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A SUPERSYMMETRY PRIMER

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I provide a pedagogical introduction to supersymmetry. The level of discussion is aimed at readers who are familiar with the Standard Model and quantum field theory, but who have little or no prior exposure to supersymmetry. Topics covered include: motivations for supersymmetry; the construction of supersymmetric Lagrangians; supersymmetry-breaking interactions; the Minimal Supersymmetric Standard Model (MSSM); R-parity and its consequences; the origins of supersymmetry breaking; the mass spectrum of the MSSM; decays of supersymmetric particles; experimental signals for supersymmetry; and some extensions of the minimal framework. This is an extended version of a contribution to the book *Perspectives on Supersymmetry*, edited by G. L. Kane (World Scientific, Singapore 1998).

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1 Introduction

The Standard Model of high energy physics provides a remarkably successful description of presently known phenomena. The experimental high-energy frontier has advanced into the hundreds of GeV range with no confirmed deviations from Standard Model predictions and few unambiguous hints of additional structure. Still, it seems quite clear that the Standard Model is a work in progress and will have to be extended to describe physics at arbitrarily high energies. Certainly a new framework will be

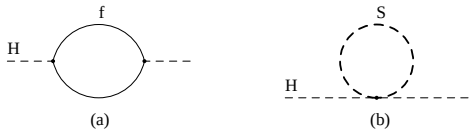

Figure 1: Quantum corrections to the Higgs (mass)^2.

required at the reduced Planck scale $M_{\rm P} = (8\pi G_{\rm Newton})^{-1/2} = 2.4 \times 10^{18}$ GeV, where quantum gravitational effects become important. Based only on a proper respect for the power of Nature to surprise us, it seems nearly as obvious that new physics exists in the 16 orders of magnitude in energy between the presently explored territory and the Planck scale.

The mere fact that the ratio $M_{\rm P}/M_W$ is so huge is already a powerful clue to the character of physics beyond the Standard Model, because of the infamous "hierarchy problem".[1] This is not really a difficulty with the Standard Model itself, but rather a disturbing sensitivity of the Higgs potential to new physics in almost any imaginable extension of the Standard Model. The electrically neutral part of the Standard Model Higgs field is a complex scalar H with a classical potential given by

$$V = m_H^2 |H|^2 + \lambda |H|^4 \,. \tag{1.1}$$

The Standard Model requires a non-vanishing vacuum expectation value (VEV) for H at the minimum of the potential. This will occur if $m_H^2 < 0$, resulting in $\langle H \rangle = \sqrt{-m_H^2/2\lambda}$. Since we know experimentally that $\langle H \rangle = 174$ GeV from measurements of the properties of the weak interactions, it must be that m_H^2 is very roughly of order $-(100 \text{ GeV})^2$. However, m_H^2 receives enormous quantum corrections from the virtual effects of every particle which couples, directly or indirectly, to the Higgs field.

For example, in Fig. 1a we have a correction to m_H^2 from a loop containing a Dirac fermion f with mass m_f. If the Higgs field couples to f with a term in the lagrangian $-\lambda_f H \overline{f} f$, then the Feynman diagram in Fig. 1a yields a correction

$$\Delta m_H^2 = \frac{|\lambda_f|^2}{16\pi^2} \Big[-2\Lambda_{\rm UV}^2 + 6 m_f^2 \ln(\Lambda_{\rm UV}/m_f) + \ldots \Big] \,. \tag{1.2}$$

Here $\Lambda_{\rm UV}$ is an ultraviolet momentum cutoff used to regulate the loop integral; it should be interpreted as the energy scale at which new physics enters to alter the high-energy behavior of the theory. The ellipses represent terms which depend on the precise manner in which the momentum cutoff is applied, and which do not get large as $\Lambda_{\rm UV}$ does. Each of the leptons and quarks of the Standard Model can play the role of f; for quarks, eq. (1.2) should be multiplied by 3 to account for color. The largest correction comes when f is the top quark with $\lambda_f \approx 1$. The problem is that if $\Lambda_{\rm UV}$ is of order $M_{\rm P}$, say, then this quantum correction to m_H^2 is some 30 orders of magnitude larger than the aimed-for value of $m_H^2 \sim -(100 \text{ GeV})^2$. This is only directly a problem for corrections to the Higgs scalar boson (mass)^2, because quantum

corrections to fermion and gauge boson masses do not have the quadratic sensitivity to $\Lambda_{\rm UV}$ found in eq. (1.2). However, the quarks and leptons and the electroweak gauge bosons Z^0, $W^\pm$ of the Standard Model all owe their masses to $\langle H \rangle$, so that the entire mass spectrum of the Standard Model is directly or indirectly sensitive to the cutoff $\Lambda_{\rm UV}$.

One could imagine that the solution is to simply pick an ultraviolet cutoff $\Lambda_{\rm UV}$ which is not too large. However, one still has to concoct some new physics at the scale $\Lambda_{\rm UV}$ which not only alters the propagators in the loop, but actually cuts off the loop integral. This is not easy to do in a theory whose lagrangian does not contain more than two derivatives, and higher derivative theories generally suffer from a loss of unitarity. In string theories, loop integrals are cut off at high Euclidean momentum p by factors $e^{-p^2/\Lambda_{\rm UV}^2}$, but then $\Lambda_{\rm UV}$ is a string scale which is usually thought to be not very far below $M_{\rm P}$. Furthermore, there is a contribution similar to eq. (1.2) from the virtual effects of any arbitrarily heavy particles which might exist. For example, suppose there exists a heavy complex scalar particle S with mass m_S which couples to the Higgs with a lagrangian term $-\lambda_S |H|^2 |S|^2$. Then the Feynman diagram in Fig. 1b gives a correction

$$\Delta m_H^2 = \frac{\lambda_S}{16\pi^2} \Big[\Lambda_{\rm UV}^2 - 2 m_S^2 \ln(\Lambda_{\rm UV}/m_S) + \ldots \Big] \,. \tag{1.3}$$

If one rejects a physical interpretation of $\Lambda_{\rm UV}$ and uses dimensional regularization on the loop integral instead of a momentum cutoff, then there will be no $\Lambda_{\rm UV}^2$ piece. However, even then the term proportional to m_S^2 cannot be eliminated without the physically unjustifiable tuning of a counter-term specifically for that purpose. So m_H^2 is sensitive to the masses of the *heaviest* particles that H couples to; if m_S is very large, its effects on the Standard Model do not decouple, but instead make it very difficult to understand why m_H^2 is so small.

This problem arises even if there is no direct coupling between the Standard Model Higgs boson and the unknown heavy particles. For example, suppose that there exists a heavy fermion F which, unlike the quarks and leptons of the Standard Model, has vector-like quantum numbers and therefore gets a large mass m_F without coupling to the Higgs field. [In other words, an arbitrarily large mass term of the form $m_F \overline{F} F$ is not forbidden by any symmetry, including $SU(2)_L$.] In that case, no diagram like Fig. 1a exists for F. Nevertheless there will be a correction to m_H^2 as long as F shares some gauge interactions with the Standard Model Higgs field; these may be the familiar electroweak interactions, or some unknown gauge forces which are broken at a very high energy scale inaccessible to experiment. In any case, the two-loop Feynman diagrams in Fig. 2 yield a correction

$$\Delta m_H^2 = x \left(\frac{g^2}{16\pi^2} \right)^2 \Big[a \Lambda_{\rm UV}^2 + 48 m_F^2 \ln(\Lambda_{\rm UV}/m_F) + \ldots \Big] \,, \tag{1.4}$$

where g is the gauge coupling in question, and x is a group theory factor of order 1. (Specifically, x is the product of the quadratic Casimir invariant of H and the Dynkin index of F for the gauge group in question.) The coefficient a depends on the precise

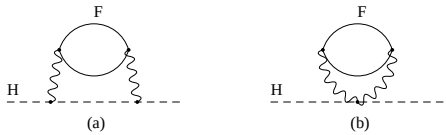

Figure 2: Two-loop corrections to the Higgs (mass)^2 due to a heavy fermion.

method of cutting off the momentum integrals. It does not arise at all if one rejects the possibility of a physical interpretation for $\Lambda_{\rm UV}$ and uses dimensional regularization, but the m_F^2 contribution is always present. The numerical factor $(g^2/16\pi^2)^2$ may be quite small (of order 10^{-5} for electroweak interactions), but the important point is that these contributions to Δm_H^2 are sensitive to the largest masses and/or ultraviolet cutoff in the theory, presumably of order $M_{\rm P}$. The "natural" (mass)^2 of a fundamental Higgs scalar, including quantum corrections, seems to be more like $M_{\rm P}^2$ than the experimentally favored value! Even very indirect contributions from Feynman diagrams with three or more loops can give unacceptably large contributions to Δm_H^2. If the Higgs boson is a fundamental particle, we have two options: either we must make the rather bizarre assumption that there do not exist *any* heavy particles which couple (even indirectly or extremely weakly) to the Higgs scalar field, or some rather striking cancellation is needed between the various contributions to Δm_H^2.

The systematic cancellation of the dangerous contributions to Δm_H^2 can only be brought about by the type of conspiracy which is better known to physicists as a symmetry. It is apparent from comparing eqs. (1.2), (1.3) that the new symmetry ought to relate fermions and bosons, because of the relative minus sign between fermion loop and boson loop contributions to Δm_H^2. (Note that λ_S must be positive if the scalar potential is to be bounded from below.) If each of the quarks and leptons of the Standard Model is accompanied by two complex scalars with $\lambda_S = |\lambda_f|^2$, then the $\Lambda_{\rm UV}^2$ contributions of Figs. 1a and 1b will neatly cancel.[2] Clearly, more restrictions on the theory will be necessary to ensure that this success persists to higher orders, so that, for example, the contributions in Fig. 2 and eq. (1.4) from a very heavy fermion are cancelled by the two-loop effects of some very heavy bosons. Fortunately, conditions for cancelling all such contributions to scalar masses are not only possible, but are actually unavoidable once we merely assume that a symmetry relating fermions and bosons, called a *supersymmetry*, should exist.

A supersymmetry transformation turns a bosonic state into a fermionic state, and vice versa. The operator Q which generates such transformations must be an anticommuting spinor, with

$$Q|{\rm Boson}\rangle = |{\rm Fermion}\rangle; \qquad Q|{\rm Fermion}\rangle = |{\rm Boson}\rangle. \tag{1.5}$$

Spinors are intrinsically complex objects, so $Q^\dagger$ (the hermitian conjugate of Q) is also a symmetry generator. Because Q and $Q^\dagger$ are fermionic operators, they carry spin angular momentum 1/2, so it is clear that supersymmetry must be a spacetime symmetry. The possible forms for such symmetries in an interacting quantum field theory

are highly restricted by the Haag-Lopuszanski-Sohnius extension of the Coleman-Mandula theorem.[3] For realistic theories which, like the Standard Model, have chiral fermions (i.e., fermions whose left- and right-handed pieces transform differently under the gauge group) and thus the possibility of parity-violating interactions, this theorem implies that the generators Q and $Q^\dagger$ must satisfy an algebra of anticommutation and commutation relations with the schematic form

$$\{Q, Q^\dagger\} = P^\mu \tag{1.6}$$
$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0 \tag{1.7}$$
$$[P^\mu, Q] = [P^\mu, Q^\dagger] = 0 \tag{1.8}$$

where P^μ is the momentum generator of spacetime translations. Here we have ruthlessly suppressed the spinor indices on Q and $Q^\dagger$; after developing some notation we will, in section 3.1, derive the precise version of eqs. (1.6)-(1.8) with indices restored. In the meantime, we simply note that the appearance of P^μ on the right-hand side of eq. (1.6) is unsurprising, since it transforms under Lorentz boosts and rotations as a spin-1 object while Q and $Q^\dagger$ on the left-hand side each transform as spin-1/2 objects.

The single-particle states of a supersymmetric theory fall naturally into irreducible representations of the supersymmetry algebra which are called *supermultiplets*. Each supermultiplet contains both fermion and boson states, which are commonly known as *superpartners* of each other. By definition, if $|\Omega\rangle$ and $|\Omega'\rangle$ are members of the same supermultiplet, then $|\Omega'\rangle$ is proportional to some combination of Q and $Q^\dagger$ operators acting on $|\Omega\rangle$, up to a spacetime translation or rotation. The (mass)^2 operator $-P^2$ commutes with the operators Q, $Q^\dagger$, and with all spacetime rotation and translation operators, so it follows immediately that particles which inhabit the same irreducible supermultiplet must have equal eigenvalues of $-P^2$, and therefore equal masses.

The supersymmetry generators $Q, Q^\dagger$ also commute with the generators of gauge transformations. Therefore particles in the same supermultiplet must also be in the same representation of the gauge group, and so must have the same electric charges, weak isospin, and color degrees of freedom.

Each supermultiplet contains an equal number of fermion and boson degrees of freedom. To prove this, consider the operator $(-1)^{2s}$ where s is the spin angular momentum. By the spin-statistics theorem, this operator has eigenvalue $+1$ acting on a bosonic state and eigenvalue -1 acting on a fermionic state. Any fermionic operator will turn a bosonic state into a fermionic state and vice versa. Therefore $(-1)^{2s}$ must anticommute with every fermionic operator in the theory, and in particular with Q and $Q^\dagger$. Now consider the subspace of states $|i\rangle$ in a supermultiplet which have the same eigenvalue p^μ of the four-momentum operator P^μ. In view of eq. (1.8), any combination of Q or $Q^\dagger$ acting on $|i\rangle$ will give another state $|i'\rangle$ which has the same four-momentum eigenvalue. Therefore one has a completeness relation $\sum_i |i\rangle\langle i| = 1$ within this subspace of states. Now one can take a trace over all such states of the operator $(-1)^{2s}P^\mu$ (including each spin helicity state separately):

$$\sum_i \langle i|(-1)^{2s}P^\mu|i\rangle \;=\; \sum_i \langle i|(-1)^{2s}QQ^\dagger|i\rangle + \sum_i \langle i|(-1)^{2s}Q^\dagger Q|i\rangle$$

$$
\begin{aligned}
&= \sum_i \langle i|(-1)^{2s} Q Q^\dagger |i\rangle + \sum_i \sum_j \langle i|(-1)^{2s} Q^\dagger |j\rangle\langle j|Q|i\rangle \\
&= \sum_i \langle i|(-1)^{2s} Q Q^\dagger |i\rangle + \sum_j \langle j|Q(-1)^{2s} Q^\dagger |j\rangle \\
&= \sum_i \langle i|(-1)^{2s} Q Q^\dagger |i\rangle - \sum_j \langle j|(-1)^{2s} Q Q^\dagger |j\rangle \\
&= 0. \qquad (1.9)
\end{aligned}
$$

The first equality follows from the supersymmetry algebra relation eq. (1.6); the second and third from use of the completeness relation; and the fourth from the fact that $(-1)^{2s}$ must anticommute with Q. Now $\sum_i \langle i|(-1)^{2s} P^\mu |i\rangle = p^\mu \ \mathrm{Tr}[(-1)^{2s}]$ is just proportional to the number of bosonic degrees of freedom n_B minus the number of fermionic degrees of freedom n_F in the trace, so that

$$n_B = n_F \qquad (1.10)$$

must hold for a given $p^\mu \neq 0$ in each supermultiplet.

The simplest possibility for a supermultiplet which is consistent with eq. (1.10) has a single Weyl fermion (with two helicity states, so $n_F = 2$) and two real scalars (each with $n_B = 1$). It is natural to assemble the two real scalar degrees of freedom into a complex scalar field; as we will see below this provides for convenient formulation of the supersymmetry algebra, Feynman rules, supersymmetry violating effects, etc. This combination of a two-component Weyl fermion and a complex scalar field is called a *chiral* or *matter* or *scalar* supermultiplet.

The next simplest possibility for a supermultiplet contains a spin-1 vector boson. If the theory is to be renormalizable this must be a gauge boson which is massless, at least before the gauge symmetry is spontaneously broken. A massless spin-1 boson has two helicity states, so the number of bosonic degrees of freedom is $n_B = 2$. Its superpartner is therefore a massless spin-1/2 Weyl fermion, again with two helicity states, so $n_F = 2$. (If one tried instead to use a massless spin-3/2 fermion, the theory would not be renormalizable.) Gauge bosons must transform as the adjoint representation of the gauge group, so their fermionic partners, called *gauginos*, must also. Since the adjoint representation of a gauge group is always its own conjugate, this means in particular that these fermions must have the same gauge transformation properties for left-handed and for right-handed components. Such a combination of spin-1/2 gauginos and spin-1 gauge bosons is called a *gauge* or *vector* supermultiplet.

There are other possible combinations of particles with spins which can satisfy eq. (1.10). However, these are always reducible to combinations of chiral and gauge supermultiplets if they have renormalizable interactions, except in certain theories with "extended" supersymmetry. Theories with extended supersymmetry have more than one distinct copy of the supersymmetry generators $Q, Q^\dagger$. Such theories are mathematically amusing, but evidently do not have any phenomenological prospects. The reason is that extended supersymmetry in four-dimensional field theories cannot allow for chiral fermions or parity violation as observed in the Standard Model. So we will not discuss such possibilities further, although extended supersymmetry in higher

dimensional field theories might describe the real world if the extra dimensions are compactified, and extended supersymmetry in four dimensions provides interesting toy models. The ordinary, non-extended, phenomenologically-viable type of supersymmetric model is sometimes called $N = 1$ supersymmetry, with N referring to the number of supersymmetries (the number of distinct copies of $Q, Q^\dagger$).

In a supersymmetric extension of the Standard Model,[4,5,6] each of the known fundamental particles must therefore be in either a chiral or gauge supermultiplet and have a superpartner with spin differing by 1/2 unit. The first step in understanding the exciting phenomenological consequences of this prediction is to decide how the known particles fit into supermultiplets, and to give them appropriate names. A crucial observation here is that only chiral supermultiplets can contain fermions whose left-handed parts transform differently under the gauge group than their right-handed parts. All of the Standard Model fermions (the known quarks and leptons) have this property, so they must be members of chiral supermultiplets.[†] The names for the spin-0 partners of the quarks and leptons are constructed by prepending an "s", which is short for scalar. Thus generically they are called *squarks* and *sleptons* (short for "scalar quark" and "scalar lepton"). The left-handed and right-handed pieces of the quarks and leptons are separate two-component Weyl fermions with different gauge transformation properties in the Standard Model, so each must have its own complex scalar partner. The symbols for the squarks and sleptons are the same as for the corresponding fermion, but with a tilde used to denote the superpartner of a Standard Model particle. For example, the superpartners of the left-handed and right-handed parts of the electron Dirac field are called left- and right-handed selectrons, and are denoted $\widetilde{e}_L$ and $\widetilde{e}_R$. It is important to keep in mind that the "handedness" here does not refer to the helicity of the selectrons (they are spin-0 particles) but to that of their superpartners. A similar nomenclature applies for smuons and staus: $\widetilde{\mu}_L$, $\widetilde{\mu}_R$, $\widetilde{\tau}_L$, $\widetilde{\tau}_R$. In the Standard Model the neutrinos are always left-handed, so the sneutrinos are denoted generically by $\widetilde{\nu}$, with a possible subscript indicating which lepton flavor they carry: $\widetilde{\nu}_e$, $\widetilde{\nu}_\mu$, $\widetilde{\nu}_\tau$. Finally, a complete list of the squarks is $\widetilde{q}_L$, $\widetilde{q}_R$ with $q = u, d, s, c, b, t$. The gauge interactions of each of these squark and slepton field are the same as for the corresponding Standard Model fermion; for instance, a left-handed squark like $\widetilde{u}_L$ will couple to the W boson while $\widetilde{u}_R$ will not.

