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On D→ ππ and $K\bar{K}$ Decays

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Abstract

We have analysed the recent data on D→ ππ and $K\bar{K}$ decays from ARGUS, CLEO and E691.

(i) We find that Bauer, Stech and Wirbel (BSW) model does fairly well in explaining all data if we assume that final state interactions simply rotate the amplitudes in the complex plane. However, to get a good fit one needs to raise D→ K transition amplitudes by (5-10)% over the BSW values and lower D→ π amplitudes by a similar amount.

(ii) We show that all the data can also be understood in a coupled channel inelastic model.

(iii) We also find that the Isgur, Scora, Grinstein and Wise (ISGW) model - form factors, if extrapolated to small momentum transfers (t) by a single pole formula, yield branching ratios too small in comparison to data. We propose an alternate scheme to interpolate between BSW form factors at t = 0 and ISGW form factors at $t = t_{max}$.

I. Introduction

Considerable new data on Cabibbo-angle-suppressed $D^o \to \pi\pi$ and $K\overline{K}$ decays have recently emerged from ARGUS, CLEO and E691 experiments which we summarize below: (In arriving at these branching ratios, in percent, we have used [1] $B(D^o \to K^-\pi^+) = (4.2\pm0.6)\%$ and added all errors in quadrature.)

$$\begin{aligned}
B(D^o \to \pi^+\pi^-) &= (0.17\pm0.05)\ \text{ARGUS}\ [2,3] \\
&= (0.21\pm0.05)\ \text{CLEO}\ [1,4,5] \\
&= (0.25\pm0.06)\ \text{E691}\ [6] \\
B(D^o \to \pi^o\pi^o) &= (0.09\pm0.03)\ \text{CLEO}\ [4,5,7] \\
B(D^o \to K^+K^-) &= (0.42\pm0.11)\ \text{ARGUS}\ [2,3] \\
&= (0.49\pm0.08)\ \text{CLEO}\ [1,5] \\
&= (0.48\pm0.09)\ \text{E691}\ [6] \\
B(D^o \to K^o\overline{K}^o) &= (0.13\ ^{+\ 0.07}_{-\ 0.05})\ \text{CLEO}\ [1,5]\ . \qquad (1)
\end{aligned}$$

In (1), above, we have not listed the older data [8,9,10] where $B(D^o \to \pi^+\pi^-)$ were generally lower. In contrast, the earlier measurements of $B(D^o \to K^+K^-)$ [8,9,10] do not differ significantly from the recent data listed in (1) above. This has led to a lowering of the ratio $B(D^o \to K^+K^-)/B(D^o \to \pi^+\pi^-)$ over the years, to a value closer to 2 in contrast to older measurements of the ratio of about 3.5 [8].

Now that $B(D^o \to \pi^+\pi^-)$ and $B(D^o \to \pi^o\pi^o)$ are available, a measurement of $B(D^+ \to \pi^+\pi^o)$ would enable us to carry out an amplitude analysis of $D \to \pi\pi$. We emphasize the need for this measurement.

Theoretically $D^o \to \pi\pi$ and $K\overline{K}$ decays have been extensively studied, with the inclusion of Final State Interactions (FSI, henceforth) in refs. [11,12,13], albeit before the new data, particularly those from CLEO, were available. Since the availability of these data, particularly $D^o \to \pi^o\pi^o$, Chau and Cheng [14] have discussed this problem in their diagrammatic approach and

concluded that FSI phases and some SU(3) breaking effects, allow one to understand $D\to\pi\pi$ and $K\overline{K}$ branching ratios.

Corroborative evidence for this conclusion was also arrived at in refs. [11,12] where it was shown that if one assumes *SU(3) symmetry* and that FSI *simply rotate the amplitude in the complex plane*, then one could generate $B(D^o\to K^+K^-)/B(D^o\to\pi^+\pi^-)$ as high as ≈ 2 but in so doing $B(D^o\to K^o\overline{K}^o)$ would have to be $< 0.02\%$. Since then CLEO data [1,5] have shown that $B(D^o\to K^o\overline{K}^o)$ is considerably larger. This implies that *SU(3) symmetry* with FSI phases which *simply rotate* the amplitude would not satisfy all of $D\to\pi\pi$ and $K\overline{K}$ data.

We wish to clarify one other point. There is a common misconception in the literature that *elastic* FSI do not change the magnitude of the weak decay amplitudes but simply rotate them in the complex plane. This is not so [13]. The consequence of an elastic FSI is to make the *phase of the decay amplitude equal to the phase of the scattering amplitude.* The assumption that the magnitude of the weak decay amplitude does not change is *extra* to the assumption of elastic FSI and would follow from the assumption that the scattering is "weak", i.e., the scattering phases are small [13].

In [13] the decays $D\to\pi\pi$ and $K\overline{K}$ were studied in a coupled channel formalism and it was shown that constraining the parameters of the scattering K-matrix to fit $\pi\pi$-*scattering 'data'* around the D-mass, one could understand all the data *except* $B(D^o\to\pi^o\pi^o)$ which was not available at that time. Later, in this paper, we describe a re-analysis of the inelastic FSI model introduced in [13] and show that the data on $D\to\pi\pi$ and $K\overline{K}$ can also be understood in an inelastic FSI model.

We have organized the discussion of $D\to\pi\pi$ and $K\overline{K}$ problem as follows: In Section II we discuss the problem in the factorization model of Bauer, Stech and Wirbel (BSW, henceforth) [15], which has built-in SU(3) breaking, with the assumption that FSI simply rotate the BSW isospin amplitudes. The material in Section II is a restatement and reanalysis of the relevant parts of [13].

In Section III we reanalyse the inelastic FSI model for $D\to\pi\pi$ and $K\overline{K}$ decays employed in [13] and show that all the data can be understood in an inelastic FSI model too.

