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Mass and Energy Shifts as Particle ID

Consider the two body decay $D^0 \to h^- h^+$, where h stands for hadron. In this note, we review how shifts in the reconstructed mass of the $h^- h^+$ system allow one to perform particle identification (ID, or PID) for the two hadrons, using only information from the tracking spectrometer.

The measurements made by the tracking spectrometer in CLEO (the DR/VD/SVX + magnet; pulseheight in the DR is *not* included here) are in effect of the momenta of the two hadrons, $\vec{p}_-$ and $\vec{p}_+$: each has three components, so there are six measurements in all, and one can reconstruct from those measurements six parameters that describe the parent particle and its decay. Three of these parameters are dull: the two angles that describe the D^0 direction, and the one azimuth that describes the orientation of the D^0 decay plane about the D^0 direction.

The fourth parameter is of more interest: it is the magnitude of the total momentum of the D^0: $|\vec{p}_{D^0}| = |\vec{p}_- + \vec{p}_+|$. In CLEO, one tends to benefit by looking at D^0 decays with relatively large $p_{D^0} = |\vec{p}_{D^0}|$, where the combinatoric background under the D^0 tends to be small.

The remaining two parameters are of the most interest, but, unfortunately, they cannot be reconstructed without some assumption as to the masses of the two hadrons, m_- and m_+. After making a mass assumption, one can reconstruct the most interesting parameter of the decay, namely, the invariant mass, M:

$$M^2 = m_-^2 + m_+^2 + 2\left\{\sqrt{m_-^2 + p_-^2}\sqrt{m_+^2 + p_+^2} - p_- p_+ \cos\theta\right\} \tag{1}$$

where $p_- = |\vec{p}_-|$, $p_+ = |\vec{p}_+|$, and θ is the angle between $\vec{p}_-$ and $\vec{p}_+$. If one makes the correct assumption for m_- and m_+, $M = M_{D^0}$, to within tracking resolution error. *Incorrect* assumptions result in $M \neq M_{D^0}$; the main purpose of this note is to derive expressions for the magnitude of the mass shift, $\delta M = M - M_{D^0}$, due to incorrect mass assumptions.

The remaining parameter is the angle θ^* in the parent rest frame of the direction of the h^+, say, with respect to the direction of the boost back into the lab frame. In decays of a spin-0 meson such as the D^0, the decays are distributed uniformly in $\cos\theta^*$, and it might be thought that θ^* is dull also.

In fact, θ^* governs the relative momenta, $R = p_+/p_-$ of the decay: roughly, R is large when $\theta^* \sim 0$; R is near one when $\theta^* \sim 90°$, and finally R is small when $\theta^* \sim 180°$. Because R is directly accessible from the measurements without assumptions about m_- and m_+, R is a bit easier to use than θ^*.

More importantly, though, it is the relative momentum R that governs the size of the shift in δM brought about by *incorrect* mass assumptions in equation 1, at least in the ultrarelativistic limit, which usually pertains for CLEO:

$$\begin{aligned}
\sqrt{m_-^2+p_-^2} \approx p_- + \frac{m_-^2}{2p_-} \quad &, \quad \sqrt{m_+^2+p_+^2} \approx p_+ + \frac{m_+^2}{2p_+} \\
2\sqrt{m_-^2+p_-^2}\sqrt{m_+^2+p_+^2} &\approx \left(\frac{p_+}{p_-}m_-^2 + \frac{p_-}{p_+}m_+^2\right) + 2p_-p_+ \\
M^2 &= m_-^2\,(1+R) + m_+^2\left(1+\frac{1}{R}\right) + 4p_-p_+\sin^2\frac{\theta}{2} \qquad (2)
\end{aligned}$$

One can see from equation 2 that the change in M^2 brought by a change in m_- is largest when R is largest, meaning p_- is smallest, and the change in M^2 brought by a change in m_+ is largest when $1/R$ is largest, or R is smallest, meaning p_+ is smallest.

As a concrete example, consider the most common decay of the form $D^0 \to h^-h^+$, the Cabibbo-favored $D^0 \to K^-\pi^+$. What happens when that decay actually occurs, and one makes the correct assumption that $m_- = m_{K^-} = 493.7\,\mathrm{MeV}$, but one makes the incorrect assumption that $m_+ = m_{K^-} = 493.7\,\mathrm{MeV}$, instead of the correct assumption that $m_+ = m_{\pi^+} = 139.6\,\mathrm{MeV}$? In other words, one reconstructs a true $D^0 \to K^-\pi^+$ as $D^0 \to K^-K^+$.