It seems clear that the Higgs scalar boson must reside in a chiral supermultiplet, since it has spin 0. Actually, it turns out that one chiral supermultiplet is not enough. One way to see this is to note that if there were only one Higgs chiral supermultiplet, the electroweak gauge symmetry would suffer a triangle gauge anomaly, and would be inconsistent as a quantum theory. This is because the conditions for cancellation of gauge anomalies include $\mathrm{Tr}[Y^3] = \mathrm{Tr}[T_3^2 Y] = 0$, where T_3 and Y are the third component of weak isospin and the weak hypercharge, respectively, in a normalization where the ordinary electric charge is $Q_{\rm EM} = T_3 + Y$. The traces run over all of the left-handed Weyl fermionic degrees of freedom in the theory. In the Standard Model, these conditions are already satisfied, somewhat miraculously, by the known quarks

[†] In particular, one cannot attempt to make a spin-1/2 neutrino be the superpartner of the spin-1 photon; the neutrino is in a doublet, and the photon neutral, under weak isospin.

Table 1: Chiral supermultiplets in the Minimal Supersymmetric Standard Model.

Names		spin 0	spin 1/2	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
squarks, quarks	Q	$(\tilde{u}_L\ \ \tilde{d}_L)$	$(u_L\ \ d_L)$	$(\ \mathbf{3},\ \mathbf{2}\ ,\ \frac{1}{6})$
($\times 3$ families)	$\overline{u}$	$\tilde{u}_R^*$	$u_R^\dagger$	$(\ \overline{\mathbf{3}},\ \mathbf{1},\ -\frac{2}{3})$
	$\overline{d}$	$\tilde{d}_R^*$	$d_R^\dagger$	$(\ \overline{\mathbf{3}},\ \mathbf{1},\ \frac{1}{3})$
sleptons, leptons	L	$(\tilde{\nu}\ \ \tilde{e}_L)$	$(\nu\ \ e_L)$	$(\ \mathbf{1},\ \mathbf{2}\ ,\ -\frac{1}{2})$
($\times 3$ families)	$\overline{e}$	$\tilde{e}_R^*$	$e_R^\dagger$	$(\ \mathbf{1},\ \mathbf{1},\ 1)$
Higgs, higgsinos	H_u	$(H_u^+\ \ H_u^0)$	$(\widetilde{H}_u^+\ \ \widetilde{H}_u^0)$	$(\ \mathbf{1},\ \mathbf{2}\ ,\ +\frac{1}{2})$
	H_d	$(H_d^0\ \ H_d^-)$	$(\widetilde{H}_d^0\ \ \widetilde{H}_d^-)$	$(\ \mathbf{1},\ \mathbf{2}\ ,\ -\frac{1}{2})$

and leptons. Now, a fermionic partner of a Higgs chiral supermultiplet must be a weak isodoublet with weak hypercharge $Y = 1/2$ or $Y = -1/2$. In either case alone, such a fermion will make a non-zero contribution to the traces and spoil the anomaly cancellation. This can be avoided if there are two Higgs supermultiplets, one with each of $Y = \pm 1/2$. In that case the total contribution to the anomaly traces from the two fermionic members of the Higgs chiral supermultiplets will vanish. As we will see in section 5.1, both of these are also necessary for another completely different reason: because of the structure of supersymmetric theories, only a $Y = +1/2$ Higgs chiral supermultiplet can have the Yukawa couplings necessary to give masses to charge $+2/3$ up-type quarks (up, charm, top), and only a $Y = -1/2$ Higgs can have the Yukawa couplings necessary to give masses to charge $-1/3$ down-type quarks (down, strange, bottom) and to charged leptons. We will call the $SU(2)_L$-doublet complex scalar fields corresponding to these two cases H_u and H_d respectively.[‡] The weak isospin components of H_u with $T_3 = (+1/2,\ -1/2)$ have electric charges 1, 0 respectively, and are denoted $(H_u^+,\ H_u^0)$. Similarly, the $SU(2)_L$-doublet complex scalar H_d has $T_3 = (+1/2,\ -1/2)$ components $(H_d^0,\ H_d^-)$. The neutral scalar that corresponds to the physical Standard Model Higgs boson is in a linear combination of H_u^0 and H_d^0; we will discuss this further in section 7.2. The generic nomenclature for a spin-1/2 superpartner is to append "-ino" to the name of the Standard Model particle, so the fermionic partners of the Higgs scalars are called higgsinos. They are denoted by $\widetilde{H}_u$, $\widetilde{H}_d$ for the $SU(2)_L$-doublet left-handed Weyl spinor fields, with weak isospin components $\widetilde{H}_u^+$, $\widetilde{H}_u^0$ and $\widetilde{H}_d^0$, $\widetilde{H}_d^-$.

We have now found all of the chiral supermultiplets of a minimal phenomenologically viable extension of the Standard Model. They are summarized in Table 1, classified according to their transformation properties under the Standard Model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, which combines u_L, d_L and ν, e_L degrees of

[‡]Other notations which are popular in the literature have $H_d, H_u \to H_1, H_2$ or $H, \overline{H}$. The one used here has the virtue of making it easy to remember which Higgs is responsible for giving masses to which quarks.

Table 2: Gauge supermultiplets in the Minimal Supersymmetric Standard Model.

Names	spin 1/2	spin 1	$SU(3)_C,\ SU(2)_L,\ U(1)_Y$
gluino, gluon	$\tilde{g}$	g	$(\ \mathbf{8},\ \mathbf{1}\ ,\ 0)$
winos, W bosons	$\widetilde{W}^\pm\ \ \widetilde{W}^0$	$W^\pm\ \ W^0$	$(\ \mathbf{1},\ \mathbf{3}\ ,\ 0)$
bino, B boson	$\widetilde{B}^0$	B^0	$(\ \mathbf{1},\ \mathbf{1}\ ,\ 0)$

freedom into $SU(2)_L$ doublets. Here we have followed the standard convention that all chiral supermultiplets are defined in terms of left-handed Weyl spinors, so that the *conjugates* of the right-handed quarks and leptons (and their superpartners) appear in Table 1. This protocol for defining chiral supermultiplets turns out to be very useful for constructing supersymmetric lagrangians, as we will see in section 3. It is useful also to have a symbol for each of the chiral supermultiplets as a whole; these are indicated in the second column of Table 1. Thus for example Q stands for the $SU(2)_L$-doublet chiral supermultiplet containing $\tilde{u}_L, u_L$ (with weak isospin component $T_3 = +1/2$), and $\tilde{d}_L, d_L$ (with $T_3 = -1/2$), while $\overline{u}$ stands for the $SU(2)_L$-singlet supermultiplet containing $\tilde{u}_R^*, u_R^\dagger$. There are three families for each of the quark and lepton supermultiplets, but we have used first-family representatives in Table 1. Below, a family index $i = 1, 2, 3$ will be affixed to the chiral supermultiplet names (Q_i, $\overline{u}_i, \ldots$) when needed, e.g. $(\overline{e}_1, \overline{e}_2, \overline{e}_3) = (\overline{e}, \overline{\mu}, \overline{\tau})$. The bar on $\overline{u}$, $\overline{d}$, $\overline{e}$ fields is part of the name, and does not denote any kind of conjugation.

It is interesting to note that the Higgs chiral supermultiplet H_d (containing H_d^0, H_d^-, $\widetilde{H}_d^0$, $\widetilde{H}_d^-$) has exactly the same Standard Model gauge quantum numbers as the left-handed sleptons and leptons L_i, e.g. $(\tilde{\nu}, \tilde{e}_L, \nu, e_L)$. Naively one might therefore suppose that we could have been more economical in our assignment by taking a neutrino and a Higgs scalar to be superpartners, instead of putting them in separate supermultiplets. This would amount to the proposal that the Higgs boson and a sneutrino should be the same particle. This is a nice try which played a key role in some of the first attempts to connect supersymmetry to phenomenology,[4] but it is now known not to work. Even ignoring the anomaly cancellation problem mentioned above, many insoluble phenomenological problems would result, including lepton number violation and a mass for at least one of the neutrinos in gross violation of experimental bounds. Therefore, all of the superpartners of Standard Model particles are really new particles, and cannot be identified with some other Standard Model state.

The vector bosons of the Standard Model clearly must reside in gauge supermultiplets. Their fermionic superpartners are generically referred to as gauginos. The $SU(3)_C$ color gauge interactions of QCD are mediated by the gluon, whose spin-1/2 color-octet supersymmetric partner is the gluino. As usual, a tilde is used to denote the supersymmetric partner of a Standard Model state, so the symbols for the gluon and gluino are g and $\tilde{g}$ respectively. The electroweak gauge symmetry $SU(2)_L \times U(1)_Y$ has associated with it spin-1 gauge bosons W^+, W^0, W^- and B^0, with spin-1/2 super-

partners $\widetilde{W}^+, \widetilde{W}^0, \widetilde{W}^-$ and $\tilde{B}^0$, called *winos* and *bino*. After electroweak symmetry breaking, the W^0, B^0 gauge eigenstates mix to give mass eigenstates Z^0 and γ. The corresponding gaugino mixtures of $\widetilde{W}^0$ and $\tilde{B}^0$ are called zino ($\widetilde{Z}^0$) and photino ($\widetilde{\gamma}$); if supersymmetry were unbroken, they would be mass eigenstates with masses m_Z and 0. Table 2 summarizes the gauge supermultiplets of a minimal supersymmetric extension of the Standard Model.

The chiral and gauge supermultiplets in Tables 1 and 2 make up the particle content of the Minimal Supersymmetric Standard Model (MSSM). The most obvious and interesting feature of this theory is that none of the superpartners of the Standard Model particles has been discovered as of this writing. If supersymmetry were unbroken, then there would have to be selectrons $\widetilde{e}_L$ and $\widetilde{e}_R$ with masses exactly equal to $m_e = 0.511...$ MeV. A similar statement applies to each of the other sleptons and squarks, and there would also have to be a massless gluino and photino. These particles would have been extraordinarily easy to detect long ago. Clearly, therefore, *supersymmetry is a broken symmetry* in the vacuum state chosen by nature.

A very important clue as to the nature of supersymmetry breaking can be obtained by returning to the motivation provided by the hierarchy problem. Supersymmetry forced us to introduce two complex scalar fields for each Standard Model Dirac fermion, which is just what is needed to enable a cancellation of the quadratically divergent ($\Lambda_{\rm UV}^2$) pieces of eqs. (1.2) and (1.3). This sort of cancellation also requires that the associated dimensionless couplings should be related (e.g. $\lambda_S = |\lambda_f|^2$). The necessary relationships between couplings indeed occur in unbroken supersymmetry, as we will see in section 3. In fact, unbroken supersymmetry guarantees that the quadratic divergences in scalar squared masses must vanish to all orders in perturbation theory.[§] Now, if broken supersymmetry is still to provide a solution to the hierarchy problem, then the relationships between dimensionless couplings which hold in an unbroken supersymmetric theory must be maintained. Otherwise, there would be quadratically divergent radiative corrections to the Higgs scalar masses of the form

$$\Delta m_H^2 = \frac{1}{8\pi^2}(\lambda_S - |\lambda_f|^2)\Lambda_{\rm UV}^2 + \ldots . \tag{1.11}$$

We are therefore led to consider "soft" supersymmetry breaking. This means that the effective lagrangian of the MSSM can be written in the form

$$\mathcal{L} = \mathcal{L}_{\rm SUSY} + \mathcal{L}_{\rm soft}, \tag{1.12}$$

where $\mathcal{L}_{\rm SUSY}$ preserves supersymmetry invariance, and $\mathcal{L}_{\rm soft}$ violates supersymmetry but contains only mass terms and couplings with *positive* mass dimension. Without further justification, soft supersymmetry breaking might seem like a rather arbitrary requirement. Fortunately, we will see in section 6 that theoretical models for supersymmetry breaking can indeed yield effective lagrangians with just such terms for $\mathcal{L}_{\rm soft}$. If the largest mass scale associated with the soft terms is denoted $m_{\rm soft}$, then

[§] A simple way to understand this is to note that unbroken supersymmetry requires the degeneracy of scalar and fermion masses. Radiative corrections to fermion masses are known to diverge at most logarithmically, so the same must be true for scalar masses in unbroken supersymmetry.

the additional non-supersymmetric corrections to the Higgs scalar (mass)^2 must vanish in the $m_{\rm soft} \to 0$ limit, so by dimensional analysis they cannot be proportional to $\Lambda_{\rm UV}^2$. More generally, these models maintain the cancellation of quadratically divergent terms in the radiative corrections of all scalar masses, to all orders in perturbation theory. The corrections also cannot go like $\Delta m_H^2 \sim m_{\rm soft}\Lambda_{\rm UV}$, because in general the loop momentum integrals always diverge either quadratically or logarithmically, not linearly, as $\Lambda_{\rm UV} \to \infty$. So they must be of the form

$$\Delta m_H^2 = m_{\rm soft}^2 \left[\frac{\lambda}{16\pi^2}\ln(\Lambda_{\rm UV}/m_{\rm soft}) + \ldots\right]. \tag{1.13}$$

Here λ is schematic for various dimensionless couplings, and the ellipses stand both for terms which are independent of $\Lambda_{\rm UV}$ and for higher loop corrections (which depend on $\Lambda_{\rm UV}$ through powers of logarithms).

Since the mass splittings between the known Standard Model particles and their superpartners are just determined by the parameters $m_{\rm soft}$ appearing in $\mathcal{L}_{\rm soft}$, eq. (1.13) tells us that the superpartner masses cannot be too huge. Otherwise, we would lose our successful cure for the hierarchy problem since the $m_{\rm soft}^2$ corrections to the Higgs scalar (mass)^2 would be unnaturally large compared to the electroweak breaking scale of 174 GeV. The top and bottom squarks and the winos and bino give especially large contributions to $\Delta m_{H_u}^2$ and $\Delta m_{H_d}^2$, but the gluino mass and all the other squark and slepton masses also feed in indirectly, through radiative corrections to the top and bottom squark masses. Furthermore, in most viable models of supersymmetry breaking that are not unduly contrived, the superpartner masses do not differ from each other by more than about an order of magnitude. Using $\Lambda_{\rm UV} \sim M_{\rm P}$ and $\lambda \sim 1$ in eq. (1.13), one finds that roughly speaking $m_{\rm soft}$, and therefore the masses of at least the lightest few superpartners, should be at the most about 1 TeV or so, in order for the MSSM scalar potential to provide a Higgs VEV resulting in $m_W, m_Z = 80.4$, 91.2 GeV without miraculous cancellations. This is the best reason for the optimism among many theorists that supersymmetry will be discovered at LEP2, the Tevatron, the LHC, or a next generation lepton linear collider.

However, it is useful to keep in mind that the hierarchy problem was *not* the historical motivation for the development of supersymmetry in the early 1970's. The supersymmetry algebra and supersymmetric field theories were originally concocted independently in various disguises [7,8,9,10] which bear little resemblance to the MSSM. It is quite impressive that a theory which was developed for quite different reasons, including purely aesthetic ones, can later be found to provide a solution for the hierarchy problem.

One might also wonder if there is any good reason why all of the superpartners of the Standard Model particles should be heavy enough to have avoided discovery so far. There is. All of the particles in the MSSM which have been discovered so far have something in common; they would necessarily be massless in the absence of electroweak symmetry breaking. In particular, the masses of the $W^\pm, Z^0$ bosons and all quarks and leptons are equal to dimensionless coupling constants times the Higgs VEV $\sim$ 174 GeV, while the photon and gluon are required to be massless

by electromagnetic and QCD gauge invariance. Conversely, all of the undiscovered particles in the MSSM have exactly the opposite property, since each of them can have a lagrangian mass term in the absence of electroweak symmetry breaking. For the squarks, sleptons, and Higgs scalars this follows from a general property of complex scalar fields that a mass term $m^2|\phi|^2$ is always allowed by all gauge symmetries. For the higgsinos and gauginos, it follows from the fact that they are fermions in a real representation of the gauge group. So, from the point of view of the MSSM, the discovery of the top quark in 1995 marked a quite natural milestone; the already-discovered particles are precisely those which had to be light, based on the principle of electroweak gauge symmetry. There is a single exception: one neutral Higgs scalar boson should be lighter than about 150 GeV if supersymmetry is correct, for reasons to be discussed in section 7.2.

A very important feature of the MSSM is that the superpartners listed in Tables 1 and 2 are not necessarily the mass eigenstates of the theory. This is because after electroweak symmetry breaking and supersymmetry breaking effects are included, there can be mixing between the electroweak gauginos and the higgsinos, and within the various sets of squarks and sleptons and Higgs scalars which have the same electric charge. The lone exception is the gluino, which is a color octet fermion and therefore does not have the appropriate quantum numbers to mix with any other particle. The masses and mixings of the superpartners are obviously of paramount importance to experimentalists. It is perhaps slightly less obvious that these phenomenological issues are all quite directly related to one central question which is also the focus of much of the theoretical work in supersymmetry: "How is supersymmetry broken?" The reason for this is that most of what we do not already know about the MSSM has to do with $\mathcal{L}_{\rm soft}$. The structure of supersymmetric lagrangians allows very little arbitrariness, as we will see in section 3. In fact, all of the dimensionless couplings and all but one mass term in the supersymmetric part of the MSSM lagrangian correspond directly to some parameter in the ordinary Standard Model which has already been measured by experiment. For example, we will find out that the supersymmetric coupling of a gluino to a squark and a quark is determined by the QCD coupling constant α_S. In contrast, the supersymmetry-breaking part of the lagrangian apparently contains many unknown parameters and a considerable amount of arbitrariness. Each of the mass splittings between Standard Model particles and their superpartners correspond to terms in the MSSM lagrangian which are purely supersymmetry-breaking in their origin and effect. These soft supersymmetry-breaking terms can also introduce a large number of mixing angles and CP-violating phases not found in the Standard Model. Fortunately, as we will see in section 5.4, there is already rather strong evidence that the supersymmetry-breaking terms in the MSSM are actually not arbitrary at all. Furthermore, the additional parameters will be measured and constrained as the superpartners are detected. From a theoretical perspective, the challenge is to explain all of these parameters with a model for supersymmetry breaking.

The rest of our discussion is organized as follows. Section 2 provides a list of important notations. In section 3, we will learn how to construct lagrangians for supersymmetric field theories. Soft supersymmetry-breaking couplings are described in

section 4. In section 5, we will apply the preceding general results to the special case of the MSSM, introduce the concept of R-parity, and emphasize the importance of the structure of the soft terms. Section 6 outlines some considerations for understanding the origin of supersymmetry breaking, and the consequences of various proposals. In section 7, we will study the mass and mixing angle patterns of the new particles predicted by the MSSM. Their decay modes are considered in section 8, and some of the qualitative features of experimental signals for supersymmetry are reviewed in section 9. Section 10 describes some sample variations on the standard MSSM picture. The discussion will be lacking in historical accuracy or perspective, for which the author apologizes in advance. The reader is encouraged to consult the many outstanding textbooks,[11–18] review articles,[19–34] and the reprint volume,[35] which contain a much more consistent guide to the original literature.