In Section IV we discuss the calculation of the hadronic matrix elements relevant to $D\to\pi\pi$ and $D\to K\bar{K}$ decays in the Isgur, Scora, Grinstein and Wise [ISGW, henceforth] model [16] and make comments on the predictions of this model. We conclude with Section V.

II. $D\to\pi\pi$ and $K\bar{K}$ in the BSW model with FSI phases

Many of the details of the material in this sub-section can be found in [13]. The weak decay Hamiltonian relevant to these Cabibbo-angle-suppressed decays, in BSW notation, is

$$H_w = \frac{G_F}{\sqrt{2}}\left[V_{ud}V_{cd}^*\{a_1(\bar{u}d)_H(\bar{d}c)_H + a_2(\bar{u}c)_H(\bar{d}d)_H\} + V_{us}V_{cs}^*\{a_1(\bar{u}s)_H(\bar{s}c)_H + a_2(\bar{u}c)_H(\bar{s}s)_H\}\right] \quad (2)$$

where we choose the QCD coefficients to be

$$a_1 = 1.2 \ , \qquad a_2 = -0.5 \ . \quad (3)$$

Wirbel [17] assigns them an error of ±0.1 each. The color-singlet quark bilinear brackets stand for (V–A) bilinears and the subscript H instructs us to treat the quark bilinears as interpolating fields for hadrons.

In the factorization model one finds the following decay amplitudes in absence of FSI (note that the annihilation terms vanish due to conserved vector current hypothesis (CVC) and a common factor of $G_F/\sqrt{2}\ \sin\theta_c \cos\theta_c$, where θ_c is the Cabibbo angle with $\sin\theta_c = 0.23$, is being omitted),

$$A(D^o\to\pi^+\pi^-) = -\ a_1F_\pi f_+^{D\to\pi}(m_\pi^2)\ (m_D^2 - m_\pi^2)$$

$$A(D^o\to\pi^o\pi^o) = \ a_2F_\pi f_+^{D\to\pi}(m_\pi^2)\ (m_D^2 - m_\pi^2)$$

$$A(D^o\to K^+K^-) = a_1F_K f_+^{D\to K}(m_K^2)\ (m_D^2 - m_K^2)$$

$$A(D^o\to K^o\bar{K}^o) \ = \ 0 \quad (4)$$

where we use $F_\pi = 133$ MeV, $F_K = 162$ MeV. The contribution of the form factor $f_-(q^2)$ is being ignored since it appears multiplied by a small mass m_π^2 or m_K^2 depending on the decay mode. In BSW [15] model the form factor $f_+(q^2)$ is calculated at $q^2 = 0$ and extrapolated via a single pole,

$$f_+(q^2) = f_+(0)/(1 - q^2/m_{1^-}^2) \qquad (5)$$

with [15]

$$f_+^{D\to\pi}(0) = 0.692 \ , \qquad f_+^{D\to K}(0) = 0.762 \ , \qquad (6)$$

m_{1^-} in (5) is the D* mass at 2.01 GeV for $f_+^{D\to\pi}$ and D_s^* mass at 2.11 GeV for $f_+^{D\to K}$. The prescription for extrapolating $f_+(q^2)$ to $q^2 \neq 0$ is not too important in these decays since the form factors are needed at $q^2 = m_\pi^2$ for $f_+^{D\to\pi}(q^2)$ and $q^2 = m_K^2$ for $f_+^{D\to K}(q^2)$, both of which are rather small excursions from $q^2 = 0$.

The decay amplitudes in (4) could also have been expressed in terms of the form factor $F_0(q^2)$ introduced in BSW [15] by a simple replacement of $f_+(q^2)$ by $F_0(q^2)$. With $F_0(0) = f_+(0)$, a single-pole extrapolation of $F_0(q^2)$ to finite q^2 would entail using a scalar charmed meson mass, with and without strangeness. Such scalar particles are as yet unobserved but their masses are anticipated to be $\approx$ 2.5–2.6 GeV [15]. Numerically it would make little difference to $D^o \to \pi\pi$ amplitudes but will depress $D^o \to K\bar{K}$ amplitudes slightly since $F_0(m_K^2) < f_+(m_K^2)$ because $m_{0^+} > m_{1^-}$.

We define the isospin decomposition of the (complex) decay amplitudes as follows [13],

$$A(D^o\to\pi^+\pi^-) = \frac{1}{\sqrt{6}} A_o^{\pi\pi} e^{i\delta_o^{\pi\pi}} + \frac{1}{\sqrt{12}} A_2^{\pi\pi} e^{i\delta_2^{\pi\pi}}$$

$$A(D^o\to\pi^o\pi^o) = \frac{1}{\sqrt{6}} A_o^{\pi\pi} e^{i\delta_o^{\pi\pi}} - \frac{1}{\sqrt{3}} A_2^{\pi\pi} e^{i\delta_2^{\pi\pi}}$$

$$A(D^+\to\pi^+\pi^o) = \frac{\sqrt{3}}{2\sqrt{2}} A_2^{\pi\pi} e^{i\delta_2^{\pi\pi}}$$

$$A(D^o\to K^+K^-) = \frac{1}{2}(A_o^{K\bar{K}} e^{i\delta_o^{K\bar{K}}} + A_1^{K\bar{K}} e^{i\delta_1^{K\bar{K}}})$$

$$A(D^o\to K^o\bar{K}^o) = \frac{1}{2}(A_o^{K\bar{K}} e^{i\delta_o^{K\bar{K}}} - A_1^{K\bar{K}} e^{i\delta_1^{K\bar{K}}}) \ . \qquad (7)$$

