Formulas

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April 22, 1998

Formulas

These are appropriate if there is no CP violation.

$$\begin{aligned}
\tau_D &= \text{mean life of } D^0 \text{ mesons} \\
\overline{\Gamma} &= 1/\tau_D = \text{mean rate of decay of } D^0 \text{ mesons]} \\
&= \frac{1}{2}(\Gamma_+ + \Gamma_-) \\
\Gamma_+ &= \text{rate of decay of } D^0 \text{ to CP+ eigenstates (e.g. } \pi^+\pi^-,\ K^+K^-) \\
\Gamma_- &= \text{rate of decay of } D^0 \text{ to CP$-$ eigenstates (e.g. } K_{\rm S}\pi^0) \\
\Delta\Gamma &= \Gamma_+ - \Gamma_- \\
&= i \times \text{mixing amplitude through real intermediate states} \\
y &= \frac{\Delta\Gamma}{2\overline{\Gamma}} \\
&= +1 \text{ If } D^0 \text{ width dominated by CP+ states, like } K^+K^-, \pi^+\pi^- \\
&= -1 \text{ If } D^0 \text{ width dominated by CP$-$ states, like } K_{\rm S}\pi^0 \\
\Delta M &= \text{mixing amplitude through virtual intermediate states} \\
x &= \frac{\Delta M}{\overline{\Gamma}} \\
\phi_{\rm SW} &= \tan^{-1}\left(-2\frac{\Delta M}{\Delta\Gamma}\right) \\
\delta_s &= \text{Strong phase between amplitudes for } D^0 \to K^+\pi^- \text{ and } \overline{D^0} \to K^+\pi^- \\
&= \text{small, by theoretical bias} \\
t &= \text{proper time of decay} \\
\tau &= \overline{\Gamma} t = t/\tau_D \\
N_{\rm R} &= \text{number of right sign decays} \\
N_{\rm W} &= \text{number of wrong sign decays}
\end{aligned}$$

Then, pretty much everyone simplifies to, for the wrong-sign decay rate as a function of time:

$$w_{\rm raw}(\tau) = (a + b\tau + c\tau^2)e^{-\tau}, \tag{1}$$

with a, b, c dependent on the parameterization. Normalizing this to the number of wrong and right sign decays gives,

$$w(\tau) = \frac{N_{\mathrm{W}}}{N_{\mathrm{R}}} \frac{a + b\tau + c\tau^2}{a + b + 2c} \, e^{-\tau}. \tag{2}$$

Then, I think, in Ted Liu's parameterization,

$$w(\tau) = (R_{\mathrm{DCSD}} + \sqrt{2 R_{\mathrm{DCSD}} R_{\mathrm{mixing}}} \cos\phi \, \tau + \frac{1}{2} R_{\mathrm{mixing}} \, \tau^2) \, e^{-\tau}. \tag{3}$$

To connect with the parameterization involving $\Delta\Gamma$ and ΔM,

$$\begin{aligned}
R_{\mathrm{mixing}} &= \frac{1}{2}(x^2 + y^2) \\
\phi &= \phi_{\mathrm{SW}} + \delta_s \\
\Delta\Gamma &= -2\overline{\Gamma} \sqrt{2 R_{\mathrm{mixing}}} \cos\phi \;\; \text{if} \;\; \delta_s = 0 \\
\Delta M &= \overline{\Gamma} \sqrt{2 R_{\mathrm{mixing}}} \sqrt{1 - \cos^2\phi} \;\; \text{if} \;\; \delta_s = 0
\end{aligned}$$