2 Interlude: Notations and Conventions

Before proceeding to discuss the construction of supersymmetric lagrangians, we need to specify our notations. It is overwhelmingly convenient to employ two-component Weyl notation for fermions, rather than four component Dirac or Majorana spinors. The lagrangian of the Standard Model (and supersymmetric extensions of it) violates parity; each Dirac fermion has left-handed and right-handed parts with completely different electroweak gauge interactions. If one used four-component notation, one would therefore have to include clumsy left- and right-handed projection operators

$$P_{L,R} = (1 \pm \gamma_5)/2 \tag{2.1}$$

all over the place. The two-component Weyl fermion notation has the advantage of treating fermionic degrees of freedom with different gauge quantum numbers separately from the start (as Nature intended for us to do). But an even better reason for using two-component notation here is that in supersymmetric models the minimal building blocks of matter are chiral supermultiplets, each of which contains a single two-component Weyl fermion.

Since two-component fermion notation may be unfamiliar to some readers, we will specify our conventions [†] by showing how they correspond to the four-component fermion language. A four-component Dirac fermion Ψ_D with mass M is described by the lagrangian

$$\mathcal{L}_{\rm Dirac} = -i\overline{\Psi}_D\gamma^\mu\partial_\mu\Psi_D - M\overline{\Psi}_D\Psi_D\,. \tag{2.2}$$

We use a spacetime metric $\eta_{\mu\nu} = {\rm diag}(-1,1,1,1)$. For our purposes it is convenient to use the specific representation of the 4×4 gamma matrices given in 2×2 blocks by

$$\gamma_\mu = \begin{pmatrix} 0 & \sigma_\mu \\ \overline{\sigma}_\mu & 0 \end{pmatrix}; \qquad \gamma_5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \tag{2.3}$$

[†] The conventions used here are the same as in Ref.[11], except that we use a dagger rather than a bar to indicate hermitian conjugation for Weyl spinors.

where

$$\sigma_0 = \overline{\sigma}_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \qquad \sigma_1 = -\overline{\sigma}_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix};$$
$$\sigma_2 = -\overline{\sigma}_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}; \qquad \sigma_3 = -\overline{\sigma}_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2.4}$$

In this basis, a four component Dirac spinor is written in terms of 2 two-component, complex, anticommuting objects $(\xi)_\alpha$ with $\alpha = 1, 2$ and $(\chi^\dagger)^{\dot\alpha}$ with $\dot\alpha = 1, 2$:

$$\Psi_{D} = \begin{pmatrix} \xi_\alpha \\ \chi^{\dagger\dot\alpha} \end{pmatrix}; \qquad \overline{\Psi}_{D} = \begin{pmatrix} \chi^\alpha & \xi^\dagger_{\dot\alpha} \end{pmatrix}. \tag{2.5}$$

The undotted (dotted) indices are used for the first (last) two components of a Dirac spinor. The heights of these indices are important; for example, comparing eqs. (2.2)-(2.5), we observe that the matrices $(\sigma^\mu)_{\alpha\dot\alpha}$ and $(\overline{\sigma}^\mu)^{\dot\alpha\alpha}$ defined by eq. (2.4) carry indices with the heights as indicated. The spinor indices are raised and lowered using the antisymmetric symbol $\epsilon^{12} = -\epsilon^{21} = \epsilon_{21} = -\epsilon_{12} = 1$; $\epsilon_{11} = \epsilon_{22} = \epsilon^{11} = \epsilon^{22} = 0$, according to

$$\xi_\alpha = \epsilon_{\alpha\beta}\xi^\beta; \qquad \xi^\alpha = \epsilon^{\alpha\beta}\xi_\beta; \qquad \chi^\dagger_{\dot\alpha} = \epsilon_{\dot\alpha\dot\beta}\chi^{\dagger\dot\beta}; \qquad \chi^{\dagger\dot\alpha} = \epsilon^{\dot\alpha\dot\beta}\chi^\dagger_{\dot\beta}. \tag{2.6}$$

This is consistent since $\epsilon_{\alpha\beta}\epsilon^{\beta\gamma} = \epsilon^{\gamma\beta}\epsilon_{\beta\alpha} = \delta^\gamma_\alpha$ and $\epsilon_{\dot\alpha\dot\beta}\epsilon^{\dot\beta\dot\gamma} = \epsilon^{\dot\gamma\dot\beta}\epsilon_{\dot\beta\dot\alpha} = \delta^{\dot\gamma}_{\dot\alpha}$. The field ξ is called a "left-handed Weyl spinor" and $\chi^\dagger$ is a "right-handed Weyl spinor". The names fit, because

$$P_L\Psi_{D} = \begin{pmatrix} \xi_\alpha \\ 0 \end{pmatrix}; \qquad P_R\Psi_{D} = \begin{pmatrix} 0 \\ \chi^{\dagger\dot\alpha} \end{pmatrix}. \tag{2.7}$$

The hermitian conjugate of a left-handed Weyl spinor is a right-handed Weyl spinor $(\psi_\alpha)^\dagger = (\psi^\dagger)_{\dot\alpha}$ and vice versa $(\psi^{\dagger\dot\alpha})^\dagger = \psi^\alpha$. Therefore any particular fermionic degrees of freedom can be described equally well using a Weyl spinor which is left-handed (with an undotted index) or by one which is right-handed (with a dotted index). By convention, all names of fermion fields are chosen so that left-handed Weyl spinors do not carry daggers and right-handed Weyl spinors do carry daggers, as in eq. (2.5).

It is useful to abbreviate expressions with two spinor fields by suppressing undotted indices contracted like ${}^\alpha{}_\alpha$ and dotted indices contracted like ${}_{\dot\alpha}{}^{\dot\alpha}$. In particular,

$$\xi\chi \equiv \xi^\alpha\chi_\alpha = \xi^\alpha\epsilon_{\alpha\beta}\chi^\beta = -\chi^\beta\epsilon_{\alpha\beta}\xi^\alpha = \chi^\beta\epsilon_{\beta\alpha}\xi^\alpha = \chi^\beta\xi_\beta \equiv \chi\xi \tag{2.8}$$

with, conveniently, no minus sign in the end. [A minus sign appeared in eq. (2.8) from exchanging the order of anticommuting spinors, but it disappeared due to the antisymmetry of the ϵ symbol.] Likewise, $\xi^\dagger\chi^\dagger$ and $\chi^\dagger\xi^\dagger$ are equivalent abbreviations for $\chi^\dagger_{\dot\alpha}\xi^{\dagger\dot\alpha} = (\xi\chi)^*$, the complex conjugate of $\xi\chi$. In a similar way,

$$\xi^\dagger\overline{\sigma}^\mu\chi = -\chi\sigma^\mu\xi^\dagger = (\chi^\dagger\overline{\sigma}^\mu\xi)^* = -(\xi\sigma^\mu\chi^\dagger)^* \tag{2.9}$$

stands for $\xi^\dagger_{\dot\alpha}(\overline{\sigma}^\mu)^{\dot\alpha\alpha}\chi_\alpha$, etc. With these conventions, the Dirac lagrangian eq. (2.2) can now be rewritten:

$$\mathcal{L}_{\rm Dirac} = -i\overline{\Psi}_{D}\gamma^\mu\partial_\mu\Psi_{D} - M\overline{\Psi}_{D}\Psi_{D} \tag{2.10}$$
$$= -i\xi^\dagger\overline{\sigma}^\mu\partial_\mu\xi - i\chi^\dagger\overline{\sigma}^\mu\partial_\mu\chi - M(\xi\chi + \xi^\dagger\chi^\dagger) \tag{2.11}$$

where we have dropped a total derivative piece $i\partial_\mu(\chi^\dagger\overline{\sigma}^\mu\chi)$ which does not affect the action.

A four-component Majorana spinor can be obtained from the Dirac spinor of eq. (2.5) by imposing the constraint $\chi = \xi$, so that

$$\Psi_{\rm M} = \begin{pmatrix} \xi_\alpha \\ \xi^{\dagger\dot\alpha} \end{pmatrix}; \qquad \overline{\Psi}_{\rm M} = \begin{pmatrix} \xi^\alpha & \xi^\dagger_{\dot\alpha} \end{pmatrix}. \tag{2.12}$$

The lagrangian for a Majorana fermion with mass M

$$\mathcal{L}_{\rm Majorana} = -\frac{i}{2}\overline{\Psi}_{\rm M}\gamma^\mu\partial_\mu\Psi_{\rm M} - \frac{1}{2}M\overline{\Psi}_{\rm M}\Psi_{\rm M} \tag{2.13}$$

in the four-component Majorana spinor form can therefore be rewritten

$$\mathcal{L}_{\rm Majorana} = -i\xi^\dagger\overline{\sigma}^\mu\partial_\mu\xi - \frac{1}{2}M(\xi\xi + \xi^\dagger\xi^\dagger) \tag{2.14}$$

in the more economical two-component Weyl spinor representation. [Note that even though ξ_α is anticommuting, $\xi\xi$ and its complex conjugate $\xi^\dagger\xi^\dagger$ do not vanish, because of the suppressed ϵ symbol, see eq. (2.8).]

More generally, any theory involving spin-1/2 fermions can *always* be written down in terms of a collection of left-handed Weyl spinors ψ_i with

$$\mathcal{L} = -i\psi^{\dagger i}\overline{\sigma}^\mu\partial_\mu\psi_i + \ldots \tag{2.15}$$

where the ellipses represent possible mass terms, gauge interactions, and Yukawa interactions with scalar fields. Here the index i runs over the appropriate gauge and flavor indices of the fermions; it is raised or lowered by hermitian conjugation. There is a different ψ_i for the left-handed piece and for the hermitian conjugate of the right-handed piece of a Dirac fermion. If one has any expression involving bilinears in four-component spinors

$$\Psi_1 = \begin{pmatrix} \xi_1 \\ \chi_1^\dagger \end{pmatrix} \qquad \text{and} \qquad \Psi_2 = \begin{pmatrix} \xi_2 \\ \chi_2^\dagger \end{pmatrix}, \tag{2.16}$$

then one can translate into two-component Weyl spinor language (or vice versa) using the dictionary:

$$\overline{\Psi}_1 P_L\Psi_2 = \chi_1\xi_2; \qquad \overline{\Psi}_1 P_R\Psi_2 = \xi_1^\dagger\chi_2^\dagger; \tag{2.17}$$
$$\overline{\Psi}_1\gamma^\mu P_L\Psi_2 = \xi_1^\dagger\overline{\sigma}^\mu\xi_2; \qquad \overline{\Psi}_1\gamma^\mu P_R\Psi_2 = \chi_1\sigma^\mu\chi_2^\dagger \tag{2.18}$$

etc. We will introduce a few other Weyl spinor identities in the following as they are needed.

Let us now see how the Standard Model quarks and leptons are described in this notation. The complete list of left-handed Weyl spinors can be given names corresponding to the chiral supermultiplets in Table 1:

$$Q_i = (u\ d),\ (c\ s),\ (t\ b) \tag{2.19}$$

$$\overline{u}_i = \overline{u},\ \overline{c},\ \overline{t} \qquad \overline{d}_i = \overline{d},\ \overline{s},\ \overline{b} \tag{2.20}$$

$$L_i = (\nu_e\ e),\ (\nu_\mu\ \mu),\ (\nu_\tau\ \tau) \tag{2.21}$$

$$\overline{e}_i = \overline{e},\ \overline{\mu},\ \overline{\tau}. \tag{2.22}$$

Here $i = 1, 2, 3$ is a family index. The bars on these fields are part of the names of the fields, and do *not* denote any kind of conjugation. Rather, the unbarred fields are the left-handed pieces of a Dirac spinor, while the barred fields are the names given to the conjugates of the right-handed piece of a Dirac spinor. For example, e is the same thing as e_L in Table 1, and $\overline{e}$ is the same as $e_R^\dagger$. Together they form a Dirac spinor:

$$\begin{pmatrix} e \\ \overline{e}^\dagger \end{pmatrix} \equiv \begin{pmatrix} e_L \\ e_R \end{pmatrix} \tag{2.23}$$

with similar equations for all of the other quark and charged lepton Dirac spinors. (The neutrinos of the Standard Model are not part of a Dirac spinor.) The fields Q_i and L_i are weak isodoublets which always go together when one is constructing interactions invariant under the full Standard Model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. Suppressing all color and weak isospin indices, the purely kinetic part of the Standard Model fermion lagrangian density is then

$$\mathcal{L} = -iQ^{\dagger i}\overline{\sigma}^\mu \partial_\mu Q_i - i\overline{u}^{\dagger i}\overline{\sigma}^\mu \partial_\mu \overline{u}_i - i\overline{d}^{\dagger i}\overline{\sigma}^\mu \partial_\mu \overline{d}_i - iL^{\dagger i}\overline{\sigma}^\mu \partial_\mu L_i - i\overline{e}^{\dagger i}\overline{\sigma}^\mu \partial_\mu \overline{e}_i \tag{2.24}$$

with the family index $i = 1, 2, 3$ summed over.

3 Supersymmetric lagrangians

In this section we will describe the construction of supersymmetric lagrangians. Our aim is to arrive at a sort of recipe which will allow us to write down the allowed interactions and mass terms of a general supersymmetric theory, so that later we can apply the results to the special case of the MSSM. We will not use the superfield language,[36] which is often more elegant and efficient for those who know it, but which might seem rather cabalistic to some readers. Our approach is therefore intended to be rather complementary to the superfield derivations given in Refs.[11–18] We begin by considering the simplest example of a supersymmetric theory in four dimensions.

3.1 The simplest supersymmetric model: a free chiral supermultiplet

The minimum fermion content of any theory in four dimensions consists of a single left-handed two-component Weyl fermion ψ. Since this is an intrinsically complex object, it seems sensible to choose as its superpartner a complex scalar field ϕ. The

simplest action we can write down for these fields just consists of kinetic energy terms for each:

$$S = \int d^4x\ (\mathcal{L}_{\rm scalar} + \mathcal{L}_{\rm fermion}) \tag{3.1}$$

$$\mathcal{L}_{\rm scalar} = -\partial^\mu \phi^* \partial_\mu \phi; \qquad \mathcal{L}_{\rm fermion} = -i\psi^\dagger \overline{\sigma}^\mu \partial_\mu \psi. \tag{3.2}$$

This is called the massless, non-interacting *Wess-Zumino model*,[9] and it corresponds to a single chiral supermultiplet as discussed in the Introduction.

A supersymmetry transformation should turn the scalar boson ϕ into something involving the fermion ψ_α. The simplest possibility for the transformation of the scalar field is

$$\delta\phi = \epsilon\psi; \qquad \delta\phi^* = \epsilon^\dagger \psi^\dagger \tag{3.3}$$

where ϵ^α is an infinitesimal, anticommuting, two-component Weyl fermion object which parameterizes the supersymmetry transformation. Until section 6.2, we will be discussing global supersymmetry, which means that ϵ^α is a constant, satisfying $\partial_\mu \epsilon^\alpha = 0$. Since ψ has dimensions of $(\text{mass})^{3/2}$ and ϕ has dimensions of (mass), it must be that ϵ has dimensions of $(\text{mass})^{-1/2}$. Using eq. (3.3), we find that the scalar part of the lagrangian transforms as

$$\delta\mathcal{L}_{\rm scalar} = -\epsilon\partial^\mu\psi\ \partial_\mu\phi^* - \epsilon^\dagger\partial^\mu\psi^\dagger\ \partial_\mu\phi. \tag{3.4}$$

We would like for this to be cancelled by $\delta\mathcal{L}_{\rm fermion}$, at least up to a total derivative, so that the action will be invariant under the supersymmetry transformation. Comparing eq. (3.4) with $\mathcal{L}_{\rm fermion}$, we see that for this to have any chance of happening, $\delta\psi$ should be linear in $\epsilon^\dagger$ and in ϕ and contain one spacetime derivative. Up to a multiplicative constant, there is only one possibility to try:

$$\delta\psi_\alpha = i(\sigma^\mu\epsilon^\dagger)_\alpha\ \partial_\mu\phi; \qquad \delta\psi^\dagger_{\dot\alpha} = -i(\epsilon\sigma^\mu)_{\dot\alpha}\ \partial_\mu\phi^*. \tag{3.5}$$

With this guess, one immediately obtains

$$\delta\mathcal{L}_{\rm fermion} = -\epsilon\sigma^\mu\overline{\sigma}^\nu\partial_\nu\psi\ \partial_\mu\phi^* + \psi^\dagger\overline{\sigma}^\nu\sigma^\mu\epsilon^\dagger\ \partial_\mu\partial_\nu\phi\,. \tag{3.6}$$

This can be put in a slightly more useful form by employing the Pauli matrix identities

$$[\sigma^\mu\overline{\sigma}^\nu + \sigma^\nu\overline{\sigma}^\mu]^\beta_\alpha = -2\eta^{\mu\nu}\delta^\beta_\alpha; \qquad [\overline{\sigma}^\mu\sigma^\nu + \overline{\sigma}^\nu\sigma^\mu]^{\dot\beta}_{\dot\alpha} = -2\eta^{\mu\nu}\delta^{\dot\beta}_{\dot\alpha} \tag{3.7}$$

and using the fact that partial derivatives commute ($\partial_\mu\partial_\nu = \partial_\nu\partial_\mu$). Equation (3.6) then becomes

$$\begin{aligned}\delta\mathcal{L}_{\rm fermion} &= \epsilon\partial^\mu\psi\ \partial_\mu\phi^* + \epsilon^\dagger\partial^\mu\psi^\dagger\ \partial_\mu\phi \\ &\quad -\partial_\mu\left(\epsilon\sigma^\nu\overline{\sigma}^\mu\psi\ \partial_\nu\phi^* + \epsilon\psi\ \partial^\mu\phi^* + \epsilon^\dagger\psi^\dagger\ \partial^\mu\phi\right).\end{aligned} \tag{3.8}$$

The first two terms here just cancel against $\delta\mathcal{L}_{\rm scalar}$, while the remaining contribution is a total derivative. So we arrive at

$$\delta S = \int d^4x\ \ (\delta\mathcal{L}_{\rm scalar} + \delta\mathcal{L}_{\rm fermion}) = 0, \tag{3.9}$$

justifying our guess of the numerical multiplicative factor made in eq. (3.5).

We are not quite finished in demonstrating that the theory described by eq. (3.1) is supersymmetric. We must also show that the supersymmetry algebra closes; in other words, that the commutator of two supersymmetry transformations is another symmetry of the theory. Using eq. (3.5) in eq. (3.3), one finds

$$(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})\phi = i(\epsilon_1\sigma^\mu\epsilon_2^\dagger - \epsilon_2\sigma^\mu\epsilon_1^\dagger)\,\partial_\mu\phi. \tag{3.10}$$

This is a remarkable result; in words, we have found that the commutator of two supersymmetry transformations gives us back the derivative of the original field. Since ∂_μ just corresponds to the generator of spacetime translations P_μ, eq. (3.10) implies the form of the supersymmetry algebra which was foreshadowed in eq. (1.6) of the Introduction. (We will make this statement more explicit before the end of this section.)