If we assume that FSI *simply rotate* the decay amplitudes in the complex plane, we can evaluate the isospin amplitudes by setting all the phases in (7) equal to zero and equating (7) with (4). We find (an overall factor of $G_F/\sqrt{2}\,\sin\theta_c\cos\theta_c$ is being omitted),

$$A_o^{\pi\pi} = -\left(\frac{2}{3}\right)^{1/2}(2a_1 - a_2)\,F_\pi\,f_+^{D\to\pi}(m_\pi^2)\,(m_D^2 - m_\pi^2)$$

$$A_2^{\pi\pi} = \frac{2}{\sqrt{3}}(a_1 + a_2)\,F_\pi\,f_+^{D\to\pi}(m_\pi^2)\,(m_D^2 - m_\pi^2)$$

$$A_o^{K\bar{K}} = A_1^{K\bar{K}} = a_1\,F_K\,f_+^{D\to K}(m_K^2)\,(m_D^2 - m_K^2)\ . \qquad (8)$$

Using a_1 and a_2 from (4) we find

$$\left|A_2^{\pi\pi}/A_o^{\pi\bar{\pi}}\right| = 0.34$$

$$\left|A_1^{K\bar{K}}/A_o^{K\bar{K}}\right| = 1.0\ . \qquad (9)$$

Once we substitute (8) in (7), it is evident that independent of $\delta_2^{\pi\pi}$, the fact that $|A_2^{\pi\pi}| << |A_o^{\pi\pi}|$ leads one to expect $B(D^o\to\pi^o\pi^o)/B(D^o\to\pi^+\pi^+) \approx 0.5$, as the CLEO data [4,5,7] require.

In Table I we show the calculated branching ratios as functions of $\delta_o^{\pi\pi} - \delta_2^{\pi\pi}$ and $\delta_o^{K\bar{K}} - \delta_1^{K\bar{K}}$. We have also indicated that BSW prediction for $B(D^+\to\pi^+\pi^o)$ is 0.14%. SU(3) symmetry [11,12] yields $B(D^+\to\pi^+\pi^o) \approx 0.1\%$. From Table I we find that CLEO data [4,5,7] on $D^o\to\pi^+\pi^-$ and $\pi^o\pi^o$ require $\delta_o^{\pi\pi} - \delta_2^{\pi\pi}$ in the vicinity of 90°. Chau and Cheng [14] have argued that this angle is ≈ 83°. Thus we are in agreement here. From Table I we also find that $D^o\to K^+K^-$ and $K^o\bar{K}^o$ data require $\delta_o^{K\bar{K}} - \delta_1^{K\bar{K}}$ in the vicinity of 60° which is also in agreement with Chau and Cheng [14] who have estimated this angle to be ≈ 56° though the errors in this ought to be quite large.

In absence of a measurement of $B(D^+\to\pi^+\pi^o)$ a complete amplitude analysis of $D\to\pi\pi$ decay is not possible. However, we have analysed the CLEO data on $B(D^o\to\pi^+\pi^-)$ and $B(D^o\to\pi^o\pi^o)$ with an *added theoretical constraint* from (9), $\left|A_2^{\pi\pi}/A_o^{\pi\bar{\pi}}\right| = 0.34\pm0.07$, the errors

coming from assigning an error of ± 0.1 to both a_1 and a_2 [17]. We found that CLEO data [1,4,5,7] constrain $\delta_o^{\pi\pi} - \delta_2^{\pi\pi}$ to lie in the range: $50° \leq \delta_o^{\pi\pi} - \delta_2^{\pi\pi} \leq 95°$. This range would be narrower if the theoretical error in $\left| A_2^{\pi\pi}/A_o^{\pi\bar{\pi}} \right|$ would be smaller. It was pointed out in [13] that the following sums of branching ratios are independent of FSI phases which in the BSW model are (we have used (4) for a_1 and a_2),

$$B(D^o \to \pi^+\pi^-) + B(D^o \to \pi^o\pi^o) = 0.37\%$$
$$B(D^o \to K^+K^-) + B(D^o \to K^o\bar{K}^o) = 0.52\% \ . \qquad (10)$$

CLEO data [1,4,5,7] give $(0.3\pm0.06)\%$ and $(0.62\pm0.10)\%$ respectively. One could raise the theoretical sum $B(D \to K^+K^-+K^o\bar{K}^o)$ by raising the QCD coefficient a_1 but that would also raise $B(D \to \pi^+\pi^-+\pi^o\pi^o)$. Within the BSW model an agreement of (10) with CLEO data would require a larger SU(3) breaking than that implied by the BSW calculation of $f_+^{D\to\pi}(0)$ and $f_+^{D\to K}(0)$; one would need to *raise* $f_+^{D\to K}(0)$ by (5–10)% and *lower* $f_+^{D\to\pi}(0)$ by a similar amount.

An alternative would be to consider inelastic effects so that, loosely speaking, some of the $\pi\pi$ rates could be channeled into $K\bar{K}$ decays. In this scenario one could use the BSW calculation of hadronic matrix elements but now interchannel mixing would alter the magnitudes of the weak decay amplitudes and one would be free of the constraints implied by (10). We discuss this inelastic FSI scenario in the next section.

III. A coupled channel FSI model with BSW weak amplitudes.

A. Introductory Remarks

In presence of inelastic FSI, not only will the magnitudes of the weak decay amplitudes get modulated but the weak amplitude phases, $\delta_o^{\pi\pi}$, $\delta_o^{K\bar{K}}$ etc. in (7), would, in general, not be the same as the *scattering phases* entering the description of $\pi\pi$ and $K\bar{K}$ scattering. We consider the following inelastic FSI scenario:

(i) $\pi\pi$ and $K\bar{K}$ channels couple in the final state with I=0, L=0. In this isospin-angular momentum state the known [18] $\pi\pi$ scattering phase shift is used to fix the parameters of the

scattering **K**-matrix [13] which in turn is used to unitarize the BSW weak decay amplitudes as described in [13]. In this approach the phases, $\delta_o^{\pi\pi}$ and $\delta_o^{K\bar{K}}$, in (7), of the weak decay amplitudes, become calculable quantities.