From equation 2, the invariant mass will shift by an amount, $\delta M_{\pi\to K}$,

$$\delta M_{\pi\to K} = \frac{m_{K^-}+m_{\pi^+}}{2M_{D^0}}(m_{K^-}-m_{\pi^+})\left(1+\frac{1}{R}\right), \qquad (3)$$

which is at least 60.1 MeV, for $R \to \infty$; for the typical value of $R=1$, $\delta M_{\pi\to K} \approx 120\,\mathrm{MeV}$.

As a second example, suppose again that the true decay is $D^0 \to K^-\pi^+$, but one makes the incorrect assumption that $m_- = m_{\pi^-} = 139.6\,\mathrm{MeV}$, and the correct assumption that $m_+ = m_{\pi^+} = 139.6\,\mathrm{MeV}$? In other words, one reconstructs a true $D^0 \to K^-\pi^+$ as $D^0 \to \pi^-\pi^+$.

Again from equation 2, there will be a negative mass shift, $\delta M_{K\to\pi}$:

$$\delta M_{K\to\pi} = -\frac{m_{K^-}+m_{\pi^+}}{2M_{D^0}}(m_{K^-}-m_{\pi^+})\,(1+R)\,, \qquad (4)$$

As a final D^0 example, suppose one makes the wrong assumption about *both* masses, and reconstructs a true $D^0 \to K^-\pi^+$ as $D^0 \to \pi^-K^+$; the mass shift δM_2 will be the sum of equations 3 and 4:

$$\delta M_2 = \frac{m_{K^-}+m_{\pi^+}}{2M_{D^0}}(m_{K^-}-m_{\pi^+})\left(\frac{1}{R}-R\right) \qquad (5)$$

From equation 5, one can see that when $R=1$, or the decay is symmetric, both the $D^0 \to K^-\pi^+$ and $D^0 \to \pi^-K^+$ assumptions result in the same reconstructed mass. As a consequence, in the search for $D^0 \to K^+\pi^-$, one wants to exclude a swath of near-symmetric decays.

Equations 3, 4, and 5 describe the mass shifts due to some possible changes in mass assignments. However, what is CLEO's resolution in mass at the D^0? Ideally, now that the Kalman fitter is working, one should compute that error on an event-by-event basis from the track error matrices. That method correctly nails dependences on number of hits on the track, and on θ. To get a feeling for the underlying effects, here is a rough calculation.

To lowest order, from equation 2, it is straightforward to work out that the resolution in mass, $\sigma_{M_{D^0}}$ is:

$$\sigma_{M_{D^0}} = \frac{1}{2} M_{D^0} \sqrt{\left[\frac{\sigma_p(p_-)}{p_-}\right]^2 + \left[\frac{\sigma_p(p_+)}{p_+}\right]^2} \tag{6}$$

where $\sigma_p(p_-)$ is the momentum resolution at momentum p_-, etc. Roughly,

$$\begin{aligned} \left[\frac{\sigma_p(p)}{p}\right]^2 &= A^2 + (B\,p)^2 \quad \text{so,} \\ \sigma_{M_{D^0}} &= \frac{1}{2} M_{D^0} \sqrt{2A^2 + B^2(p_-^2 + p_+^2)} \end{aligned} \tag{7}$$

where A originates in multiple coloumb scattering, primarily in the DR gas and wires, and B originates in the spatial resolution of the tracking system measurements.

For CLEO-II/SVX, Jeff Gronberg has measured $B \sim 0.16\%/\mathrm{GeV}$, and we believe $A \sim 0.45\%$, which gives $\sigma_{M_{D^0}} \approx 7\,\mathrm{MeV}$, as observed in the CLEO-II/SVX data. A true fit of the data to equation 7 has yet to be done, however. The momentum at which the multiple scattering and spatial resolution contributions become equal is about $p = 2.8\,\mathrm{GeV}$, which is somewhat above the typical momentum, $p = 1.7\,\mathrm{GeV}$, in a D^0 decay. Multiple scattering usually contributes 85% of the $\sigma_{M_{D^0}} \approx 7\,\mathrm{MeV}$.

For CLEO-II, prior to the recompress, $\sigma_{M_{D^0}} \approx 10\,\mathrm{MeV}$, and after the recompress, $\sigma_{M_{D^0}} \approx 8.8\,\mathrm{MeV}$. These are the numbers for relatively fast D^0's, and so they are comparable to the CLEO-II/SVX result of $\sigma_{M_{D^0}} \approx 7\,\mathrm{MeV}$. The main reason for the improvement in the recompress, prior to addition of the SVX, is the better weighting of information in the recompress by the Kalman fitter, particularly in the presence of multiple scattering.