All of this will be for naught if we do not find the same result for the fermion ψ, however. Using eq. (3.3) in eq. (3.5), we find

$$(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})\psi_\alpha = i(\sigma^\mu\epsilon_1^\dagger)_\alpha\,\epsilon_2\partial_\mu\psi - i(\sigma^\mu\epsilon_2^\dagger)_\alpha\,\epsilon_1\partial_\mu\psi. \tag{3.11}$$

We can put this into a more useful form by applying the Fierz identity

$$\chi_\alpha\,(\xi\eta) = -\xi_\alpha\,(\eta\chi) - \eta_\alpha\,(\chi\xi) \tag{3.12}$$

with $\chi = \sigma^\mu\epsilon_1^\dagger$, $\xi = \epsilon_2$, $\eta = \partial_\mu\psi$, and again with $\chi = \sigma^\mu\epsilon_2^\dagger$, $\xi = \epsilon_1$, $\eta = \partial_\mu\psi$, followed in each case by an application of the identity eq. (2.9). The result is

$$\begin{aligned}(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})\psi_\alpha \;&=\; i(\epsilon_1\sigma^\mu\epsilon_2^\dagger - \epsilon_2\sigma^\mu\epsilon_1^\dagger)\,\partial_\mu\psi_\alpha \\ &\quad -i\epsilon_{1\alpha}\,\epsilon_2^\dagger\overline{\sigma}^\mu\partial_\mu\psi + i\epsilon_{2\alpha}\,\epsilon_1^\dagger\overline{\sigma}^\mu\partial_\mu\psi.\end{aligned} \tag{3.13}$$

The last two terms in (3.13) vanish on-shell; that is, if the equation of motion $\overline{\sigma}^\mu\partial_\mu\psi = 0$ following from the action is enforced. The remaining piece is exactly the same spacetime translation that we found for the scalar field.

The fact that the supersymmetry algebra only closes on-shell (when the classical equations of motion are satisfied) might be somewhat worrisome, since we would like the symmetry to hold even quantum mechanically. This can be fixed by a trick. We invent a new complex scalar field F which does not have a kinetic term. Such fields are called *auxiliary*, and they are really just book-keeping devices which allow the symmetry algebra to close off-shell. The lagrangian density for F and its complex conjugate is just

$$\mathcal{L}_{\rm auxiliary} = F^*F\,. \tag{3.14}$$

The dimensions of F are $(\text{mass})^2$, unlike an ordinary scalar field which has dimensions of (mass). Equation (3.14) leads to the not-very-exciting equations of motion $F = F^* = 0$. However, we can use the auxiliary fields to our advantage by including them

in the supersymmetry transformation rules. In view of eq. (3.13), a plausible thing to do is to make F transform into a multiple of the equation of motion for ψ:

$$\delta F = i\epsilon^\dagger\overline{\sigma}^\mu\partial_\mu\psi; \qquad\qquad \delta F^* = -i\partial_\mu\psi^\dagger\overline{\sigma}^\mu\epsilon. \tag{3.15}$$

Once again we have chosen the overall factor on the right hand side by virtue of foresight. Now the auxiliary part of the lagrangian density transforms as

$$\delta\mathcal{L}_{\rm auxiliary} = i\epsilon^\dagger\overline{\sigma}^\mu\partial_\mu\psi\;F^* - i\partial_\mu\psi^\dagger\overline{\sigma}^\mu\epsilon\;F \tag{3.16}$$

which vanishes on-shell, but not for arbitrary off-shell field configurations. It is easy to see that by adding an extra term to the transformation law for ψ and $\psi^\dagger$:

$$\delta\psi_\alpha = i(\sigma^\mu\epsilon^\dagger)_\alpha\,\partial_\mu\phi + \epsilon_\alpha F; \qquad \delta\psi_{\dot\alpha}^\dagger = -i(\epsilon\sigma^\mu)_{\dot\alpha}\,\partial_\mu\phi^* + \epsilon_{\dot\alpha}^\dagger F^* \tag{3.17}$$

one obtains an additional contribution to $\delta\mathcal{L}_{\rm fermion}$ which just cancels with $\delta\mathcal{L}_{\rm auxiliary}$, up to a total derivative term. So our "modified" theory with $\mathcal{L} = \mathcal{L}_{\rm scalar} + \mathcal{L}_{\rm fermion} + \mathcal{L}_{\rm auxiliary}$ is still invariant under supersymmetry transformations. Proceeding as before, one now obtains for each of the fields $X = \phi, \phi^*, \psi, \psi^\dagger, F, F^*$,

$$(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})X = i(\epsilon_1\sigma^\mu\epsilon_2^\dagger - \epsilon_2\sigma^\mu\epsilon_1^\dagger)\,\partial_\mu X \tag{3.18}$$

using eqs. (3.3), (3.15), and (3.17), but without resorting to any of the equations of motion. So we have succeeded in showing that supersymmetry is a valid symmetry of the lagrangian off-shell.

In retrospect, one can see why we needed to introduce the auxiliary field F in order to get the supersymmetry algebra to work off-shell. On-shell, the complex scalar field ϕ has two real propagating degrees of freedom, which match with the two spin polarization states of ψ. Off-shell, however, the Weyl fermion ψ is a complex two-component object, so it has four real degrees of freedom. (Going on-shell eliminates half of the propagating degrees of freedom for ψ, because the lagrangian is linear in time derivatives, so that the canonical momenta can be reexpressed in terms of the configuration variables without time derivatives and are not independent phase space coordinates.) To make the numbers of bosonic and fermionic degrees of freedom match off-shell as well as on-shell, we had to introduce two more real scalar degrees of freedom in the complex field F, which are eliminated when one goes on-shell. The auxiliary field formulation is especially useful when discussing spontaneous supersymmetry breaking, as we will see in section 6.

Invariance of the action under a symmetry transformation always implies the existence of a conserved current, and supersymmetry is no exception. The *supercurrent* J_α^μ is an anticommuting four-vector which also carries a spinor index, as befits the current associated with a symmetry with fermionic generators.[37] By the usual Noether procedure, one finds for the supercurrent (and its hermitian conjugate) in terms of the variations of the fields $X = \phi, \phi^*, \psi, \psi^\dagger, F, F^*$:

$$\epsilon J_\mu + \epsilon^\dagger J_\mu^\dagger \equiv \sum_X \delta X\;\frac{\delta\mathcal{L}}{\delta(\partial^\mu X)} - K_\mu, \tag{3.19}$$

where K_μ is the object whose divergence is the variation of the lagrangian density under the supersymmetry transformation, $\partial^\mu K_\mu = \delta\mathcal{L}$. A little work reveals that

$$J^\mu_\alpha = (\sigma^\nu\overline{\sigma}^\mu\psi)_\alpha\,\partial_\nu\phi^*; \qquad J^{\dagger\mu}_{\dot\alpha} = (\psi^\dagger\overline{\sigma}^\mu\sigma^\nu)_{\dot\alpha}\,\partial_\nu\phi. \tag{3.20}$$

The supercurrent and its hermitian conjugate are separately conserved:

$$\partial_\mu J^\mu_\alpha = 0; \qquad \partial_\mu J^{\dagger\mu}_{\dot\alpha} = 0 \tag{3.21}$$

as can be verified by use of the equations of motion. From these currents one constructs the conserved charges

$$Q_\alpha = \sqrt{2}\int d^3x\; J^0_\alpha; \qquad Q^\dagger_{\dot\alpha} = \sqrt{2}\int d^3x\; J^{\dagger 0}_{\dot\alpha} \tag{3.22}$$

which are the generators of supersymmetry transformations. (The factor of $\sqrt{2}$ normalization is included to agree with an arbitrary historical convention.) As quantum mechanical operators, they satisfy

$$\left[\epsilon Q + \epsilon^\dagger Q^\dagger, X\right] = -i\sqrt{2}\,\delta X \tag{3.23}$$

for any field X, up to terms which vanish on-shell. This can be verified explicitly by using the canonical equal-time commutation and anticommutation relations

$$[\phi(x), \pi(y)] \;=\; [\phi^*(x), \pi^*(y)] \;=\; i\delta^{(3)}(x-y); \tag{3.24}$$

$$\{\psi^\dagger_{\dot\alpha}(x), \psi_\alpha(y)\} \;=\; -\sigma^0_{\alpha\dot\alpha}\,\delta^{(3)}(x-y) \tag{3.25}$$

derived from the free field theory lagrangian eq. (3.1). Here $\pi = \partial_0\phi^*$ and $\pi^* = \partial_0\phi$ are the momenta conjugate to ϕ and ϕ^* respectively. Now the content of eq. (3.18) can be expressed in terms of canonical commutators as

$$\left[\epsilon_2 Q + \epsilon_2^\dagger Q^\dagger,\, [\epsilon_1 Q + \epsilon_1^\dagger Q^\dagger,\, X]\right] - \left[\epsilon_1 Q + \epsilon_1^\dagger Q^\dagger,\, [\epsilon_2 Q + \epsilon_2^\dagger Q^\dagger,\, X]\right] = 2(\epsilon_2\sigma^\mu\epsilon_1^\dagger - \epsilon_1\sigma^\mu\epsilon_2^\dagger)\, i\partial_\mu X \tag{3.26}$$

up to terms which vanish on-shell. The spacetime momentum operator P^μ is given in terms of the canonical variables by $P^0 = \pi\pi^* + \partial_j\phi\partial^j\phi^* + i\psi^\dagger\overline{\sigma}^j\partial_j\psi$ and $P^j = -\pi\,\partial^j\phi - \pi^*\partial^j\phi^* + i\psi^\dagger\overline{\sigma}^0\partial^j\psi$, where j is the spacetime vector index restricted to the three spatial dimensions. It generates spacetime translations on the fields X according to

$$[P_\mu, X] = i\partial_\mu X. \tag{3.27}$$

By rearranging the terms in eq. (3.26) using the Jacobi identity, we therefore have

$$\left[[\epsilon_2 Q + \epsilon_2^\dagger Q^\dagger,\, \epsilon_1 Q + \epsilon_1^\dagger Q^\dagger],\, X\right] = 2(\epsilon_2\sigma^\mu\epsilon_1^\dagger - \epsilon_1\sigma^\mu\epsilon_2^\dagger)\,[P_\mu, X], \tag{3.28}$$

for any X, so it must be that

$$[\epsilon_2 Q + \epsilon_2^\dagger Q^\dagger,\, \epsilon_1 Q + \epsilon_1^\dagger Q^\dagger] = 2(\epsilon_2\sigma^\mu\epsilon_1^\dagger - \epsilon_1\sigma^\mu\epsilon_2^\dagger)\,P_\mu \tag{3.29}$$

up to terms which vanish on-shell. Now by expanding out eq. (3.29), one obtains the non-schematic form of the supersymmetry algebra relations

$$\{Q_\alpha, Q^\dagger_{\dot\alpha}\} = 2\sigma^\mu_{\alpha\dot\alpha}P_\mu, \tag{3.30}$$

$$\{Q_\alpha, Q_\beta\} = \{Q^\dagger_{\dot\alpha}, Q^\dagger_{\dot\beta}\} = 0 \tag{3.31}$$

as promised in the Introduction. [The commutator in eq. (3.29) turns into anticommutators in eqs. (3.30) and (3.31) in the process of extracting the anticommuting spinors ϵ_1 and ϵ_2.] The results $[Q_\alpha, P_\mu] = 0$ and $[Q^\dagger_{\dot\alpha}, P_\mu] = 0$ follow immediately from eq. (3.27) and the fact that the supersymmetry transformations are global (independent of position in spacetime). This demonstration of the supersymmetry algebra in terms of the canonical generators Q and $Q^\dagger$ requires the use of the Hamiltonian equations of motion, but the symmetry itself is valid off-shell at the level of the lagrangian, as we have already shown.

3.2 Interactions of chiral supermultiplets

In a realistic theory like the MSSM, there are many chiral supermultiplets which have both gauge and non-gauge interactions. In this subsection, our task is to construct the most general possible theory of masses and non-gauge interactions for particles that live in chiral supermultiplets. In the MSSM these are the quarks, squarks, leptons, sleptons, Higgs scalars and higgsino fermions. We will find that the form of the non-gauge couplings, including mass terms, is highly restricted by the requirement that the action is invariant under supersymmetry transformations. (Gauge interactions will be dealt with in the following subsections.)

Our starting point is the lagrangian density for a collection of free chiral supermultiplets labelled by an index i which runs over all gauge and flavor degrees of freedom. Since we will want to construct an interacting theory with supersymmetry closing off-shell, each supermultiplet contains a complex scalar ϕ_i and a left-handed Weyl fermion ψ_i as physical degrees of freedom, plus a complex auxiliary field F_i which does not propagate. The results of the previous subsection tell us that the free part of the Lagrangian is

$$\mathcal{L}_{\rm free} = -\partial^\mu\phi^{*i}\partial_\mu\phi_i - i\psi^{\dagger i}\overline{\sigma}^\mu\partial_\mu\psi_i + F^{*i}F_i \tag{3.32}$$

where we sum over repeated indices i (not to be confused with the suppressed spinor indices), with the convention that fields ϕ_i and ψ_i always carry lowered indices, while their conjugates always carry raised indices. It is invariant under the supersymmetry transformation

$$\delta\phi_i = \epsilon\psi_i \qquad \delta\phi^{*i} = \epsilon^\dagger\psi^{\dagger i} \tag{3.33}$$

$$\delta(\psi_i)_\alpha = i(\sigma^\mu\epsilon^\dagger)_\alpha\,\partial_\mu\phi_i + \epsilon_\alpha F_i \qquad \delta(\psi^{\dagger i})_{\dot\alpha} = -i(\epsilon\sigma^\mu)_{\dot\alpha}\,\partial_\mu\phi^{*i} + \epsilon^\dagger_{\dot\alpha}F^{*i} \tag{3.34}$$

$$\delta F_i = i\epsilon^\dagger\overline{\sigma}^\mu\partial_\mu\psi_i \qquad \delta F^{*i} = -i\partial_\mu\psi^{\dagger i}\overline{\sigma}^\mu\epsilon\,. \tag{3.35}$$

As we will now argue, the most general set of renormalizable interactions for these fields can be written in the simple form

$$\mathcal{L}_{\rm int} = -\frac{1}{2}W^{ij}\psi_i\psi_j + W^i F_i + \text{c.c.}, \tag{3.36}$$

where W^{ij} and W^i are some functions of the bosonic fields with dimensions of (mass) and (mass)^2 respectively, and "c.c." henceforth stands for complex conjugate. At this point, we are not assuming that W^{ij} and W^i are related to each other in any way whatsoever. However, soon we will find out that they *are* related, which is why we have chosen the same letter for them. Notice that eq. (2.8) tells us that W^{ij} is symmetric under $i \leftrightarrow j$. Now, let us require the lagrangian to be renormalizable by power counting, so that each term has field content with mass dimension ≤ 4. It follows immediately that we do not need to consider the possibility of W^{ij} or W^i being functions of the fermionic or auxiliary fields. For the same reason, we can take W^i to be at most a quadratic polynomial, and W^{ij} linear, in the fields ϕ_i and ϕ^{*i}. Also, we do not need to consider including in $\mathcal{L}_{\rm int}$ any term which is a function of the scalar fields ϕ_i, ϕ^{*i} only. If there were such a term, then under a supersymmetry transformation eq. (3.33) it would go into another function of the scalar fields only, multiplied by $\epsilon\psi_i$ or $\epsilon^\dagger\psi^{\dagger i}$, and with no spacetime derivatives or F_i, F^{*i} fields. It is easy to see from eqs. (3.33)-(3.36) that nothing of this form can possibly be cancelled by the supersymmetry transformation of any other term in the lagrangian. So eq. (3.36) is indeed the most general possibility!

We must now require that $\mathcal{L}_{\rm int}$ is invariant under the supersymmetry transformations, since $\mathcal{L}_{\rm free}$ was already invariant by itself. It is easiest to divide the variation of $\mathcal{L}_{\rm int}$ into several parts which must cancel separately. First, we consider the part which contains four spinors:

$$\delta\mathcal{L}_{\rm int}|_{4-\rm spinor} \;=\; -\frac{1}{2}\frac{\delta W^{ij}}{\delta\phi_k}(\epsilon\psi_k)(\psi_i\psi_j) - \frac{1}{2}\frac{\delta W^{ij}}{\delta\phi^{*k}}(\epsilon^\dagger\psi^{\dagger k})(\psi_i\psi_j) + \text{c.c.} \tag{3.37}$$

The term proportional to $(\epsilon\psi_k)(\psi_i\psi_j)$ cannot cancel against any other term. Fortunately, however, the Fierz identity eq. (3.12) implies

$$(\epsilon\psi_i)(\psi_j\psi_k) + (\epsilon\psi_j)(\psi_k\psi_i) + (\epsilon\psi_k)(\psi_i\psi_j) = 0, \tag{3.38}$$

which allows this contribution to $\delta\mathcal{L}_{\rm int}$ to vanish identically if and only if $\delta W^{ij}/\delta\phi_k$ is totally symmetric under interchange of i, j, k. There is no such identity available for the term proportional to $(\epsilon^\dagger\psi^{\dagger k})(\psi_i\psi_j)$. Since it cannot cancel with any other term, requiring it to be absent just tells us that W^{ij} cannot contain ϕ^{*k}. In other words, W^{ij} is *analytic* (or *holomorphic*) in the complex fields ϕ_k.

So far, what we have learned is that we can write

$$W^{ij} = M^{ij} + y^{ijk}\phi_k \tag{3.39}$$

where M^{ij} is a symmetric mass matrix for the fermion fields, and y^{ijk} is a Yukawa coupling of a scalar ϕ_k and two fermions $\psi_i\psi_j$ which must be totally symmetric under interchange of i, j, k. It is convenient to write

$$W^{ij} = \frac{\delta^2}{\delta\phi_i\delta\phi_j}W \tag{3.40}$$

where we have introduced a very useful object

$$W = \frac{1}{2}M^{ij}\phi_i\phi_j + \frac{1}{6}y^{ijk}\phi_i\phi_j\phi_k \tag{3.41}$$

which is called the *superpotential*. This is not a scalar potential in the ordinary sense; in fact, it is not even real. It is instead an analytic function of the scalar fields ϕ_i treated as complex variables.

Continuing on our vaunted quest, we next consider the parts of $\delta\mathcal{L}_{\rm int}$ which contain a spacetime derivative:

$$\delta\mathcal{L}_{\rm int}|_\partial \;=\; -iW^{ij}\partial_\mu\phi_j\,\psi_i\sigma^\mu\epsilon^\dagger - iW^i\,\partial_\mu\psi_i\sigma^\mu\epsilon^\dagger + \text{c.c.} \tag{3.42}$$

Here we have used the identity eq. (2.9) on the second term, which came from $(\delta F_i)W^i$. Now we can use eq. (3.40) to observe that

$$W^{ij}\partial_\mu\phi_j = \partial_\mu\left(\frac{\delta W}{\delta\phi_i}\right). \tag{3.43}$$

Then it is clear that eq. (3.42) will be a total derivative if and only if

$$W^i = \frac{\delta W}{\delta\phi_i} = M^{ij}\phi_j + \frac{1}{2}y^{ijk}\phi_j\phi_k\,, \tag{3.44}$$

which explains why we chose its name the way we did. The remaining terms in $\delta\mathcal{L}_{\rm int}$ are all linear in F_i or F^{*i}, and it is easy to show that they cancel, given the results for W^i and W^{ij} that we have already found.