(ii) $\pi\pi$ scattering in I=2, L=0 state is assumed to be "weak" and elastic. A "weak" rescattering (with a small scattering phase shift) would imply that the magnitude of the weak decay amplitude, $A_2^{\pi\pi}$ in (7), would not be modulated appreciably and the assumption of elasticity would imply that we can use the scattering phase shift in I=2, L=0 state [19] in place of $\delta_2^{\pi\pi}$ in (7).

(iii) Not much is known about the $K\bar{K}$-scattering phase in I=1, L=0 state. It is however expected to be inelastic since this channel can couple to $\eta\pi$ channel, say, via $a_0(980)$ [8]. Though some theoretical discussion of the $K\bar{K}$- scattering phase shift in I=1, L=0 state can be found in the literature [20], we have chosen to assume that FSI do not affect the magnitude of $A_1^{K\bar{K}}$ significantly and have used its phase, $\delta_1^{K\bar{K}}$, as an input parameter.

B. Model and Method

We start with a discussion of the coupled channel scattering problem for $\pi\pi$ and $K\bar{K}$ systems in I=0, L=0 state, eventually leading up to a calculation of FSI corrected amplitudes $A_o^{\pi\pi}$ and $A_o^{K\bar{K}}$ and their phases $\delta_o^{\pi\pi}$ and $\delta_o^{K\bar{K}}$. In the following we describe the scattering problem in terms of a **K**-matrix, whose parameters are fixed through the $\pi\pi$ scattering 'data' [18]. Once this **K**-matrix is determined, we use it to unitarize the BSW amplitudes through [13],

$$\mathbf{A}^U = (\mathbf{1} - i\mathbf{k}\mathbf{K})^{-1T}\,\mathbf{A} \qquad (11)$$

where T refers to 'transpose' and **k** is a diagonal matrix with entries k_1 and kk_2, the center of mass momenta in $\pi\pi$ and $K\bar{K}$ systems. **A** is a column with the BSW amplitudes $A_o^{\pi\pi}$ and $A_o^{K\bar{K}}$ as entries and $\mathbf{A}^U$ is a column with the unitarized amplitudes, $A_o^{\pi\pi,U}$ and $A_o^{K\bar{K},U}$ as entries. The phases $\delta_o^{\pi\pi}$ and $\delta_o^{K\bar{K}}$ in (7) are the phases that emerge from (11).

A constant **K**-matrix is used [13] to describe the coupled $\pi\pi$ and $K\bar{K}$ scattering in a limited energy range around the centre of mass energy $\approx m_D$:

$$\mathbf{K} = \begin{array}{c} \\ \pi\pi \\ K\bar{K} \end{array} \begin{array}{c} \begin{array}{cc} \pi\pi & K\bar{K} \end{array} \\ \begin{pmatrix} a & b \\ b & c \end{pmatrix} \end{array} \tag{12}$$

The scattering **T**-matrix is then generated through,

$$\mathbf{T} = (8\pi W)\,\mathbf{K}\,(1 - i\mathbf{k}\mathbf{K})^{-1} \tag{13}$$

where W = total center of mass energy. The **S**-matrix is related to the **T**-matrix through [13],

$$\rho^{1/2}\,\mathbf{T}\,\rho^{1/2} = (\mathbf{S} - 1)/2i \tag{14}$$

where ρ is a diagonal matrix with entries $k_1/8\pi W$ and $k_2/8\pi W$. The **S**-matrix is finally parametrized in terms of the scattering phase shifts δ_1 and δ_2 in $\pi\pi$ and $K\bar{K}$ channels respectively and the elasticity η,

$$\mathbf{S} = \begin{pmatrix} \eta e^{2i\delta_1}, & i\,(1-\eta^2)^{1/2} e^{i(\delta_1+\delta_2)} \\ i\,(1-\eta^2)^{1/2} e^{i(\delta_1+\delta_2)} & \eta e^{2i\delta_2} \end{pmatrix} . \tag{15}$$

The parameters of the **K**-matrix are fixed by fitting the $\pi\pi$ scattering phase shift δ_1 [18]. Once the K-matrix is determined, the unitarized BSW amplitudes are generated by (11).

Next we describe how the scattering data, δ_1, δ_2 and η, are related to the **T**-matrix and the **K**-matrix elements.

From (14) and (15), one gets

$$\tan 2\delta_i = \frac{\rho_i \,\mathrm{Re}\, T_{ii}}{0.5 - \rho_i \,\mathrm{Im}\, T_{ii}} \quad , \quad (i = 1,2) \tag{16}$$

and

$$\begin{aligned} \eta^2 &= 1 - 4\rho_1 \,\mathrm{Im}\, T_{11} + 4\rho_1^2 \,|T_{11}|^2 \\ &= 1 - 4\rho_2 \,\mathrm{Im}\, T_{22} + 4\rho_2^2 \,|T_{22}|^2 \\ &= 1 - 4\rho_1\rho_2 |T_{12}|^2 \quad . \end{aligned} \tag{17}$$

To decide which quadrant to put $2\delta_1$ in, one has to look at the signs of ρ_i Re T_{ii} and $(0.5 - \rho_i \,\mathrm{Im}\, T_{ii})$. There are four such combinations: (+,+), (+,-), (-,-) and (-,+), where the signs

refer to the signs of ρ_i Re T_{ii} and $(0.5 - \rho_i$ Im $T_{ii})$ respectively. The four combinations above put $2\delta_i$ in the first, second, third or the fourth quadrant respectively.