The improvement from $\sigma_{M_{D^0}} \approx 8.8\,\mathrm{MeV}$ in the recompress to $\sigma_{M_{D^0}} \approx 7\,\mathrm{MeV}$ in CLEO-II/SVX is due to the replacement of Argon-Ethane in the DR by Helium-Propane, a gas which causes less multiple scattering. The number of radiation lengths per meter declined from 1.0% in the CLEO-II DR to 0.61% in the CLEO-II/SVX DR, and so we expect that A in CLEO-II was $\sim 0.45\%/\sqrt{0.61} \sim 0.58\%$, and hence $\sigma_{M_{D^0}} \approx 8.4\,\mathrm{MeV}$, which is consistent to within errors with the observed 8.8 MeV.

It is interesting to note that of the 0.61%/meter of material in the CLEO-II/SVX DR, only 0.18%/meter of it is in the gas: the remaining 0.43%/meter of the material is in the drift chamber wires, mostly the field wires. Typically only 1-2 wire(s) are struck by any track, and a kink from a multiple scatter could be in principle detected and fit. Such an algorithm might reduce the A term to $0.45\% \times \sqrt{0.18/0.61} = 0.24\%$, and thus the mass resolution to $\sigma_{M_{D^0}} \approx 4.8\,\mathrm{MeV}$.

Currently, the minimum mass shift of 60.1 MeV from equation 3 constitutes a separation between $D^0 \to K^-\pi^+$ and either $D^0 \to K^-K^+$ or $D^0 \to \pi^-\pi^+$ of about $60.1/7.0 \approx 8.6\,\sigma$. Particle ID by mass shift is powerful.

For the double misidentification described by equation 5, one needs to select events with $R > 1.25$ or $R < 0.8$ in order to achieve 4σ separation, which is the level appropriate for suppression of $D^0 \to K^-\pi^+$ down to the level of the doubly Cabibbo suppressed $D^0 \to K^+\pi^-$.

Turning to the case $\overline{B^0} \to h^- h^+$, there is a key difference with $D^0 \to h^- h^+$. In particular, there is prior knowledge of the energy of the $\overline{B^0}$, from the CESR beam energy: $E_{\rm beam} = 5290\,{\rm MeV}$, while for the D^0, there is no such knowledge.

So, the reconstruction strategy for $\overline{B^0} \to h^- h^+$ has always differed from that for $D^0 \to h^- h^+$. In particular, the added information from the beam energy affords one an opportunity to devise two new parameters for the $\overline{B^0} \to h^- h^+$, which replace the invariant mass for $D^0 \to h^- h^+$. For one of the $\overline{B^0} \to h^- h^+$ variables, the beam-constrained mass, M_B, *no mass assignments are necessary for* m_- and m_+. The definition of M_B is given below in equation 9. The second parameter, ΔE, is the deficit between the beam energy $E_{\rm beam}$ and the total energy of the $\overline{B^0}$ decay products, for which mass assignments are necessary. The expression is simpler than equation 1 for the 2-body mass:

$$\begin{aligned} \Delta E &= E_{\rm beam} - \sqrt{m_-^2 + p_-^2} - \sqrt{m_+^2 + p_+^2} \\ &\approx E_{\rm beam} - p_- - p_+ - \frac{m_-^2}{2p_-} - \frac{m_+^2}{2p_+} \end{aligned} \quad (8)$$

Taking the baseline decay as $\overline{B^0} \to \pi^- \pi^+$ instead of the $D^0 \to K^- \pi^+$ the energy shift induced by a change of m_- from m_{π^-} to m_{K^-}, $\delta[\Delta E]_{\pi\to K}$, will be:

$$\begin{aligned} \delta[\Delta E]_{\pi\to K} &\approx -\frac{m_{K^-} + m_{\pi^+}}{2p_-}(m_{K^-} - m_{\pi^-}) \\ &\approx -42.4\,{\rm MeV}. \end{aligned}$$

Then, from equation 8, the resolution in ΔE, $\sigma_{\Delta E}$, will be:

$$\begin{aligned} \sigma_{\Delta E} &\approx \sqrt{A^2(p_-^2 + p_+^2) + B^2(p_-^4 + p_+^4)} \\ &\approx 23.1\,{\rm MeV},\ \text{CLEO-II/SVX}. \end{aligned}$$

So, the separation in CLEO-II/SVX between $\overline{B^0} \to \pi^- \pi^+$ and $\overline{B^0} \to K^- \pi^+$ will be $42.4/23.1 = 1.83\,\sigma$. Fitting of kinks at wires would reduce $\sigma_{\Delta E}$ to 18.3 MeV, and increase the separation to $2.32\,\sigma$.