To recap, we have found that the most general non-gauge interactions for chiral supermultiplets are determined by a single analytic function of the complex scalar fields, the superpotential W. The auxiliary fields F_i and F^{*i} can be eliminated using their classical equations of motion. The part of $\mathcal{L}_{\rm free} + \mathcal{L}_{\rm int}$ that contains the auxiliary fields is $F_iF^{*i} + W^iF_i + W_i^*F^{*i}$, leading to the equations of motion

$$F_i = -W_i^*; \qquad\qquad F^{*i} = -W^i\,. \tag{3.45}$$

Thus the auxiliary fields are expressible algebraically (without any derivatives) in terms of the scalar fields. After making the replacement eq. (3.45) in $\mathcal{L}_{\rm free} + \mathcal{L}_{\rm int}$, we obtain the lagrangian density

$$\mathcal{L} = -\partial^\mu\phi^{*i}\partial_\mu\phi_i - i\psi^{\dagger i}\overline{\sigma}^\mu\partial_\mu\psi_i - \frac{1}{2}\left(W^{ij}\psi_i\psi_j + W^{*ij}\psi^{\dagger i}\psi^{\dagger j}\right) - W^iW_i^*. \tag{3.46}$$

(Since F_i and F^{*i} appear only quadratically in the action, the result of instead doing a functional integral over them at the quantum level has precisely the same effect.) Now that the non-propagating fields F_i, F^{*i} have been eliminated, it is clear from eq. (3.46) that the scalar potential for the theory is just given in terms of the superpotential by (recall $\mathcal{L}$ contains $-V$):

$$\begin{aligned} V(\phi,\phi^*) &= W^iW_i^* = F_iF^{*i} = M_{ik}^*M^{kj}\phi^{*i}\phi_j \\ &\quad + \frac{1}{2}M^{in}y_{jkn}^*\phi_i\phi^{*j}\phi^{*k} + \frac{1}{2}M_{in}^*y^{jkn}\phi^{*i}\phi_j\phi_k + \frac{1}{4}y^{ijn}y_{kln}^*\phi_i\phi_j\phi^{*k}\phi^{*l}\,. \end{aligned} \tag{3.47}$$

This scalar potential is automatically bounded from below; in fact, since it is a sum of squares of absolute values (of the W^i), it is always non-negative. If we substitute the general form for the superpotential eq. (3.41) into eq. (3.46), we obtain for the full lagrangian density

$$\begin{aligned}\mathcal{L} &= -\partial^\mu \phi^{*i} \partial_\mu \phi_i - i\psi^{\dagger i}\overline{\sigma}^\mu \partial_\mu \psi_i \\ &\quad -\frac{1}{2}M^{ij}\psi_i\psi_j - \frac{1}{2}M^*_{ij}\psi^{\dagger i}\psi^{\dagger j} - V(\phi,\phi^*) \\ &\quad -\frac{1}{2}y^{ijk}\phi_i\psi_j\psi_k - \frac{1}{2}y^*_{ijk}\phi^{*i}\psi^{\dagger j}\psi^{\dagger k}. \end{aligned} \tag{3.48}$$

Now we can compare the masses of the fermions and scalars by looking at the linearized equations of motion:

$$\partial^\mu\partial_\mu\phi_i = M^*_{ik}M^{kj}\phi_j + \ldots; \tag{3.49}$$

$$-i\overline{\sigma}^\mu\partial_\mu\psi_i = M^*_{ij}\psi^{\dagger j} + \ldots; \qquad -i\sigma^\mu\partial_\mu\psi^{\dagger i} = M^{ij}\psi_j + \ldots. \tag{3.50}$$

One can eliminate ψ in terms of $\psi^\dagger$ and vice versa in eq. (3.50), obtaining [after use of the identity eq. (3.7)]

$$\partial^\mu\partial_\mu\psi_i = M^*_{ik}M^{kj}\psi_j + \ldots; \qquad \partial^\mu\partial_\mu\psi^{\dagger j} = \psi^{\dagger i}M^*_{ik}M^{kj} + \ldots. \tag{3.51}$$

Therefore, the fermions and the bosons satisfy the same wave equation with exactly the same (mass)^2 matrix with real non-negative eigenvalues, namely $(M^2)_i{}^j = M^*_{ik}M^{kj}$. It follows that diagonalizing this matrix gives a collection of chiral supermultiplets each of which contains a mass-degenerate complex scalar and Weyl fermion, in agreement with the general argument in the Introduction.

3.3 Lagrangians for gauge supermultiplets

The propagating degrees of freedom in a gauge supermultiplet are a massless gauge boson field A^a_μ and a two-component Weyl fermion gaugino λ^a. The index a here runs over the adjoint representation of the gauge group ($a = 1\ldots 8$ for $SU(3)_C$ color gluons and gluinos; $a = 1,2,3$ for $SU(2)_L$ weak isospin; $a = 1$ for $U(1)_Y$ weak hypercharge). The gauge transformations of the vector supermultiplet fields are then

$$\delta_{\rm gauge}A^a_\mu = -\partial_\mu\Lambda^a + g f^{abc}A^b_\mu\Lambda^c \tag{3.52}$$

$$\delta_{\rm gauge}\lambda^a = g f^{abc}\lambda^b\Lambda^c \tag{3.53}$$

where Λ^a is an infinitesimal gauge transformation parameter, g is the gauge coupling, and f^{abc} are the totally antisymmetric structure constants which define the gauge group. (The special case of an abelian group like $U(1)_Y$ is obtained by just setting $f^{abc} = 0$; in particular the corresponding gaugino is a gauge singlet in that case.)

The on-shell degrees of freedom for A^a_μ and λ^a_α amount to two bosonic and two fermionic helicity states (for each a), as required by supersymmetry. However, off-shell λ^a_α consists of two complex, or four real, fermionic degrees of freedom, while A^a_μ

only has three real bosonic degrees of freedom; one degree of freedom is removed by the inhomogeneous gauge transformation eq. (3.52). So, we will need one real bosonic auxiliary field, traditionally called D^a, in order for supersymmetry to be consistent off-shell. This field also transforms as an adjoint of the gauge group [i.e., like eq. (3.53) with $\lambda \to D$] and satisfies $(D^a)^* = D^a$. Like the chiral auxiliary fields F_i, it has dimensions of (mass)^2 and thus no kinetic term, so that it can be eliminated on-shell using its algebraic equation of motion.

Therefore, the lagrangian density for a gauge supermultiplet ought to be

$$\mathcal{L}_{\rm gauge} = -\frac{1}{4}F^a_{\mu\nu}F^{\mu\nu a} - i\lambda^{\dagger a}\overline{\sigma}^\mu D_\mu\lambda^a + \frac{1}{2}D^aD^a \tag{3.54}$$

where

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc}A^b_\mu A^c_\nu \tag{3.55}$$

is the usual Yang-Mills field strength, and

$$D_\mu\lambda^a = \partial_\mu\lambda^a - g f^{abc}A^b_\mu\lambda^c \tag{3.56}$$

is the covariant derivative of the gaugino field. One can infer the appropriate form for the supersymmetry transformation of the fields, up to multiplicative constants, from the requirements that they should be linear in the infinitesimal parameters $\epsilon, \epsilon^\dagger$ with dimensions of (mass)$^{-1/2}$; that δA^a_μ is real; and that δD^a should be real and proportional to the field equations for the gaugino, in analogy with the role of the auxiliary field F in the chiral supermultiplet case. Thus one can guess, up to multiplicative factors,

$$\delta A^a_\mu = -\frac{1}{\sqrt{2}}\left[\epsilon^\dagger\overline{\sigma}_\mu\lambda^a + \lambda^{\dagger a}\overline{\sigma}_\mu\epsilon\right] \tag{3.57}$$

$$\delta\lambda^a_\alpha = -\frac{i}{2\sqrt{2}}(\sigma^\mu\overline{\sigma}^\nu\epsilon)_\alpha\, F^a_{\mu\nu} + \frac{1}{\sqrt{2}}\epsilon_\alpha\, D^a \tag{3.58}$$

$$\delta D^a = \frac{i}{\sqrt{2}}\left[\epsilon^\dagger\overline{\sigma}^\mu D_\mu\lambda^a - D_\mu\lambda^{\dagger a}\overline{\sigma}^\mu\epsilon\right]. \tag{3.59}$$

The factors of $\sqrt{2}$ are chosen [†] so that the action obtained by integrating $\mathcal{L}_{\rm gauge}$ is invariant. It is now a little bit tedious, but straightforward, to check that eq. (3.18) is modified to

$$(\delta_{\epsilon_2}\delta_{\epsilon_1} - \delta_{\epsilon_1}\delta_{\epsilon_2})X = i(\epsilon_1\sigma^\mu\epsilon_2^\dagger - \epsilon_2\sigma^\mu\epsilon_1^\dagger)D_\mu X \tag{3.60}$$

for X equal to any of the gauge-covariant fields $F^a_{\mu\nu}$, λ^a, $\lambda^{\dagger a}$, D^a, as well as arbitrary covariant derivatives acting on them. This ensures that the supersymmetry algebra

[†] For future convenience in treating the MSSM, we have chosen complex phases so that our λ^a, $\lambda^{\dagger a}$ are equal to $-i$, i times the gaugino spinors in Ref.[11]

eqs. (3.30)-(3.31) is realized on gauge-invariant combinations of fields in gauge supermultiplets, as they were on the chiral supermultiplets.[‡] These calculations require the use of identities

$$\xi\sigma^\mu\overline{\sigma}^\nu\chi = \chi\sigma^\nu\overline{\sigma}^\mu\xi = (\chi^\dagger\overline{\sigma}^\nu\sigma^\mu\xi^\dagger)^* = (\xi^\dagger\overline{\sigma}^\mu\sigma^\nu\chi^\dagger)^*; \tag{3.61}$$

$$\overline{\sigma}^\mu\sigma^\nu\overline{\sigma}^\rho = \eta^{\mu\rho}\overline{\sigma}^\nu - \eta^{\nu\rho}\overline{\sigma}^\mu - \eta^{\mu\nu}\overline{\sigma}^\rho - i\epsilon^{\mu\nu\rho\kappa}\overline{\sigma}_\kappa; \tag{3.62}$$

$$\sigma^\mu_{\alpha\dot{\alpha}}\overline{\sigma}_\mu^{\dot{\beta}\beta} = -2\delta_\alpha^\beta\delta_{\dot{\alpha}}^{\dot{\beta}}. \tag{3.63}$$

If we had not included the auxiliary field D^a, then the supersymmetry algebra eq. (3.60) would hold only after using the equations of motion for λ^a and $\lambda^{\dagger a}$. The auxiliary fields just satisfy the equations of motion $D^a = 0$, but this is no longer true if one couples the gauge supermultiplets to chiral supermultiplets, as we now do.

3.4 Supersymmetric gauge interactions

Finally we are ready to consider a general lagrangian density for a supersymmetric theory with both chiral and gauge supermultiplets. Suppose that the chiral supermultiplets transform under the gauge group in a representation with hermitian matrices $(T^a)_i{}^j$ satisfying $[T^a, T^b] = if^{abc}T^c$. [For example, if the gauge group is $SU(2)$, then $f^{abc} = \epsilon^{abc}$, and the T^a are 1/2 times the Pauli matrices for a chiral supermultiplet transforming in the fundamental representation.] Thus

$$\delta_{\rm gauge}X_i = ig\Lambda^a(T^aX)_i \tag{3.64}$$

for $X_i = \phi_i, \psi_i, F_i$; since supersymmetry and gauge transformations commute, the scalar, fermion, and auxiliary fields must be in the same representation of the gauge group. To have a gauge-invariant lagrangian, we need to turn the ordinary derivatives in eq. (3.32) into covariant derivatives:

$$\partial_\mu\phi_i \to D_\mu\phi_i = \partial_\mu\phi_i + igA^a_\mu(T^a\phi)_i \tag{3.65}$$

$$\partial_\mu\phi^{*i} \to D_\mu\phi^{*i} = \partial_\mu\phi^{*i} - igA^a_\mu(\phi^*T^a)^i \tag{3.66}$$

$$\partial_\mu\psi_i \to D_\mu\psi_i = \partial_\mu\psi_i + igA^a_\mu(T^a\psi)_i. \tag{3.67}$$

Naively, this simple procedure achieves the goal of coupling the vector bosons in the gauge supermultiplet to the scalars and fermions in the chiral supermultiplets. However, we also have to consider whether there are any other interactions allowed by gauge invariance involving the gaugino and D^a fields which might have to be included to make a supersymmetric lagrangian.

In fact, there are three such possibilities which are renormalizable (of mass dimension ≤ 4), namely

$$(\phi^*T^a\psi)\lambda^a, \qquad \lambda^{\dagger a}(\psi^\dagger T^a\phi) \qquad \text{and} \qquad (\phi^*T^a\phi)D^a. \tag{3.68}$$

[‡]The supersymmetry transformations eqs. (3.57)-(3.59) are non-linear for non-abelian gauge symmetries, because of the gauge fields contained in the covariant derivatives acting on the gaugino fields and in the field strength $F^a_{\mu\nu}$. By adding even more auxiliary fields besides D^a, one can make the supersymmetry transformations linear in the fields. The version given here in which those extra auxiliary fields have been removed by gauge transformations is called "Wess-Zumino gauge".[38]

Now one can add them, with arbitrary dimensionless coupling coefficients, to the lagrangians for the chiral and gauge supermultiplets and demand that the whole mess be real and invariant under supersymmetry transformations, up to a total derivative. Not surprisingly, this is possible only if one modifies the supersymmetry transformation laws for the matter fields to include gauge-covariant rather than ordinary derivatives (and to include one strategically-chosen extra term in δF_i):

$$\delta\phi_i = \epsilon\psi_i \tag{3.69}$$

$$\delta(\psi_i)_\alpha = i(\sigma^\mu\epsilon^\dagger)_\alpha D_\mu\phi_i + \epsilon_\alpha F_i \tag{3.70}$$

$$\delta F_i = i\epsilon^\dagger\overline{\sigma}^\mu D_\mu\psi_i + \sqrt{2}g(T^a\phi)_i\,\epsilon^\dagger\lambda^{\dagger a}. \tag{3.71}$$

After some algebra one can now fix the coefficients for the terms in eq. (3.68), so that the full lagrangian density for a renormalizable supersymmetric theory is

$$\begin{aligned}\mathcal{L} \;=\; & \mathcal{L}_{\rm gauge} + \mathcal{L}_{\rm chiral} \\ & -\sqrt{2}g\left[(\phi^*T^a\psi)\lambda^a + \lambda^{\dagger a}(\psi^\dagger T^a\phi)\right] \\ & +g(\phi^*T^a\phi)D^a.\end{aligned} \tag{3.72}$$

Here $\mathcal{L}_{\rm chiral}$ means the chiral supermultiplet lagrangian found in section 3.2 [e.g., eq. (3.46) or (3.48)], but with ordinary derivatives replaced everywhere by gauge-covariant derivatives, and $\mathcal{L}_{\rm gauge}$ was given in eq. (3.54). To prove that eq. (3.72) is invariant under the supersymmetry transformations, one must use the identity

$$W^i(T^a)_i{}^j\phi_j = 0. \tag{3.73}$$

This is precisely the condition that must be satisfied anyway in order for the superpotential (and thus $\mathcal{L}_{\rm chiral}$) to be gauge invariant, since the left side is proportional to $\delta_{\rm gauge}W$.

The last two lines in eq. (3.72) are interactions whose strengths are fixed to be gauge couplings by the requirements of supersymmetry, even though they are not gauge interactions from the point of view of an ordinary field theory. The second line is a direct coupling of gauginos to matter fields which is the "supersymmetrization" of the usual gauge boson coupling to matter fields. The last line combines with the $(1/2)D^aD^a$ term in $\mathcal{L}_{\rm gauge}$ to provide an equation of motion

$$D^a = -g(\phi^*T^a\phi). \tag{3.74}$$

Like the auxiliary fields F_i and F^{*i}, the D^a are expressible purely algebraically in terms of the scalar fields. Replacing the auxiliary fields in eq. (3.72) using eq. (3.74), one finds that the complete scalar potential is (recall $\mathcal{L} \supset -V$):

$$V(\phi,\phi^*) = F^{*i}F_i + \frac{1}{2}\sum_a D^aD^a = W_i^*W^i + \frac{1}{2}\sum_a g_a^2(\phi^*T^a\phi)^2. \tag{3.75}$$

The two types of terms in this expression are called "F-term" and "D-term" contributions, respectively. In the second term in eq. (3.75), we have now written an explicit

sum $\sum_a$ to cover the case that the gauge group has several distinct factors with different gauge couplings g_a. [For instance, in the MSSM the three factors $SU(3)_C$, $SU(2)_L$ and $U(1)_Y$ have different gauge couplings g_3, g and g'.] Since $V(\phi,\phi^*)$ is a sum of squares, it is always greater than or equal to zero for every field configuration. It is a very interesting and unique feature of supersymmetric theories that the scalar potential is completely determined by the *other* interactions in the theory. The F-terms are fixed by Yukawa couplings and fermion mass terms, and the D-terms are fixed by the gauge interactions.

By using Noether's procedure [see eq. (3.19)], one finds the conserved supercurrent

$$J^\mu_\alpha = (\sigma^\nu\overline{\sigma}^\mu\psi_i)_\alpha\, D_\nu\phi^{*i} - i(\sigma^\mu\psi^{\dagger i})_\alpha\, W_i^* \\ -\frac{1}{2\sqrt{2}}(\sigma^\nu\overline{\sigma}^\rho\sigma^\mu\lambda^{\dagger a})_\alpha\, F^a_{\nu\rho} - \frac{i}{\sqrt{2}}g\phi^* T^a\phi\,(\sigma^\mu\lambda^{\dagger a})_\alpha, \qquad (3.76)$$

generalizing the expression given in eq. (3.20) for the Wess-Zumino model. This expression will be useful when we discuss certain aspects of spontaneous supersymmetry breaking in section 6.2.

3.5 Summary: How to build a supersymmetric model

In a renormalizable supersymmetric field theory, the interactions and masses of all particles are determined just by their gauge transformation properties and by the superpotential W. By construction, we found that W had to be an analytic function of the complex scalar fields ϕ_i, which are always defined to transform under supersymmetry into *left*-handed Weyl fermions. We should mention that in an equivalent language, W is said to be a function of chiral *superfields*.[36] A superfield is a single object which contains as components all of the bosonic, fermionic, and auxiliary fields within the corresponding supermultiplet, e.g. $\Phi_i \supset (\phi_i, \psi_i, F_i)$. (This is analogous to the way in which one often describes a weak isospin doublet or color triplet by a multicomponent field.) The gauge quantum numbers and mass dimension of a chiral superfield are the same as that of its scalar component. In the superfield formulation, one writes instead of eq. (3.41)

$$W = \frac{1}{2}M^{ij}\Phi_i\Phi_j + \frac{1}{6}y^{ijk}\Phi_i\Phi_j\Phi_k \qquad (3.77)$$

which means exactly the same thing. While this entails no difference in practical results, the fancier version eq. (3.77) at least serves to remind us that W determines not only the scalar interactions in the theory, but the fermion masses and Yukawa couplings as well. The derivation of all of our preceding results can be obtained somewhat more elegantly using superfield methods, which have the advantage of making invariance under supersymmetry transformations manifest. We have avoided this extra layer of notation on purpose, in favor of the more pedestrian but hopefully more familiar component field approach. The latter is at least more appropriate for making contact with phenomenology in a universe with supersymmetry breaking. The only (occasional) use we will make of superfield notation is the purely cosmetic one of following the common practice of specifying superpotentials like eq. (3.77) rather than

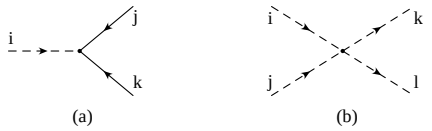

Figure 3: The dimensionless non-gauge interaction vertices in a supersymmetric theory: (a) scalar-fermion-fermion Yukawa interaction y^{ijk}, (b) quartic scalar interaction $y^{ijn}y^*_{kln}$.