The relation between the quantities in the **K**-matrix, (12), and the scattering parameters introduced in (16) and (17) are,

$$T_{11} = \frac{8\pi W}{\Delta}(a - ik_2 \det \mathbf{K})$$
$$T_{12} = T_{21} = \frac{8\pi W}{\Delta} b$$
$$T_{22} = \frac{8\pi W}{\Delta}(c - ik_1 \det \mathbf{K}) \qquad (18)$$

where det **K** is the determinant of the **K**-matrix and $\Delta \equiv \det(\mathbf{1} - i\mathbf{k}\mathbf{K})$. The real and imaginary parts of T_{ii} from (18) are then used to calculate tan $2\delta_i$. The elasticity, η, is given by

$$\eta^2 = 1 - \frac{4k_1^2 b^2}{|\Delta|^2} \quad . \qquad (19)$$

We constrained the parameters of the **K**-matrix by demanding that the $\pi\pi$ scattering phase shift in I=0, L=0 state, δ_1, have a zero (modulo 2π) at W = 1.6 GeV and reach a value of 45° (modulo 2π) at W = 1.87 GeV [18]. These two conditions constrain two of the three parameters, a, b and c; we chose to use det K $\equiv$ ac - b^2 as a free parameter. We could, in principle, also fix the third parameter by fitting the elasticity but we found that an energy-independent parametrization of the **K**-matrix does not have enough flexibility to allow us to do this. Thus the elasticity became a *calculated* quantity.

In Table II we have shown a particular fit to the $\pi\pi$ 'data' [18] with a set of parameters: det **K** = -1.65 GeV^{-2}, a = -0.86 GeV^{-1} and c = -1.32 GeV^{-1}. Clearly, we can fit $\pi\pi$ scattering phase shift, δ_1, in the energy range 1.5 GeV $\leq$ W $\leq$ 2.0 GeV. We have also calculated δ_2, the $\overline{KK}$ scattering phase shift in I=0, L=0 state. This phase (determined modulo π, i.e., $\pi+\delta_2$ is also a solution) has the correct qualitative behaviour [21]. The elasticity is probably too low in

comparison with the 'data' [18], but with an energy-independent **K**-matrix we found that it was not possible to raise η closer to the values needed by 'data' [18].

In Table III we have shown a few calculated values of the unitarized BSW weak decay amplitudes $A_o^{\pi\pi}$ and $A_o^{K\bar{K}}$. The calculated phases of these weak decay amplitudes are also shown. Typically, for the values of $A_o^{\pi\pi}$ and $A_o^{K\bar{K}}$ shown in Table III, we find that the effect of unitarization was to depress the value of $|A_o^{\pi\pi}|$ and to raise the value of $|A_o^{K\bar{K}}|$. For the values shown in Table III we found,

$$|A_o^{\pi\pi}| = (0.76 - 0.80)\,|A_o^{\pi\pi}|_{BSW}$$
$$|A_o^{K\bar{K}}| = (1.13 - 1.23)\,|A_o^{K\bar{K}}|_{BSW}\ . \qquad (20)$$

Note that the calculated phase, $\delta_o^{\pi\pi}$, of the weak decay amplitude $A_o^{\pi\pi}$ is markedly different from the $\pi\pi$ scattering phase shift δ_1 which was 45° at the D-mass. Same remarks apply to $\delta_o^{K\bar{K}}$ which is very different from δ_2, the scattering phase shift in I=0, L=0 $K\bar{K}$ scattering.

Finally, in Table IV we have listed a sampling of the calculated branching ratios. We have treated the phase, $\delta_1^{K\bar{K}}$, of the weak decay amplitude $A_1^{K\bar{K}}$, as a free parameter. We chose two values for the phase, $\delta_2^{\pi\pi}$, of the amplitude $A_2^{\pi\pi}$: -45° and -60°. 'Experimental' value of the scattering phase shift is about -30° at W = 1.2 GeV [19].

Clearly the simple inelastic model considered here does extremely well in fitting $D^o \to K^+K^-$ and $K^o\bar{K}^o$ data. The sum $B(D^o \to K^+K^- + K^o\bar{K}^o)$ also can be raised to 0.6% as CLEO data require.

$B(D^o \to \pi^+\pi^-)$ is estimated to be in agreement with CLEO [1,4,5] and ARGUS [2,3] data and approximately one standard deviation away from E691 central value [6]. We found it hard to raise $B(D^o \to \pi^o\pi^o)$ much above the values quoted in Table IV, though the calculated branching ratio falls within errors of CLEO result [4,5,7]: $B(D^o \to \pi^o\pi^o) = (0.09 \pm 0.03)\%$. Part of the reason for the low value of $B(D^o \to \pi^o\pi^o)$ is that inelastic FSI have depressed $A_o^{\pi\pi}$ by quite a large amount – see eq. (20).

Note also that with $\delta_2^{\pi\pi}$ chosen to be, say, −45° and $\delta_o^{\pi\pi}$ taken from Table III, one finds $\delta_o^{\pi\pi} - \delta_2^{\pi\pi}$ to be around (60~70°) and with $\delta_2^{\pi\pi} = -60°$, $\delta_o^{\pi\pi} - \delta_2^{\pi\pi} = (75\sim85)°$. One also finds from

Tables III and IV that $|\delta_o^{K\bar{K}} - \delta_1^{K\bar{K}}| \simeq (45\text{~}60)^\circ$ fit $D^o \to K\bar{K}$ data well. Note that the branching ratios, depending on the cosine of the phase differences, are insensitive to the sign of the phase difference.

III. Weak decay amplitudes in ISGW model

Isgur, Scora, Grinstein and Wise [16] have calculated the form factors $f_+(t)$ and $f_-(t)$, where $t \equiv q^2$, in a quark model with "mock" mesons. A "mock" meson is a loosely bound state of a quark and an antiquark with a mass approximately equal to the sum of the constituent masses of the quark and the antiquark. They argue that at maximum momentum transfer their calculation of the "mock" form factors $\tilde{f}_+(\tilde{t})$ and $\tilde{f}_-(\tilde{t})$ ("mock" quantities are characterised by tilde) is trustworthy since the initial and the final "mock" mesons are stationary. One then assumes that the "physical" $f_+(t)$ and $f_-(t)$ are equal to the "mock" $\tilde{f}_+(\tilde{t})$ and $\tilde{f}_-(\tilde{t})$ at maximum momentum transfer.

