inherently varied with time. Thus, $\gamma(t)$ was graphed using discrete time intervals (see Figure 5). An unexpected curve was shown to describe the time evolution of $\gamma(t)$. It appears to reach a maximum value near 90 seconds, which is why the author chose to use $\gamma(90s)$ for the experimental value. $\gamma(t)$ displays a near constant value for approximate range of range $75sec \leq t \leq 100sec$, only varying 0.001 $\frac{Ns^2}{m^4}$ in this range.

Figure 5 - Time evolution of the quadratic drag coefficient in air



Conclusion

The experimenter determined $\gamma_{air} = (0.212 \pm 0.049)\frac{Ns^2}{m^4}$. The methods used were simple and effective, making a time efficient procedure for roughly determining the quadratic drag coefficient in a viscous medium. However, a more precise methodology for measuring the coefficient is desired. Specifically, if one can uncouple the motion, then one can solve for the coefficient directly as a function of horizontal velocity only through the equation

$$v(t) = \frac{v_0}{1 + t\gamma D^2 v_0/m}$$

which is a result of solving the first order differential found in equation 3[4]. This method of solving of solving for γ directly was unattainable by the author do due lack of software capabilities for data analysis and time constraints.

[4] Taylor, John R.; pg. 46, *Classical Mechanics*, University Science Books, 2005