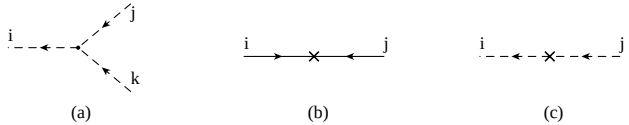

Figure 4: Supersymmetric dimensionful couplings: (a) (scalar)^3 interaction vertex $M^*_{in}y^{jkn}$, (b) fermion mass term M^{ij}, (c) scalar (mass)^2 term $M^*_{ik}M^{kj}$.

(3.41). The specification of the superpotential is really a code for the terms that it implies in the lagrangian, so the reader may feel free to think of the superpotential either as a function of the scalar fields ϕ_i or as the same function of the superfields Φ_i which contain them.

Given the supermultiplet content of the theory, the form of the superpotential is restricted by gauge invariance. In any given theory, only a subset of the couplings M^{ij} and y^{ijk} will be allowed to be non-zero. The entries of the mass matrix M^{ij} can only be non-zero for i and j such that the supermultiplets Φ_i and Φ_j transform under the gauge group in representations which are conjugates of each other. (In fact, in the MSSM there is only one such term, as we will see.) Likewise, the Yukawa couplings y^{ijk} can only be non-zero when Φ_i, Φ_j, and Φ_k transform in representations which can combine to form a singlet.

The interactions implied by the superpotential eq. (3.77) are shown [§] in Figs. 3 and 4. Those in Fig. 3 are all determined by the dimensionless parameters y^{ijk}. The Yukawa interaction in Fig. 3a corresponds to the next-to-last term in eq. (3.48). For each particular Yukawa coupling of $\phi_i\psi_j\psi_k$ with strength y^{ijk}, there must be equal couplings of $\phi_j\psi_i\psi_k$ and $\phi_k\psi_i\psi_j$, since y^{ijk} is completely symmetric under interchange of any two of its indices as shown in section 3.2. There is also a dimensionless coupling for $\phi_i\phi_j\phi^{*k}\phi^{*l}$, with strength $y^{ijn}y^*_{kln}$ as required by supersymmetry [see the last term in eq. (3.47)]. The arrows on both the fermion and scalar lines follow the chirality; i.e., one direction for propagation of ϕ and ψ and the other for the propagation of ϕ^* and $\psi^\dagger$. Thus there is a vertex corresponding to the one in Fig. 3a but with all arrows reversed, corresponding to the complex conjugate [the last term in eq. (3.48)]. The relationship between the interactions in Figs. 3a and 3b is exactly of the special type

[§] Here, the auxiliary fields have been eliminated using their equations of motion ("integrated out") as in eq. (3.48). It is quite possible instead to give Feynman rules which include the auxiliary fields, although this tends to be less useful in phenomenological applications.

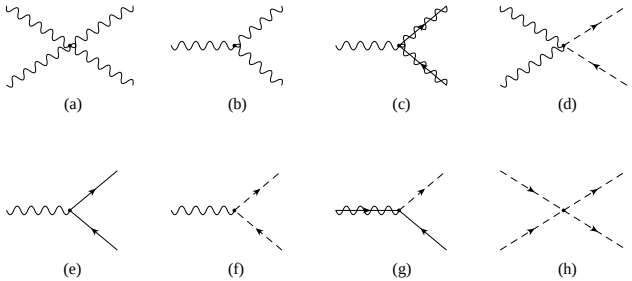

Figure 5: Supersymmetric gauge interaction vertices.

needed to cancel the quadratic divergences in quantum corrections to scalar masses, as discussed in the Introduction [compare Fig. 1].

In Fig. 4, we show the only interactions corresponding to renormalizable and supersymmetric vertices with dimensions of (mass) and (mass)^2. First, there are (scalar)^3 couplings which are entirely determined by the superpotential mass parameters M^{ij} and Yukawa couplings y^{ijk}, as indicated by the second and third terms in eq. (3.47). The propagators of the fermions and scalars in the theory are constructed in the usual way using the fermion mass M^{ij} and scalar (mass)^2 $M^*_{in}M^{nj}$. Of particular interest is the fact that the fermion mass term M^{ij} leads to a chirality-changing insertion in the fermion propagator; note the directions of the arrows in Fig. 4b. There is no such arrow-reversal for a scalar propagator in a theory with exact supersymmetry; as shown in Fig. 4c, if one treats the scalar (mass)^2 term as an insertion in the propagator, the arrow direction is preserved. Again, for each of Figures 4a and 4b there is an interaction with all arrows reversed.

In Fig. 5 we show in a similar manner the gauge interactions in a supersymmetric theory. Figures 5a,b,c occur only when the gauge group is non-abelian (e.g. for $SU(3)_C$ color and $SU(2)_L$ weak isospin in the MSSM). Figures 5a and 5b are the interactions of gauge bosons which derive from the first term in eq. (3.54). In the MSSM these are exactly the same as the well-known QCD gluon and electroweak gauge boson vertices of the Standard Model. (We do not show the interactions of ghost fields, which are necessary only for consistent loop amplitudes.) Figures 5c,d,e,f are just the standard interactions between gauge bosons and fermion and scalar fields which must occur in any gauge theory because of the form of the covariant derivative; they come from eqs. (3.56) and (3.65)-(3.67) inserted in the kinetic part of the lagrangian. Figure 5c shows the coupling of a gaugino to a gauge boson; the gaugino line in a Feynman diagram is traditionally drawn as a solid fermion line superimposed on a gauge boson squiggly line. In Fig. 5g we have the coupling of a gaugino to a chiral fermion and a complex scalar [the first term in the second line in eq. (3.72)]. One can think of this as the "supersymmetrization" of Figure 5e or 5f; any of these three vertices may be obtained from any other (up to a factor of $\sqrt{2}$) by replacing two of the particles

by their supersymmetric partners. There is also an interaction like Fig. 5g but with all arrows reversed, corresponding to the complex conjugate term in the lagrangian [the second term in the second line in eq. (3.72)]. Finally in Fig. 5h we have a scalar quartic interaction vertex [the last term in eq. (3.75)] which is also determined by the gauge coupling.

The results of this section can be used as a recipe for constructing the supersymmetric interactions for any model. In the case of the MSSM, we already know the gauge group, particle content and the gauge transformation properties, so it only remains to decide on the superpotential. This we will do in section 5.1.

4 Soft supersymmetry breaking interactions

A realistic phenomenological model must contain supersymmetry breaking. From a theoretical perspective, we expect that supersymmetry, if it exists at all, should be an exact symmetry which is spontaneously broken. In other words, the ultimate model should have a lagrangian density which is invariant under supersymmetry, but a vacuum state which is not. In this way, supersymmetry is hidden at low energies in a manner exactly analogous to the fate of the electroweak symmetry in the ordinary Standard Model.

Many models of spontaneous symmetry breaking have indeed been proposed and we will mention the basic ideas of some of them in section 6. These always involve extending the MSSM to include new particles and interactions at very high mass scales, and there is no consensus on exactly how this should be done. However, from a practical point of view, it is extremely useful to simply parameterize our ignorance of these issues by just introducing extra terms which break supersymmetry explicitly in the effective MSSM lagrangian. As was argued in the Introduction, the extra supersymmetry-breaking couplings should be soft (of positive mass dimension) in order to be able to naturally maintain a hierarchy between the electroweak scale and the Planck (or some other very large) mass scale. This means in particular that we should not consider any dimensionless supersymmetry-breaking couplings.

In the context of a general renormalizable theory, the possible soft supersymmetry-breaking terms in the lagrangian are

$$\begin{aligned}\mathcal{L}_{\rm soft} &= -\frac{1}{2}\left(M_\lambda\,\lambda^a\lambda^a + \text{c.c.}\right) - (m^2)^i_j\phi^{j*}\phi_i \\ &\quad -\left(\frac{1}{2}b^{ij}\phi_i\phi_j + \frac{1}{6}a^{ijk}\phi_i\phi_j\phi_k + \text{c.c.}\right), \qquad (4.1)\\ \mathcal{L}_{\text{maybe soft}} &= -\frac{1}{2}c_i^{jk}\phi^{*i}\phi_j\phi_k + \text{c.c.} \qquad (4.2)\end{aligned}$$

They consist of gaugino masses M_λ for each gauge group, scalar (mass)^2 terms $(m^2)^j_i$ and b^{ij}, and (scalar)^3 couplings a^{ijk} and c_i^{jk}. One might wonder why we have not included possible soft mass terms for the chiral supermultiplet fermions. The reason is that including such terms would be redundant; they can always be absorbed into a redefinition of the superpotential and the terms $(m^2)^j_i$ and c_i^{jk}. It has been shown

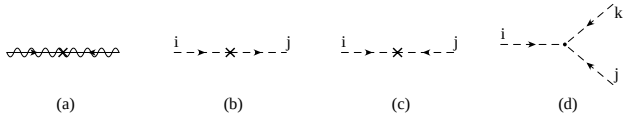

Figure 6: Soft supersymmetry-breaking terms: (a) Gaugino mass insertion M_λ; (b) non-analytic scalar (mass)^2 $(m^2)^i_j$; (c) analytic scalar (mass)^2 b^{ij}; (d) (scalar)^3 coupling a^{ijk}.

rigorously that a softly-broken supersymmetric theory with $\mathcal{L}_{\rm soft}$ as given by eq. (4.1) is indeed free of quadratic divergences in quantum corrections to scalar masses, to all orders in perturbation theory.[39] The situation is slightly more subtle if one tries to include the non-analytic (scalar)^3 couplings in $\mathcal{L}_{\rm maybe\ soft}$. If any of the chiral supermultiplets in the theory are completely uncharged under all gauge symmetries, then non-zero c_i^{jk} terms can lead to quadratic divergences, despite the fact that they are formally soft. Now, this constraint need *not* apply to the MSSM, which does not have any gauge-singlet chiral supermultiplets. Nevertheless, the possibility of c_i^{jk} terms is nearly always neglected.[40] The real reason for this is that it is extremely difficult to construct any model of spontaneous supersymmetry breaking in which the c_i^{jk} are not utterly negligibly small. Equation (4.1) is therefore usually taken to be the most general soft supersymmetry-breaking lagrangian.

It should be clear that $\mathcal{L}_{\rm soft}$ indeed breaks supersymmetry, since it involves only scalars and gauginos, and not their respective superpartners. In fact, the soft terms in $\mathcal{L}_{\rm soft}$ are capable of giving masses to all of the scalars and gauginos in a theory, even if the gauge bosons and fermions in chiral supermultiplets are massless (or relatively light). The gaugino masses M_λ are always allowed by gauge symmetry. The $(m^2)^i_j$ terms are allowed for i, j such that ϕ_i, ϕ^{j*} transform in complex conjugate representations of each other under all gauge symmetries; in particular this is true of course when $i = j$, so every scalar is eligible to get a mass in this way if supersymmetry is broken. The remaining soft terms may or may not be allowed by the symmetries. In this regard it is useful to note that the b^{ij} and a^{ijk} terms have the same form as the M^{ij} and y^{ijk} terms in the superpotential [compare eq. (4.1) to eq. (3.41) or eq. (3.77)], so they will be allowed by gauge invariance if and only if a corresponding superpotential term is allowed. The Feynman diagram interactions corresponding to the allowed soft terms in eq. (4.1) are shown in Fig. 6. As before, for each of the interactions in Figs. 6a,c,d there is one with all arrows reversed, corresponding to the complex conjugate term in the lagrangian. We will apply these general results to the specific case of the MSSM in the next section.

5 The Minimal Supersymmetric Standard Model

In sections 3 and 4, we have found a general recipe for constructing lagrangians for softly broken supersymmetric theories. We are now ready to apply these general results to the MSSM. The particle content for the MSSM was described in the Introduction. In this section we will complete the model by specifying the superpotential

and the soft-breaking terms.

5.1 The superpotential and supersymmetric interactions

The superpotential for the MSSM is given by

$$W_{\rm MSSM} = \overline{u}\mathbf{y_u}QH_u - \overline{d}\mathbf{y_d}QH_d - \overline{e}\mathbf{y_e}LH_d + \mu H_u H_d\,. \tag{5.1}$$

The objects H_u, H_d, Q, L, $\overline{u}$, $\overline{d}$, $\overline{e}$ appearing in eq. (5.1) are chiral superfields corresponding to the chiral supermultiplets in Table 1. (Alternatively, they can be just thought of as the corresponding scalar fields, as was done in section 3, but we prefer not to put the tildes on Q, L, $\overline{u}$, $\overline{d}$, $\overline{e}$ in order to reduce clutter.) The dimensionless Yukawa coupling parameters $\mathbf{y_u}, \mathbf{y_d}, \mathbf{y_e}$ are 3×3 matrices in family space. Here we have suppressed all of the gauge [$SU(3)_C$ color and $SU(2)_L$ weak isospin] and family indices. The "μ term", as it is traditionally called, can be written out as $\mu(H_u)_\alpha(H_d)_\beta\epsilon^{\alpha\beta}$, where $\epsilon^{\alpha\beta}$ is used to tie together $SU(2)_L$ weak isospin indices $\alpha, \beta = 1, 2$ in a gauge-invariant way. Likewise, the term $\overline{u}\mathbf{y_u}QH_u$ can be written out as $\overline{u}^i_a\,(\mathbf{y_u})_i{}^j\,Q^a_{j\alpha}\,(H_u)_\beta\epsilon^{\alpha\beta}$, where $i = 1, 2, 3$ is a family index, and $a = 1, 2, 3$ is a color index which is raised (lowered) in the $\mathbf{3}$ ($\overline{\mathbf{3}}$) representation of $SU(3)_C$.

The μ term in eq. (5.1) is the supersymmetric version of the Higgs boson mass in the Standard Model. It is unique, because terms $H_u^*H_u$ or $H_d^*H_d$ are forbidden in the superpotential, since it must be analytic in the chiral superfields (or equivalently in the scalar fields) treated as complex variables, as shown in section 3.2. We can also see from the form of eq. (5.1) why both H_u and H_d are needed in order to give Yukawa couplings, and thus masses, to all of the quarks and leptons. Since the superpotential must be analytic, the $\overline{u}QH_u$ Yukawa terms cannot be replaced by something like $\overline{u}QH_d^*$. Similarly, the $\overline{d}QH_d$ and $\overline{e}LH_d$ terms cannot be replaced by something like $\overline{d}QH_u^*$ and $\overline{e}LH_u^*$. The analogous Yukawa couplings would be allowed in a general non-supersymmetric two Higgs doublet model, but are forbidden by the structure of supersymmetry. So we need both H_u and H_d, even without invoking the argument based on anomaly cancellation which was mentioned in the Introduction.

The Yukawa matrices determine the masses and CKM mixing angles of the ordinary quarks and leptons, after the neutral scalar components of H_u and H_d get VEVs. Since the top quark, bottom quark and tau lepton are the heaviest fermions in the Standard Model, it is often useful to make an approximation that only the $(3,3)$ family components of each of $\mathbf{y_u}$, $\mathbf{y_d}$ and $\mathbf{y_e}$ are important:

$$\mathbf{y_u} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_t \end{pmatrix}; \qquad \mathbf{y_d} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_b \end{pmatrix}; \qquad \mathbf{y_e} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_\tau \end{pmatrix}. \tag{5.2}$$

In this limit, only the third family and Higgs fields contribute to the MSSM superpotential. It is instructive to write the superpotential in terms of the separate $SU(2)_L$ weak isospin components [$Q_3 = (t\ b)$; $L_3 = (\nu_\tau\ \tau)$; $H_u = (H_u^+\ H_u^0)$; $H_d = (H_d^0\ H_d^-)$; $\overline{u}_3 = \overline{t}$; $\overline{d}_3 = \overline{b}$; $\overline{e}_3 = \overline{\tau}$], so:

$$\begin{aligned} W_{\rm MSSM} \;\approx\; & y_t(\overline{t}tH_u^0 - \overline{t}bH_u^+) - y_b(\overline{b}tH_d^- - \overline{b}bH_d^0) - y_\tau(\overline{\tau}\nu_\tau H_d^- - \overline{\tau}\tau H_d^0) \\ & + \mu(H_u^+H_d^- - H_u^0H_d^0). \end{aligned} \tag{5.3}$$

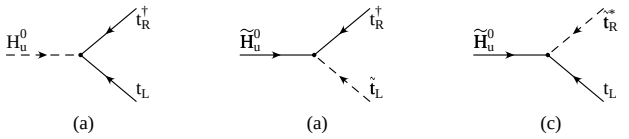

Figure 7: The top-quark Yukawa coupling (a) and its supersymmetrizations (b),(c), all of strength y_t.

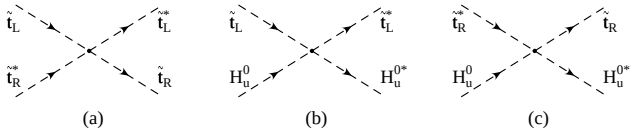

Figure 8: Some of the (scalar)^4 interactions with strength proportional to y_t^2.

The minus signs inside the parentheses appear because of the antisymmetry of the $\epsilon^{\alpha\beta}$ symbol used to tie up the $SU(2)_L$ indices. The minus signs in eq. (5.1) were chosen so that the terms $y_t \overline{t} t H_u^0$, $y_b \overline{b} b H_d^0$, and $y_\tau \overline{\tau} \tau H_d^0$, which will become the top, bottom and tau masses when H_u^0 and H_d^0 get VEVs, have positive signs in eq. (5.3).

Since the Yukawa interactions y^{ijk} in a general supersymmetric theory must be completely symmetric under interchange of i, j, k, we know that $\mathbf{y_u}$, $\mathbf{y_d}$ and $\mathbf{y_e}$ imply not only Higgs-quark-quark and Higgs-lepton-lepton couplings as in the Standard Model, but also squark-Higgsino-quark and slepton-Higgsino-lepton interactions. To illustrate this, we show in Figs. 7a,b,c some of the interactions which involve the top-quark Yukawa coupling y_t. Figure 7a is the Standard Model-like coupling of the top quark to the neutral complex scalar Higgs boson, which follows from the first term in eq. (5.3). For variety, we have used t_L and $t_R^\dagger$ in place of their synonyms t and $\overline{t}$ in Fig. 7; see the discussion in the final paragraph in section 2. In Fig. 7b, we have the coupling of the left-handed top squark $\tilde{t}_L$ to the neutral higgsino field $\widetilde{H}_u^0$ and right-handed top quark, while in Fig. 7c the right-handed top-squark field (known either as $\widetilde{\overline{t}}$ or $\widetilde{t}_R^*$ depending on taste) couples to $\widetilde{H}_u^0$ and t_L. For each of the three interactions, there is another with $H_u^0 \to H_u^+$ and $t_L \to -b_L$ (with tildes where appropriate), corresponding to the second part of the first term in eq. (5.3). All of these interactions are required by supersymmetry to have the same strength y_t. This is also an incontrovertible prediction of softly-broken supersymmetry at tree-level, since these interactions are dimensionless and can be modified by the introduction of soft supersymmetry breaking only through finite (and small) radiative corrections. A useful mnemonic is that each of Figs. 7a,b,c can be obtained from any of the others by changing two of the particles into their superpartners.