Though ISGW [16] have calculated $f_+(t_m)$ and $f_-(t_m)$, $t_m = t_{max}$, we have repeated their calculation and find (we are assuming that $f_+(t_m) = \tilde{f}_+(\tilde{t}_m)$ etc.),

$$f_+^{D\to K}(t_m) = \tilde{F}_3^{D\to K}(\tilde{t}_m)\left[1 + \frac{3}{8m_c m_s}\frac{\tilde{\beta}_D^2\tilde{\beta}_K^2}{\tilde{\beta}_{DK}^2} + (\tilde{m}_D - \tilde{m}_K)\left\{\frac{1}{2m_s} - \frac{1}{4}\frac{m_u}{\tilde{m}_K}\frac{\tilde{\beta}_D^2}{\tilde{\beta}_{DK}^2}\left(\frac{1}{m_c} + \frac{1}{m_s}\right)\right\}\right] \quad (21)$$

where

$$\tilde{F}_3^{D\to K}(\tilde{t}_m) = \left(\frac{\tilde{m}_K}{\tilde{m}_D}\right)^{1/2}\frac{\tilde{\beta}_D^{3/2}\tilde{\beta}_K^{3/2}}{\tilde{\beta}_{DK}^3} \quad . \quad (22)$$

$\tilde{\beta}$'s are the factors which appear in the Gaussian radial wavefunctions [16] and

$$\tilde{\beta}_{DK}^2 = \frac{1}{2}\left(\tilde{\beta}_D^2 + \tilde{\beta}_K^2\right) \quad . \quad (23)$$

Though $f_+^{D\to K}(t_m)$ in (21) appears to be different from that calculated in [16], it differs only in the appearance of the second term on the right hand side of (21) proportional to $3/(8m_c m_s)$ which is indeed very small and contributes a correction of the order of 3%.

We also find,

$$f_-^{D\to K}(t_m) = \tilde{F}_3^{D\to K}(\tilde{t}_m)\left[1+\frac{3}{8m_cm_s}\frac{\tilde{\beta}_D^2\tilde{\beta}_K^2}{\tilde{\beta}_{DK}^2} - (\tilde{m}_D+\tilde{m}_K)\left\{\frac{1}{2m_s}-\frac{1}{4}\left(\frac{1}{m_c}+\frac{1}{m_s}\right)\frac{m_u}{\tilde{m}_K}\frac{\tilde{\beta}_D^2}{\tilde{\beta}_{DK}^2}\right\}\right]. \quad (24)$$

Again, apart from the second term proportional to $3/(8m_cm_s)$ this agrees with ISGW [22] calculation. For the $D\to\pi$ matrix elements, we find

$$f_+^{D\to\pi}(t_m) = \tilde{F}_3^{D\to\pi}(\tilde{t}_m)\left[1+\frac{3}{8m_cm_d}\frac{\tilde{\beta}_D^2\tilde{\beta}_\pi^2}{\tilde{\beta}_{D\pi}^2} + (\tilde{m}_D-\tilde{m}_\pi)\left\{\frac{1}{2m_d}-\frac{1}{4}\frac{m_u}{\tilde{m}_\pi}\frac{\tilde{\beta}_D^2}{\tilde{\beta}_{D\pi}^2}\left(\frac{1}{\tilde{m}_c}+\frac{1}{m_d}\right)\right\}\right] \quad (25)$$

and

$$f_-^{D\to\pi}(t_m) = \tilde{F}_3^{D\to\pi}\left[1+\frac{3}{8m_cm_d}\frac{\tilde{\beta}_D^2\tilde{\beta}_\pi^2}{\tilde{\beta}_{D\pi}^2} - (\tilde{m}_D+\tilde{m}_\pi)\left\{\frac{1}{2m_d}-\frac{1}{4}\left(\frac{1}{m_c}+\frac{1}{m_d}\right)\frac{m_u}{\tilde{m}_\pi}\frac{\tilde{\beta}_D^2}{\tilde{\beta}_{D\pi}^2}\right\}\right]. \quad (26)$$

We take the attitude that ISGW method is reliable only at t_m where the hadronic matrix element is calculated between two stationary mesons. The value of the hadronic matrix elements away from t_{max} is given by an extrapolation procedure yet to be defined. In calculating $f_+(t_m)$ we have used ISGW [16] parameters,

$$m_u = m_d = 330 \text{ MeV}$$
$$m_s = 550 \text{ MeV}, \quad m_c = 1.82 \text{ GeV} \quad (27)$$

and all the "mock" meson masses as sums of the constituent quark masses. Thus $\tilde{m}_K = 0.88$ GeV and $\tilde{m}_\pi = 0.66$ GeV. The sensitivity of $\tilde{F}_3$ on $\tilde{\beta}$'s is not strong; we use flavor independent $\tilde{\beta}$'s so that all $\tilde{\beta}$'s are set equal to 0.35 GeV. With these parameters we find,

$$\text{ISGW model}: \quad \begin{aligned} f_+^{D\to K}(t_m) &= 1.23 \\ f_+^{D\to\pi}(t_m) &= 1.48 \end{aligned} \quad (28)$$

In calculating $B(D\to\pi\pi)$ and $B(D\to K\overline{K})$ the form factors are needed at small values of the momentum transfer. Suppose, now, we choose to extrapolate $f_+(t)$ with a single-pole formula

$$f_+(t) = \frac{f_+(0)}{1 - t/m_{1^-}^2} \quad . \tag{29}$$

With $m_{1^-} = m_{D_s^*}$ for $D \to K$ transition and m_{D^*} for $D \to \pi$ transition, we find in ISGW model,

$$f_+^{D\to K}(0) = 0.706 \tag{30}$$

which is to be compared with 0.762 calculated by BSW [15]. Conversely if we extrapolate the BSW value of $f_+^{D\to K}(0) = 0.762$ to $t_m = (m_D - m_K)^2$, we get $f_+^{D\to K}(t_m) = 1.324$.

