Section Exercises: Week of 05/28

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PHYS 125: Elementary Particle Physics April 2, 2014

Exercise 1. From Griffiths

(a) Construct the Hamiltonian for the Dirac equation. Begin with the Dirac equation in position space,

$$i\hbar\gamma^\mu\partial_\mu\psi - mc\psi = 0 \tag{1}$$

Transform the equation into momentum space and then solve for $H \equiv E = cp^0$.

(b) Show that the orbital angular momentum operator, $\mathbf{L} \equiv \mathbf{r} \times \mathbf{p}$, does not commute with H. What does this result tell us about conservation of orbital angular momentum under time evolution?

(c) Now introduce the spin angular momentum operator,

$$\mathbf{S} \equiv \frac{\hbar}{2}\vec{\Sigma} = \frac{\hbar}{2}\begin{pmatrix}\vec{\sigma} & 0\\ 0 & \vec{\sigma}\end{pmatrix} \tag{2}$$

Show that the total angular momentum operator, $\mathbf{J} \equiv \mathbf{L} + \mathbf{S}$, commutes with H. Is this a conserved quantity?