There are also scalar quartic interactions with strength proportional to y_t^2, as can be seen e.g. from Fig. 3b or the last term in eq. (3.47). Three of them are shown in Fig. 8. The reader is invited to check, using eq. (3.47) and eq. (5.3), that there are

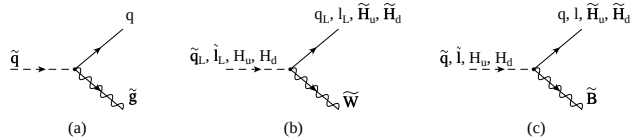

Figure 9: Couplings of the gluino, wino, and bino to MSSM (scalar, fermion) pairs.

nine more, which can be obtained by replacing $\tilde{t}_L \to \tilde{b}_L$ and/or $H_u^0 \to H_u^+$ in each vertex. This illustrates the remarkable economy of supersymmetry; there are many interactions determined by only a single parameter! In a similar way, the existence of all the other quark and lepton Yukawa couplings in the superpotential eq. (5.1) leads not only to Higgs-quark-quark and Higgs-lepton-lepton lagrangian terms as in the ordinary Standard Model, but also to squark-higgsino-quark and slepton-higgsino-lepton terms, and scalar quartic couplings [(squark)^4, (slepton)^4, (squark)^2(slepton)^2, (squark)^2(Higgs)^2, and (slepton)^2(Higgs)^2]. If needed, these can all be obtained in terms of the Yukawa matrices $\mathbf{y_u}$, $\mathbf{y_d}$, and $\mathbf{y_e}$ as outlined above.

However, it is useful to note that the dimensionless interactions determined by the superpotential are often not the most important ones of direct interest for phenomenology. This is because the Yukawa couplings are already known to be very small, except for those of the third family (top, bottom, tau). Instead, decay and especially production processes for superpartners in the MSSM are typically dominated by the supersymmetric interactions of gauge-coupling strength, as we will explore in more detail in sections 8 and 9. The couplings of the Standard Model gauge bosons (photon, $W^\pm$, Z^0 and gluons) to the MSSM particles are determined completely by the gauge invariance of the kinetic terms in the lagrangian. The gauginos also couple to (squark, quark) and (slepton, lepton) and (Higgs, higgsino) pairs as illustrated in the general case in Fig. 5g and the second line in eq. (3.72). For instance, each of the squark-quark-gluino couplings is given by $\sqrt{2} g_3 (\tilde{q} T^a q \tilde{g} + \text{c.c.})$ where T^a $(a = 1 \ldots 8)$ are the Gell-Mann matrices for $SU(3)_C$. The Feynman diagram for this interaction is shown in Fig. 9a. In Figs. 9b,c we show in a similar way the couplings of (squark, quark), (lepton, slepton) and (Higgs, higgsino) pairs to the winos and bino, with strengths proportional to the electroweak gauge couplings g and g' respectively. The winos only couple to the left-handed squarks and sleptons, and the (lepton, slepton) and (Higgs, higgsino) pairs of course do not couple to the gluino. The bino couplings for each (scalar, fermion) pair are also proportional to the weak hypercharges Y as given in Table 1. The interactions shown in Fig. 9 provide for decays $\tilde{q} \to q\tilde{g}$ and $\tilde{q} \to \widetilde{W} q'$ and $\tilde{q} \to \widetilde{B} q$ when the final states are kinematically allowed to be on-shell. However, a complication is that the $\widetilde{W}$ and $\widetilde{B}$ states are not mass eigenstates, because of mixing due to electroweak symmetry breaking, as we will see in section 7.3.

There are also various scalar quartic interactions in the MSSM which are uniquely determined by gauge invariance and supersymmetry, according to the last term in eq. (3.75) illustrated in Fig. 5h. Among them are (Higgs)^4 terms proportional to g^2 and g'^2 in the scalar potential. These are the direct generalization of the last term in

the Standard Model Higgs potential, eq. (1.1), to the case of the MSSM. We will have occasion to identify them explicitly when we discuss the minimization of the MSSM Higgs potential in section 7.2.

The dimensionful terms in the supersymmetric part of the MSSM lagrangian are all dependent on μ. Following the general result of eq. (3.48), we find that μ provides for higgsino fermion mass terms

$$\mathcal{L} \supset -\mu(\widetilde{H}_u^+ \widetilde{H}_d^- - \widetilde{H}_u^0 \widetilde{H}_d^0) + \text{c.c.}, \tag{5.4}$$

as well as Higgs (mass)^2 terms in the scalar potential

$$-\mathcal{L} \supset V \supset |\mu|^2(|H_u^0|^2 + |H_u^+|^2 + |H_d^0|^2 + |H_d^-|^2). \tag{5.5}$$

Since eq. (5.5) is positive-definite, it is clear that we cannot understand electroweak symmetry breaking without including supersymmetry-breaking (mass)^2 soft terms for the Higgs scalars, which can be negative. An explicit treatment of the Higgs scalar potential will therefore have to wait until we have introduced the soft terms for the MSSM. However, we can already see a puzzle: we expect that μ should be roughly of order 10^2 or 10^3 GeV, in order to allow a Higgs VEV of order 174 GeV without too much miraculous cancellation between $|\mu|^2$ and the negative soft (mass)^2 terms that we have not written down yet. But why should μ be so small compared to, say, $M_{\rm P}$, and in particular why should it be roughly of the same order as $m_{\rm soft}$? The scalar potential of the MSSM seems to depend on two types of dimensionful parameters which are conceptually quite distinct, namely the supersymmetry-respecting mass μ and the supersymmetry-breaking soft mass terms. Yet the observed value for the electroweak breaking scale suggests that without miraculous cancellations, both of these apparently unrelated mass scales should be within an order of magnitude or so of 100 GeV. This puzzle is called "the μ problem". Several different solutions to the μ problem have been proposed, involving extensions of the MSSM of varying intricacy. They all work in roughly the same way; the parameter μ is required or assumed to be completely absent at tree-level, and is to be replaced by the VEV(s) of some new field(s). The latter are in turn determined by minimizing a potential which depends on soft supersymmetry-breaking terms. In this way, the value of the effective parameter μ is no longer conceptually distinct from the mechanism of supersymmetry breaking; if we can explain why $m_{\rm soft} \ll M_{\rm P}$, we will also be able to understand why μ is of the same order. In section 10.2 we will describe one such mechanism. Some other attractive solutions for the μ problem are proposed in Refs.[41,42,43] From the point of view of the MSSM, however, we can just treat μ as an independent parameter.

The μ-term and the Yukawa couplings in the superpotential eq. (5.1) combine to yield (scalar)^3 couplings [see the second and third terms on the right-hand side of eq. (3.47)] of the form

$$\begin{aligned}\mathcal{L} \supset\ & \mu^*(\widetilde{\overline{u}}\mathbf{y_u}\widetilde{u}H_d^{0*} + \widetilde{\overline{d}}\mathbf{y_d}\widetilde{d}H_u^{0*} + \widetilde{\overline{e}}\mathbf{y_e}\widetilde{e}H_u^{0*} \\ & + \widetilde{\overline{u}}\mathbf{y_u}\widetilde{d}H_d^{-*} + \widetilde{\overline{d}}\mathbf{y_d}\widetilde{u}H_u^{+*} + \widetilde{\overline{e}}\mathbf{y_e}\widetilde{\nu}H_u^{+*}) + \text{c.c.}\end{aligned} \tag{5.6}$$

In Fig. 10 we show some of these couplings which are proportional to $\mu^* y_t$, $\mu^* y_b$, and $\mu^* y_\tau$ respectively. These play an important role in determining the mixing of top squarks, bottom squarks, and tau sleptons, as we will see in section 7.5.

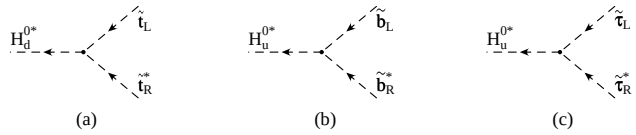

Figure 10: Some of the supersymmetric (scalar)^3 couplings proportional to $\mu^* y_t$, $\mu^* y_b$, and $\mu^* y_\tau$.

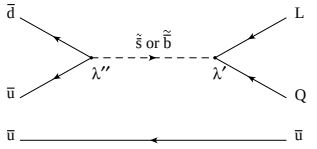

Figure 11: Squarks can mediate disastrously rapid proton decay if R-parity is violated.

5.2 R-parity (also known as matter parity) and its consequences

The superpotential eq. (5.1) is minimal in the sense that it is sufficient to produce a phenomenologically viable model. However, there are other terms that one could write down which are gauge-invariant and analytic in the chiral superfields, but are not included in the MSSM because they violate either baryon number (B) or total lepton number (L). The most general gauge-invariant and renormalizable superpotential would include not only eq. (5.1), but also the terms

$$W_{\Delta \rm L=1} = \frac{1}{2}\lambda^{ijk} L_i L_j \overline{e}_k + \lambda'^{ijk} L_i Q_j \overline{d}_k + \mu'^i L_i H_u \tag{5.7}$$

$$W_{\Delta \rm B=1} = \frac{1}{2}\lambda''^{ijk} \overline{u}_i \overline{d}_j \overline{d}_k \tag{5.8}$$

where we have restored family indices $i = 1, 2, 3$. The chiral supermultiplets carry baryon number assignments B $= +1/3$ for Q_i; B $= -1/3$ for $\overline{u}_i, \overline{d}_i$; and B $= 0$ for all others. The total lepton number assignments are L $= +1$ for L_i, L $= -1$ for $\overline{e}_i$, and L $= 0$ for all others. Therefore, the terms in eq. (5.7) violate total lepton number by 1 unit (as well as the individual lepton flavors) and those in eq. (5.8) violate baryon number by 1 unit.

The possible existence of such terms might seem rather disturbing, since B- and L-violating processes have never been seen experimentally. The most obvious experimental constraint comes from the non-observation of proton decay, which would violate both B and L by 1 unit. If both λ' and λ'' couplings were present and of order unity, then the lifetime of the proton would be measured in minutes or hours! For example, the Feynman graph in Fig. 11 would lead to $p^+ \to e^+\pi^0$ or e^+K^0 or $\mu^+\pi^0$ or μ^+K^0 or $\nu\pi^+$ or νK^+ etc. depending on which components of λ' are largest, and these processes would seem to be completely unsuppressed since the necessary couplings are all renormalizable. (The coupling λ'' must be antisymmetric in its last two flavor indices, since the color indices are contracted antisymmetrically. That is

why the squark in Fig. 11 is $\widetilde{\bar{s}}$ or $\widetilde{\bar{b}}$ but not $\widetilde{\bar{d}}$, for u, d quarks in the initial state.) In contrast, the decay time of the proton into these modes is measured to be in excess of 10^{32} years. Many other processes also give very significant constraints on the violation of lepton and baryon numbers; these are reviewed in Ref.[44]

One could simply try to take B and L conservation as a postulate in the MSSM. However, this is clearly a step backwards from the situation in the Standard Model, where the conservation of these quantum numbers is *not* assumed, but is rather a pleasantly "accidental" consequence of the fact that there are no possible renormalizable lagrangian terms which violate B or L. Furthermore, there is a quite general obstacle to treating B and L as fundamental symmetries of nature, since they are known to be necessarily violated by non-perturbative electroweak effects (even though those effects are calculably negligible for experiments at ordinary energies). Therefore, in the MSSM one adds a new symmetry which has the effect of eliminating the possibility of B and L violating terms in the renormalizable superpotential, while allowing the good terms in eq. (5.1). This new symmetry is called "R-parity" [6] or equivalently "matter parity".[45]

Matter parity is a multiplicatively conserved quantum number defined as

$$P_M = (-1)^{3(\mathrm{B-L})} \tag{5.9}$$

for each particle in the theory. It is easy to check that the quark and lepton supermultiplets all have $P_M = -1$, while the Higgs supermultiplets H_u and H_d have $P_M = +1$. The gauge bosons and gauginos of course do not carry baryon number or lepton number, so they are assigned matter parity $P_M = +1$. The symmetry principle to be enforced is that a term in the Lagrangian (or in the superpotential) is allowed only if the product of P_M for all of the fields in it is $+1$. It is easy to see that each of the terms in eq. (5.7) and (5.8) is thus forbidden, while the good and necessary terms in eq. (5.1) are allowed. This discrete symmetry commutes with supersymmetry, as all members of a given supermultiplet have the same matter parity. The advantage of matter parity is that it can in principle be an *exact* and fundamental symmetry, which B and L themselves cannot, since they are known to be violated by non-perturbative electroweak effects. So even with exact matter parity conservation in the MSSM, one expects that baryon number and total lepton number violation will occur in very tiny amounts, due to nonrenormalizable terms in the Lagrangian. However, the MSSM does not have renormalizable interactions that violate B or L, with the standard assumption of matter parity conservation.

It is sometimes useful to recast matter parity in terms of R-parity, defined for each particle as

$$P_R = (-1)^{3(\mathrm{B-L})+2s} \tag{5.10}$$

where s is the spin of the particle. Now, matter parity conservation and R-parity conservation are precisely equivalent, since the product of $(-1)^{2s}$ is of course equal to $+1$ for the particles involved in any interaction vertex in a theory that conserves angular momentum. However, particles within the same supermultiplet do not have

the same R-parity. In general, symmetries with the property that particles within the same multiplet have different charges are called R symmetries; they do not commute with supersymmetry. Continuous $U(1)$ R symmetries are often encountered in the model-building literature; they should not be confused with R-parity, which is a discrete Z_2 symmetry. In fact, the matter parity version of R-parity makes clear that there is really nothing intrinsically "R" about it; in other words it secretly does commute with supersymmetry, so its name is somewhat suboptimal. Nevertheless, the R-parity assignment is very useful for phenomenology because all of the Standard Model particles and the Higgs bosons have even R-parity ($P_R = +1$), while all of the squarks, sleptons, gauginos, and higgsinos have odd R-parity ($P_R = -1$).

The R-parity odd particles are known as "supersymmetric particles" or "sparticles" for short, and they are distinguished by a tilde (see Tables 1 and 2). If R-parity is exactly conserved, then there can be no mixing between the sparticles and the $P_R = +1$ particles. Furthermore, every interaction vertex in the theory contains an even number of $P_R = -1$ sparticles. This has three extremely important phenomenological consequences:

- The lightest sparticle with $P_R = -1$, called the "lightest supersymmetric particle" or LSP, must be absolutely stable. If the LSP is electrically neutral, it interacts only weakly with ordinary matter, and so can make an attractive candidate [46] for the non-baryonic dark matter which seems to be required by cosmology.

- Each sparticle other than the LSP must eventually decay into a state which contains an odd number of LSPs (usually just one).

- In collider experiments, sparticles can only be produced in even numbers (usually two-at-a-time).

We *define* the MSSM to conserve R-parity or equivalently matter parity. While this decision seems to be well-motivated phenomenologically by proton decay constraints and the hope that the LSP will provide a good dark matter candidate, it might appear somewhat *ad hoc* from a theoretical point of view. After all, the MSSM would not suffer any internal inconsistency if we did not impose matter parity conservation. Furthermore, it is fair to ask why matter parity should be exactly conserved, given that the known discrete symmetries in the Standard Model (ordinary parity P, charge conjugation C, time reversal T, etc.) are all known to be inexact symmetries. Fortunately, it *is* sensible to formulate matter parity as a discrete symmetry which is exactly conserved. In general, exactly conserved, or "gauged" discrete symmetries [47] can exist provided that they satisfy certain anomaly cancellation conditions [48] (much like continuous gauged symmetries). One particularly attractive way this could occur is if B−L is a continuous $U(1)$ gauge symmetry which is spontaneously broken at some very high energy scale. From eq. (5.9), we observe that P_M is actually a discrete subgroup of the continuous $U(1)_{\mathrm{B-L}}$ group. Therefore, if gauged $U(1)_{\mathrm{B-L}}$ is broken by scalar VEVs (or other order parameters) which carry only even integer values of

3(B−L), then P_M will automatically survive as an exactly conserved remnant. A variety of extensions of the MSSM in which exact R-parity arises in just this way have been proposed.[49,50] It may also be possible to have gauged discrete symmetries which do not owe their exact conservation to an underlying continuous gauged symmetry, but rather to some other structure such as can occur in string theory. It is also possible that R-parity is broken, or is replaced by some alternative discrete symmetry. We will briefly consider these as variations on the MSSM in section 10.1.

5.3 Soft supersymmetry breaking in the MSSM

To complete the description of the MSSM, we need to specify the soft supersymmetry breaking terms. In section 4, we learned how to write down the most general set of such terms in any supersymmetric theory. Applying this recipe to the MSSM, we have:

$$\begin{aligned}\mathcal{L}_{\rm soft}^{\rm MSSM} &= -\frac{1}{2}\left(M_3\widetilde{g}\widetilde{g} + M_2\widetilde{W}\widetilde{W} + M_1\widetilde{B}\widetilde{B}\right) + \text{c.c.} \\ &\quad -\left(\widetilde{\overline{u}}\,\mathbf{a_u}\,\widetilde{Q}H_u - \widetilde{\overline{d}}\,\mathbf{a_d}\,\widetilde{Q}H_d - \widetilde{\overline{e}}\,\mathbf{a_e}\,\widetilde{L}H_d\right) + \text{c.c.} \\ &\quad -\widetilde{Q}^\dagger\,\mathbf{m_Q^2}\,\widetilde{Q} - \widetilde{L}^\dagger\,\mathbf{m_L^2}\,\widetilde{L} - \widetilde{\overline{u}}\,\mathbf{m_{\overline{u}}^2}\,\widetilde{\overline{u}}^\dagger - \widetilde{\overline{d}}\,\mathbf{m_{\overline{d}}^2}\,\widetilde{\overline{d}}^\dagger - \widetilde{\overline{e}}\,\mathbf{m_{\overline{e}}^2}\,\widetilde{\overline{e}}^\dagger \\ &\quad - m_{H_u}^2 H_u^* H_u - m_{H_d}^2 H_d^* H_d - (bH_u H_d + \text{c.c.})\,. \end{aligned} \tag{5.11}$$

In eq. (5.11), M_3, M_2, and M_1 are the gluino, wino, and bino mass terms. Here, and from now on, we suppress the adjoint representation gauge indices on the wino and gluino fields, and the gauge indices on all of the chiral supermultiplet fields. The second line in eq. (5.11) contains the (scalar)^3 couplings [of the type a^{ijk} in eq. (4.1)]. Each of $\mathbf{a_u}$, $\mathbf{a_d}$, $\mathbf{a_e}$ is a complex 3×3 matrix in family space, with dimensions of (mass). They are in one-to-one correspondence with the Yukawa coupling matrices in the superpotential. The third line of eq. (5.11) consists of squark and slepton mass terms of the $(m^2)_i^j$ type in eq. (4.1). Each of $\mathbf{m_Q^2}$, $\mathbf{m_{\overline{u}}^2}$, $\mathbf{m_{\overline{d}}^2}$, $\mathbf{m_L^2}$, $\mathbf{m_{\overline{e}}^2}$ is a 3×3 matrix in family space which can have complex entries, but they must be hermitian so that the lagrangian is real. (To avoid clutter, we do not put tildes on the $\mathbf{Q}$ in $\mathbf{m_Q^2}$, etc.) Finally, in the last line of eq. (5.11) we have supersymmetry-breaking contributions to the Higgs potential; $m_{H_u}^2$ and $m_{H_d}^2$ are (mass)^2 terms of the $(m^2)_i^j$ type, while b is the only (mass)^2 term of the type b^{ij} in eq. (4.1) which can occur in the MSSM.[†] Schematically, we can write

$$M_1,\, M_2,\, M_3,\, \mathbf{a_u},\, \mathbf{a_d},\, \mathbf{a_e} \;\sim\; m_{\rm soft}; \tag{5.12}$$

$$\mathbf{m_Q^2},\, \mathbf{m_L^2},\, \mathbf{m_{\overline{u}}^2},\, \mathbf{m_{\overline{d}}^2},\, \mathbf{m_{\overline{e}}^2},\, m_{H_u}^2,\, m_{H_d}^2,\, b \;\sim\; m_{\rm soft}^2 \tag{5.13}$$

with a characteristic mass scale $m_{\rm soft}$ which is not much larger than 10^3 GeV, as argued in the Introduction. The expression eq. (5.11) is the most general soft supersymmetry-breaking Lagrangian of the form eq. (4.1) which is compatible with gauge invariance and matter parity conservation.