Though the difference between the BSW value of $f_+^{D\to K}(0)$ and the extrapolated $f_+^{D\to K}(0)$ from ISGW model is less than 10%, it is important to note that the extrapolated $f_+^{D\to K}(0)$ in ISGW model is *smaller* than the value calculated by BSW [15]. This does not help with $B(D^o \to K^+K^-+K^o\overline{K}^o)$ where we need to *raise* $f_+^{D\to K}(0)$ by (5–10)%.

The problem becomes more acute when we calculate $f_+^{D\to\pi}(t_m)$ in the ISGW model and extrapolate it down to t=0 using a single-pole formula. We find,

$$\text{ISGW model :} \qquad \begin{aligned} f_+^{D\to\pi}(t_m) &= 1.48 \\ f_+^{D\to\pi}(0) &= 0.38 \end{aligned} \quad . \tag{31}$$

Contrast this with the BSW calculation of $f_+^{D\to\pi}(0) = 0.692$. Conversely, if we were to extrapolate the BSW form factor to t_m using a single-pole formula we would get

$$\text{BSW model :} \qquad \begin{aligned} f_+^{D\to\pi}(t_m) &= 2.65 \\ f_+^{D\to\pi}(0) &= 0.692 \end{aligned} \quad . \tag{32}$$

Clearly, if we were to use $f_+^{D\to\pi}(0)$ from (31), our estimated branching ratios for $D^o \to \pi\pi$ would be too low almost by a factor of four.

Perhaps the problem lies in the extrapolation procedure. For example, if the form factors were extrapolated by the formula

$$f_+(t) = \frac{f_+(0) - c}{1 - t/m_{1^-}^2} + c \quad . \tag{33}$$

one could interpolate between the BSW value at t=0 and ISGW value at $t=t_m$ by choosing $c^{D\to\pi} = 0.41$ and $c^{D\to K} = 0.13$.

The form (33) also has the feature that $f_+(0) = 0$ does not necessarily imply $f_+(t) = 0$ for all t's. The extrapolation formula (33) could be tested in the analysis of semileptonic D-decays. If (33) is assumed to be valid for all momentum transfers it implies $f_+(\infty) = c$.

V. Conclusion

We have analysed the new data on $D\to\pi\pi$ and $K\overline{K}$ from ARGUS, CLEO and E691. We find that the hadronic matrix elements calculated in BSW [15] come very close to satisfying the data if we assume that FSI simply rotate the amplitudes in the complex plane. It appears that one needs to raise $f_+^{D\to K}(0)$ by (5–10)% and lower $f_+^{D\to\pi}(0)$ by a similar amount.

We have also looked at a coupled channel inelastic model whose parameters are constrained by $\pi\pi$ and $K\overline{K}$ scattering 'data' in I=0, L=0 state. We find that such a model can decrease $A_o^{\pi\pi}$ and raise $A_o^{K\overline{K}}$ as needed, though both of these effects are larger than what one would ideally want. However, a fit to all data can be secured in the inelastic coupled channel model also.

We have also investigated the use of ISGW model in these decays. We found that a single-pole extrapolation of the form factor $f_+(t)$ calculated at t_{max}, leads to $f_+(0)$ for $D\to K$ and $D\to\pi$ transitions which are *smaller* than those calculated in BSW model. For this reason the estimated branching ratios for $D\to\pi\pi$ and $D\to K\overline{K}$ modes would come out to be too low. The problem here does not appear to be due to the use of the "mock" meson mass. Indeed, if physical K and π masses were used the calculated hadronic matrix elements would be even smaller. If anything, one needs even a larger "mock" pion mass than we have used.

Finally we propose a formula for the form factor $f_+(t)$ which interpolates between the BSW value of $f_+(0)$ and ISGW value at maximum momentum transfer $f_+(t_m)$.

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Table I

$B(D^o \to \pi\pi)$ and $B(D^o \to K\bar{K})$ in a model where FSI simply rotates the BSW amplitudes in the complex plane. We also find $B(D^+ \to \pi^+\pi^o) = 0.14\%$.

$\delta_o^{\pi\pi} - \delta_2^{\pi\pi}$ (degree)	$B(D^o \to \pi^+\pi^-)$	$B(D^o \to \pi^o\pi^o)$	$\delta_o^{K\bar{K}} - \delta_1^{K\bar{K}}$	$B(D^o \to K^+K^-)$	$B(D^o \to K^o\bar{K}^o)$
0	0.34	0.03	0	0.52	0.0
30	0.33	0.04	30	0.49	0.03
60	0.29	0.08	60	0.39	0.13
90	0.23	0.14	90	0.26	0.26
120	0.18	0.19	120	0.13	0.39
150	0.14	0.23	150	0.03	0.49
180	0.13	0.24	180	0.0	0.52

Table II

Coupled $\pi\pi$ and $K\bar{K}$ scattering problem in I=0, L=0 state. $\delta_1 = \pi\pi$ scattering phase shift, $\delta_2 = K\bar{K}$ scattering phase shift, η = elasticity. **K**-matrix parameters: det **K** = -1.65 GeV^{-2}, a = -0.86 GeV^{-1}, c = -1.32 GeV^{-1}.

W GeV.	δ_1 (degrees)	δ_2 (mod. π) (degrees)	η
1.5	−10.0	−29.2	0.153
1.6	0.0	−39.7	0.100
1.7	18.6	−58.4	0.077
1.8	37.2	−77.1	0.091
1.9	47.4	−87.1	0.125
2.0	52.7	−92.2	0.163

Table III

Unitarized weak decay amplitudes, $A_o^{\pi\pi}$ and $A_o^{K\bar{K}}$ generated via (11). Double entries in columns $A_o^{\pi\pi}$ and $A_o^{K\bar{K}}$ represent real and imaginary parts of the weak decay amplitudes. $\delta_o^{\pi\pi}$ and $\delta_o^{K\bar{K}}$ are the calculated phases of the weak decay amplitudes $A_o^{\pi\pi}$ and $A_o^{K\bar{K}}$.