The current π/K separation provided by CLEO-II/SVX's reconstruction of dE/dx from DR pulseheights is, in the appropriate momentum region, $1.9\,\sigma$.

A last comment about the beam-constrained mass:

$$\begin{aligned} M_B &= \sqrt{E_{\rm beam}^2 - |\vec{p}_- + \vec{p}_+|^2} \\ &\approx E_{\rm beam} - \frac{|\vec{p}_- + \vec{p}_+|^2}{2E_{\rm beam}} \end{aligned} \quad (9)$$

The quantity $q = \sqrt{\xi} = |\vec{p}_- + \vec{p}_+|$, has mean value $\overline{q} \approx 320\,{\rm MeV}$ from the difference between the beam energy ($E_{\rm beam} \approx 5290\,{\rm MeV}$) and the B-hadron mass ($m_B = 5279\,{\rm MeV}$). The smearing in q about $\overline{q}$ is dominantly from the synchrotron emission induced CESR beam energy spread, which is about 0.06% or 3.2 MeV, and which converts to a B momentum spread of $\sigma_q \approx$

$(m_B/\overline{q})\times 3.2\,\mathrm{MeV}\approx 52\,\mathrm{MeV}$. Thus, the central value of $\overline{q} = 320\,\mathrm{MeV}$ is about $6.2\,\sigma$ from $q = 0$. Tracking resolution contributes only a small amount to the spread in the reconstructed q: only about $25\,\mathrm{MeV}$, for $\overline{B^0}\to\pi^+\pi^-$.

An aside: it *might* be that deviation in the annihilation energy from the mean value is correlated with the position of the annihilation in the collision volume. That is, events with a primary vertex at $x = -150\,\mu\mathrm{m}$ and $z = +0.8\,\mathrm{cm}$ might have a net energy shift, or at least a different spread in energies, than, say, events with a primary vertex at $x = +200\,\mu\mathrm{m}$ and $z = -0.4\,\mathrm{cm}$. Preliminary discussions with three different David R.'s: David Rice, David Ritson, and David Rubin, indicate that from an accelerator physics viewpoint, no such correlation should exist, although none has a plot that proves their point. In the unlikely circumstance that those knowledgeable accelerator physicists are wrong, however, one could use information from the coordinate of the primary vertex to tighten the beam constrained-mass cut from where it is now, to accommodate σ_q of $52\,\mathrm{MeV}$, to a value tightened to accommodate an error in q of $25\,\mathrm{MeV}$.

The principal point here, however, concerns the shape of the background: if the background is uniform in d^3q, as one would conjecture to lowest order, then the shape in ξ, and hence in M_B, will be:

$$\propto q^2 dq \propto \xi \frac{d\xi}{2\sqrt{\xi}} \propto \sqrt{\xi} d\xi \propto \sqrt{E_{\mathrm{beam}} - M_B}\, dM_B. \tag{10}$$

The idea here is to choose a new variable, ζ, to replace M_B, so as to obtain a roughly flat background:

$$\zeta = \left[\frac{q}{320\,\mathrm{MeV}}\right]^x \quad , \quad q = (320\,\mathrm{MeV})\zeta^{1/x} \quad , \quad dq \propto \zeta^{1/x-1} d\zeta$$

Then, the background will be distributed according to:

$$\propto q^2 dq \propto \zeta^{2/x}\zeta^{1/x-1} d\zeta \propto \zeta^{3/x-1} d\zeta \tag{11}$$

So, the choice of $x = 3$, that is

$$\zeta = \left[\frac{q}{320\,\mathrm{MeV}}\right]^3 \quad , \quad q = (320\,\mathrm{MeV})\zeta^{1/3},$$

would make the background under the B nearly flat. As discussed above, the B peak would be about $6\,\sigma$ from the endpoint of $\zeta = 0$. In principle, the endpoint should be very sharp, without smearing, because it never happens that $q^2 < 0$, and because the assumption that the background is at least smooth in elements of d^3q near $q = 0$ seems rather solid.