Det **K** GeV^{-2}	a GeV^{-1}	c GeV^{-1}	$A_o^{\pi\pi}$ GeV^3	$\delta_o^{\pi\pi}$ (degrees)	$A_o^{K\bar{K}}$ GeV^3	$\delta_o^{K\bar{K}}$ (degrees)
−1.65	-0.86	-1.32	(-0.118, -0.050)	22.8	(0.109, 0.068)	32.2
−1.75	-1.18	-1.71	(-0.123, -0.045)	20.2	(0.117, 0.062)	28.1
−1.85	-1.61	-2.20	(-0.127, -0.040)	17.5	(0.124, 0.055)	24.0
−1.95	-2.25	-2.92	(-0.130, -0.034)	14.4	(0.132, 0.047)	19.6

Table IV

Calculated branching ratios in %. $\delta_2^{\pi\pi}$ is set = −45°. Figures in brackets are calculated with $\delta_2^{\pi\pi} = -60°$. $\delta_1^{K\bar{K}}$ is assigned values shown.

det **K** GeV^{-2}	$\delta_1^{K\bar{K}}$ degrees	$B(D^o \to \pi^+\pi^-)$	$B(D^o \to \pi^o\pi^o)$	$B(D^o \to K^+K^-)$	$B(D^o \to K^o\bar{K}^o)$
-1.65	80	0.171 (0.151)	0.059 (0.079)	0.490 (0.490)	0.098 (0.098)
	85	01.71 (0.151)	0.059 (0.079)	0.470 (0.470)	0.117 (0.117)
	90	0.171 (0.151)	0.059 (0.079)	0.450 (0.450)	0.138 (0.138)
-1.75	80	0.181 (0.160)	0.058 (0.078)	0.489 (0.489)	0.119 0.119)
	85	0.181 (0.160)	0.058 (0.078)	0.468 (0.468)	0.140 (0.140)
	90	0.181 (0.160)	0.058 (0.078)	0.445 (0.445)	0.162 (0.162)
-1.85	75	0.189 (0.169)	0.056 (0.076)	0.508 (0.508)	0.120 (0.120)
	80	0.189 (0.169)	0.056 (0.076)	0.486 (0.486)	0.141 (0.141)
	85	0.189 (0.169)	0.056 (0.076)	0.463 (0.463)	0.164 (0.164)
-1.95	65	0.197 (0.177)	0.053 (0.073)	0.547 (0.547)	0.101 (0.101)
	70	0.197 (0.177)	0.053 (0.073)	0.526 (0.526)	0.122 (0.122)
	75	0.197 (0.177)	0.053 (0.073)	0.504 (0.504)	0.144 (0.144)

References

1. CLEO collaboration, J. Alexander et al., Phys.Rev. Lett. **65**, 1184 (1990).
2. M. Danibov, Plenary talk, LP-HEP 981, 25 July - 1 August, 1991, Geneva (to be published).
3. ARGUS collaboration, H. Albrecht et al., Z. Phys. C - Particles and Fields **46**, 9 (1990).
4. Wei-Ming Yao, talk at Particles and Fields 91, Vancouver, August 1991 (to be published).
5. B. Gittelman, Plenary talk at Particles and Fields 91, Vancouver, August 1991 (to be published).
6. E691 collaboration, J.C. Anjos et al., report FERMILAB-Pub-91/147 (1991).
7. R. Poling, talk at LP-HEP 91, Geneva, 1991 (to be published).
8. Particle Data Group, J.J. Hernández et al., Phys. Lett. B **239**, 1 (1990).
9. ACCMOR collaboration, S. Barlag et al., Z. Phys. C - Particles and Fields **48**, 29 (1990).
10. D. Hitlin, in **Particles and Fields 3**, Proceedings of the Banff Summer Institute, Banff, Canada, 1988, edited by A.N. Kamal and F.C. Khanna (World Scientific, Singapore, 1989), p. 607.
11. A.N. Kamal and R.C. Verma, Phys. Rev. D **35**, 3515 (1987).
12. R.C. Verma and A.N. Kamal, ibid **43**, 829 (1991).
13. A.N. Kamal, Two-body Final State Interactions with Examples, report, Alberta Thy-28-90 (in press, Int. J. Mod. Phys. A).
14. Ling-Lie Chau and Hai-Yang Cheng, Final State Interaction and SU(3)-breaking Effects in $D^0 \to \pi\pi$, $K\overline{K}$, UC Davis report, UCDPHYS-PUB-91-26 (1991).
15. M. Bauer, B. Stech and M. Wirbel, Z. Phys. C - Particles and Fields **34**, 103 (1987); M. Wirbel, B. Stech and M. Bauer, ibid C **29**, 637 (1985).
16. N. Isgur, D. Scora, B. Grinstein and M.B. Wise, Phys. Rev. D **39**, 799 (1989).
17. M. Wirbel, Prog. Part. Nucl. Phys. **21**, 33 (1988).
18. G. Grayer et al., Nucl. Phys. B **75**, 189 (1970).

19. W. Hoogland et al., Nucl. Phys. B **69**, 266 (1974); B **126**, 109 (1977); D. Cohen et al., Phys. Rev. D **7**, 661 (1973).

20. J. Weinstein and N. Isgur, Phys. Rev. D **41**, 2236 (1990).

21. D. Cohen et al., Phys. Rev. D **22**, 2595 (1980); A.B. Wicklund et al., Phys. Rev. Lett. **45**, 1469 (1980).