[†] The parameter we call b is often seen in the literature as m_{12}^2 or m_3^2 or $B\mu$.

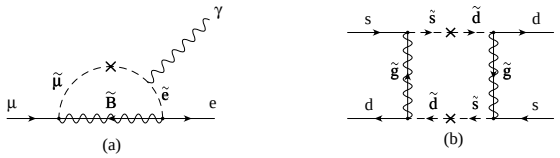

Figure 12: Diagrams which cause flavor violation in models with arbitrary soft masses.

Unlike the supersymmetry-preserving part of the lagrangian, $\mathcal{L}_{\rm soft}^{\rm MSSM}$ introduces many new parameters which were not present in the ordinary Standard Model. A careful count [51] reveals that there are 105 masses, phases and mixing angles in the MSSM lagrangian which cannot be rotated away by redefining the phases and flavor basis for the quark and lepton supermultiplets, and which have no counterpart in the ordinary Standard Model. Thus, in principle, supersymmetry (or more precisely, supersymmetry *breaking*) appears to introduce a tremendous arbitrariness in the lagrangian.

5.4 Hints of an Organizing Principle

Fortunately, there is already good experimental evidence that some sort of powerful "organizing principle" must govern the soft terms. This is because most of the new parameters in eq. (5.11) involve flavor mixing or CP violation of the type which is already severely restricted by experiment.[52] For example, suppose that $\mathbf{m_{\overline{e}}^2}$ is not diagonal in a basis $(\widetilde{e}_R, \widetilde{\mu}_R, \widetilde{\tau}_R)$ of sleptons whose superpartners are the right-handed pieces of the Standard Model mass eigenstates e, μ, τ. In that case slepton mixing occurs, and the individual lepton numbers will not be conserved. This is true even for processes which only involve the sleptons as virtual particles. A particularly strong limit on this possibility comes from the experimental constraint on $\mu \to e\gamma$, [53] which can occur via the one-loop diagram in Fig. 12a featuring a virtual bino and slepton. The cross represents an insertion of $\mathcal{L}_{\rm soft}^{\rm MSSM} \supset -(\mathbf{m_{\overline{e}}^2})_{21}\widetilde{e}_R\widetilde{\mu}_R^*$, and the slepton-bino vertices are determined by the weak hypercharge gauge coupling [see Fig. 5g and eq. (3.72)]. There are similar diagrams if the left-handed slepton mass matrix $\mathbf{m_L^2}$ has arbitrary off-diagonal entries. If $\mathbf{m_L^2}$ or $\mathbf{m_{\overline{e}}^2}$ were "random", with all entries of comparable size, then the contributions to BR$(\mu \to e\gamma)$ would be about 5 or 6 orders of magnitude larger than the current experimental upper limit of 5×10^{-11}, even if the sleptons are as heavy as 1 TeV. Therefore the form of the slepton mass matrices must be severely constrained.

There are also important experimental constraints on the squark (mass)^2 matrices. The strongest of these come from the neutral kaon system. The effective hamiltonian for $K^0 \leftrightarrow \overline{K}^0$ mixing gets contributions from the diagram in Fig. 12b, among others, if $\mathcal{L}_{\rm soft}^{\rm MSSM}$ contains (mass)^2 terms which mix down squarks and strange squarks. The gluino-squark-quark vertices in Fig. 12b are all fixed by supersymmetry to be of strong interaction strength; there are similar diagrams in which the bino and winos

are exchanged.[54] If the squark and gaugino masses are of order 1 TeV or less, one finds that limits on the parameters Δm_K and ϵ_K appearing in the neutral kaon system effective hamiltonian severely restrict the amount of down-strange squark mixing and CP-violating complex phases that one can tolerate in the soft parameters.[55] Considerably weaker, but still interesting, constraints come from the $D^0, \overline{D}^0$ and $B^0, \overline{B}^0$ neutral meson systems, and the decay $b \to s\gamma$.[56] After the Higgs scalar fields get VEVs, the $\mathbf{a_u}$, $\mathbf{a_d}$, $\mathbf{a_e}$ matrices contribute off-diagonal squark and slepton (mass)^2 terms [for example, $\widetilde{\bar{d}}\mathbf{a_d}\widetilde{Q}H_d + \text{c.c.} \to (\mathbf{a_d})_{12}\langle H_d^0\rangle \widetilde{s}_L \widetilde{d}_R^* + \text{c.c., etc.}$], so their form is also strongly constrained by flavor-changing neutral current (FCNC) limits. There are other significant constraints on CP-violating phases in the gaugino masses and (scalar)^3 soft couplings following from limits on the electric dipole moments of the neutron and electron.[57]

All of these potentially dangerous FCNC and CP-violating effects in the MSSM can be evaded if one assumes (or can explain!) that supersymmetry breaking should be suitably "universal". In particular, one can suppose that the squark and slepton (mass)^2 matrices are flavor-blind. This means that they should each be proportional to the 3×3 identity matrix in family space:

$$\mathbf{m_Q^2} = m_Q^2\mathbf{1}; \quad \mathbf{m_{\bar{u}}^2} = m_{\bar{u}}^2\mathbf{1}; \quad \mathbf{m_{\bar{d}}^2} = m_{\bar{d}}^2\mathbf{1}; \quad \mathbf{m_L^2} = m_L^2\mathbf{1}; \quad \mathbf{m_{\bar{e}}^2} = m_{\bar{e}}^2\mathbf{1}. \tag{5.14}$$

If so, then all squark and slepton mixing angles are rendered trivial, because squarks and sleptons with the same electroweak quantum numbers will be degenerate in mass and can be rotated into each other at will. Supersymmetric contributions to FCNC processes will therefore be very small in such an idealized limit, modulo the mixing due to $\mathbf{a_u}$, $\mathbf{a_d}$, $\mathbf{a_e}$. One can make the further assumption that the (scalar)^3 couplings are each proportional to the corresponding Yukawa coupling matrix:

$$\mathbf{a_u} = A_{u0}\,\mathbf{y_u}; \qquad \mathbf{a_d} = A_{d0}\,\mathbf{y_d}; \qquad \mathbf{a_e} = A_{e0}\,\mathbf{y_e}. \tag{5.15}$$

This ensures that only the squarks and sleptons of the third family can have large (scalar)^3 couplings. Finally, one can avoid disastrously large CP-violating effects with the assumption that the soft parameters do not introduce new complex phases. This is automatic for $m_{H_u}^2$ and $m_{H_d}^2$, and for m_Q^2, $m_{\bar{u}}^2$ etc. if eq. (5.14) is assumed; if they were not real numbers, the lagrangian would not be real. One can also fix μ in the superpotential and b in eq. (5.11) to be real, by an appropriate phase rotation of H_u and H_d. If one then assumes that

$$\arg(M_1),\ \arg(M_2),\ \arg(M_3),\ \arg(A_{u0}),\ \arg(A_{d0}),\ \arg(A_{e0}) = 0 \text{ or } \pi, \tag{5.16}$$

then the only CP-violating phase in the theory will be the ordinary CKM phase found in the ordinary Yukawa couplings. Together, the conditions eqs. (5.14)-(5.16) make up a rather weak version of what is often called the assumption of *soft-breaking universality*.

The soft-breaking universality relations eqs. (5.14)-(5.16) (or stronger versions of them) are presumed to be the result of some specific model for the origin of supersymmetry breaking, even though there is considerable disagreement among theorists as

to what the specific model should actually be. In any case, they are indicative of an underlying simplicity or symmetry of the lagrangian at some very high energy scale Q_0, which we will call the "input scale". If we use this lagrangian to compute masses and cross-sections and decay rates for experiments at ordinary energies near the electroweak scale, the results will involve large logarithms of order $\ln(Q_0/m_Z)$ coming from loop diagrams. As is usual in quantum field theory, the large logarithms can be conveniently resummed using renormalization group (RG) equations, by treating the couplings and masses appearing in the lagrangian as "running" parameters. Therefore, eqs. (5.14)-(5.16) should be interpreted as boundary conditions on the running soft parameters at the RG scale Q_0 which is very far removed from direct experimental probes. We must then RG-evolve all of the soft parameters, the superpotential parameters, and the gauge couplings down to the electroweak scale or comparable scales where humans perform experiments.

At the electroweak scale, eqs. (5.14) and (5.15) will no longer hold. However, RG corrections due to gauge interactions will respect eqs. (5.14) and (5.15), while RG corrections due to Yukawa interactions are quite small except for couplings involving the top squarks (stops) and possibly the bottom squarks (sbottoms) and tau sleptons (staus). In particular, the (scalar)^3 couplings should be quite negligible for the squarks and sleptons of the first two families. Furthermore, RG evolution does not introduce new CP-violating phases. Therefore, if universality can be arranged to hold at the input scale, supersymmetric contributions to FCNC and CP-violating observables can be acceptably small in comparison to present limits (although quite possibly measurable in future experiments).

One good reason to be optimistic that such a program can succeed is the celebrated apparent unification of gauge couplings in the MSSM.[58] The 1-loop RG equations for the Standard Model gauge couplings g_1, g_2, g_3 are given by

$$\frac{d}{dt}g_a = \frac{1}{16\pi^2}b_a g_a^3 \qquad \Rightarrow \qquad \frac{d}{dt}\alpha_a^{-1} = -\frac{b_a}{2\pi} \qquad (a = 1, 2, 3) \tag{5.17}$$

where $t = \ln(Q/Q_0)$ with Q the RG scale. In the Standard Model, $b_a^{\rm SM} = (41/10, -19/6, -7)$, while in the MSSM one finds instead $b_a^{\rm MSSM} = (33/5, 1, -3)$. The latter set of coefficients are larger because of the virtual effects of the extra MSSM particles in loops. The normalization for g_1 here is chosen to agree with the canonical covariant derivative for grand unification of the gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$ into $SU(5)$ or $SO(10)$. Thus in terms of the conventional electroweak gauge couplings g and g' with $e = g\sin\theta_W = g'\cos\theta_W$, one has $g_2 = g$ and $g_1 = \sqrt{5/3}g'$. The quantities $\alpha_a = g_a^2/4\pi$ have the nice property that their reciprocals run linearly with RG scale at one-loop order. In Fig. 13 we compare the RG evolution of the α_a^{-1}, including two-loop effects, in the Standard Model (dashed lines) and the MSSM (solid lines). Unlike the Standard Model, the MSSM includes just the right particle content to ensure that the gauge couplings can unify, at a scale $M_U \sim 2 \times 10^{16}$ GeV. While the apparent unification of gauge couplings at M_U could be just an accident, it may also be taken as a strong hint in favor of a grand unified theory (GUT) or superstring models, both of which indeed predict gauge coupling unification below $M_{\rm P}$. Furthermore, if we take

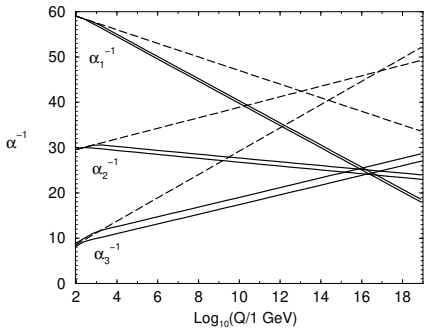

Figure 13: RG evolution of the inverse gauge couplings $\alpha_a^{-1}(Q)$ in the Standard Model (dashed lines) and the MSSM (solid lines). In the MSSM case, $\alpha_3(m_Z)$ is varied between 0.113 and 0.123, and the sparticle mass thresholds between 250 GeV and 1 TeV. Two-loop effects are included.

this hint seriously, then it means that we can reasonably expect to apply a similar RG analysis to the other MSSM couplings and soft masses as well.

We must mention that there are two other possible types of explanations for the suppression of FCNCs in the MSSM, which could replace the universality hypothesis of eqs. (5.14)-(5.16). One might refer to them as "irrelevancy" and "alignment" of the soft masses. The "irrelevancy" idea is that the sparticles masses are simply *extremely* heavy, so that their contributions to FCNC and CP-violating diagrams like Figs. 12a,b are highly suppressed. In practice, however, the degree of suppression needed often requires $m_{\rm soft} \gg 1$ TeV for at least some of the scalar masses; this seems to go directly against the motivation for supersymmetry as a cure for the hierarchy problem as discussed in the Introduction. Nevertheless, it is possible to arrange a scheme where this can work in a sensible way.[59] The "alignment" idea is that the squark (mass)^2 matrices do not have the flavor-blindness indicated in eq. (5.14), but are arranged in flavor space to be aligned with the relevant Yukawa matrices in just such a way as to avoid large FCNC effects.[40,60] The alignment models typically require rather special flavor symmetries. In any case, we will not discuss these possibilities further.

In practice, a given model for the origin of supersymmetry breaking may make predictions for the MSSM soft terms that are even stronger than eqs. (5.14)-(5.16). In the next section we will discuss the ideas that go into making such predictions, before turning to their implications for the MSSM spectrum in section 7.

6 Origins of supersymmetry breaking

6.1 General considerations for supersymmetry breaking

In the MSSM, supersymmetry breaking is simply introduced explicitly. However, we have seen that the soft parameters cannot be arbitrary. In order to understand how patterns like eqs. (5.14), (5.15) and (5.16) can emerge, it is necessary to consider models in which supersymmetry is spontaneously broken. By definition, this means that the vacuum state $|0\rangle$ is not invariant under supersymmetry transformations, so $Q_\alpha|0\rangle \neq 0$ and $Q^\dagger_{\dot\alpha}|0\rangle \neq 0$. Now, in global supersymmetry, the Hamiltonian operator H can be related to the supersymmetry generators through the algebra eq. (3.30):

$$H = P^0 = \frac{1}{4}(Q_1 Q_1^\dagger + Q_1^\dagger Q_1 + Q_2 Q_2^\dagger + Q_2^\dagger Q_2). \tag{6.1}$$

If supersymmetry is unbroken in the vacuum state, it follows that $H|0\rangle = 0$ and the vacuum has zero energy. Conversely, if supersymmetry is spontaneously broken in the vacuum state, then the vacuum must have positive energy, since

$$\langle 0|H|0\rangle = \frac{1}{4}\Big(\|Q_1|0\rangle\|^2 + \|Q_1^\dagger|0\rangle\|^2 + \|Q_2|0\rangle\|^2 + \|Q_2^\dagger|0\rangle\|^2\Big) > 0 \tag{6.2}$$

if the Hilbert space is to have positive norm. If spacetime-dependent effects and fermion condensates can be neglected, then $\langle 0|H|0\rangle = \langle 0|V|0\rangle$, where V is the scalar potential in eq. (3.75). Therefore supersymmetry will be spontaneously broken if F_i and/or D^a does not vanish in the ground state. Note that if any state exists in which all F_i and D^a vanish, then it will have zero energy, implying that supersymmetry cannot be spontaneously broken in the true ground state. Therefore the way to achieve spontaneous supersymmetry breaking is to look for models in which the equations $F_i = 0$ and $D^a = 0$ cannot be simultaneously satisfied for *any* values of the fields.

Supersymmetry breaking with non-zero D-terms can be achieved through the Fayet-Iliopoulos mechanism.[61] If the gauge symmetry includes a $U(1)$ factor, then one can introduce a term linear in the corresponding auxiliary field of the gauge supermultiplet:

$$\mathcal{L}_{\rm Fayet-Iliopoulos} = \kappa D \tag{6.3}$$

where κ is a constant parameter with dimensions of (mass)^2. This term is gauge-invariant and supersymmetric by itself. [Note that the supersymmetry transformation δD in eq. (3.59) is a total derivative for a $U(1)$ gauge symmetry.] If we include it in the lagrangian, then D may get a non-zero VEV, depending on the other interactions of the scalar fields that are charged under the $U(1)$. To see this, we can write the relevant part of the scalar potential using eqs. (3.54) and (3.72) as

$$V = \frac{1}{2}D^2 - \kappa D + g D \sum_i q_i \phi^{*i}\phi_i \tag{6.4}$$

where the q_i are the charges of the scalar fields ϕ_i under the $U(1)$ gauge group in question. The presence of the Fayet-Iliopoulos term modifies the equation of motion eq. (3.74) to

$$D = \kappa - g \sum_i q_i \phi^{*i} \phi_i. \tag{6.5}$$

Now suppose that the scalar fields ϕ_i have other interactions (such as large superpotential mass terms) which prevent them from getting VEVs. Then the auxiliary field D will be forced to get a VEV equal to κ, and supersymmetry will be broken. This mechanism cannot work for non-abelian gauge groups, however, since the analog of eq. (6.3) would not be gauge-invariant.

In the MSSM, one can imagine that the D term for $U(1)_Y$ has a Fayet-Iliopoulos term which is the principal source of supersymmetry breaking. Unfortunately, this would be an immediate disaster, because at least some of the squarks and sleptons would just get non-zero VEVs (breaking color, electromagnetism, and/or lepton number, but not supersymmetry) in order to satisfy eq. (6.5), because they do not have superpotential mass terms. This means that a Fayet-Iliopoulos term for $U(1)_Y$ must be subdominant compared to other sources of supersymmetry breaking in the MSSM, if not absent altogether. One could also attempt to trigger supersymmetry breaking with a Fayet-Iliopoulos term for some other $U(1)$ gauge symmetry which is as yet unknown because it is spontaneously broken at a very high mass scale or because it does not couple to the Standard Model particles. However, if this is the ultimate source for supersymmetry breaking, it proves difficult to give appropriate masses to all of the MSSM particles, especially the gauginos. In any case, we will not discuss D-term breaking as the ultimate origin of supersymmetry violation any further, although it may not be ruled out.[62]

Models where supersymmetry breaking is due to non-zero F-terms, called O'Raifeartaigh models,[63] may have brighter phenomenological prospects. The idea is to pick a set of chiral supermultiplets $\Phi_i \supset (\phi_i, \psi_i, F_i)$ and a superpotential W in such a way that the equations $F_i = -\delta W^*/\delta \phi^{*i} = 0$ have no simultaneous solution. Then $V = \sum_i |F_i|^2$ will have to be positive at its minimum, ensuring that supersymmetry is broken. The simplest example which does this has three chiral supermultiplets with

$$W = -k\Phi_1 + m\Phi_2\Phi_3 + \frac{y}{2}\Phi_1\Phi_3^2. \tag{6.6}$$

Note that W contains a linear term, with k having dimensions of (mass)^2. This is only possible if Φ_1 is a gauge singlet. In section 3 we cheated and did not mention such a term, because we knew that the MSSM contains no such singlet chiral supermultiplet. Nevertheless, it should be clear from retracing the derivation in section 3.2 that such a term is allowed if a gauge-singlet chiral supermultiplet is added to the theory. In fact, a linear term is absolutely necessary to achieve F-term breaking, since otherwise setting all $\phi_i = 0$ will always give a supersymmetric global minimum with all $F_i = 0$. Without loss of generality, we can choose k, m, and y to be real and positive (by a phase rotation of the fields). The scalar potential following from eq. (6.6) is

$$V = |F_1|^2 + |F_2|^2 + |F_3|^2; \tag{6.7}$$

$$F_1 = k - \frac{y}{2}\phi_3^{*2}; \qquad F_2 = -m\phi_3^*; \qquad F_3 = -m\phi_2^* - y\phi_1^*\phi_3^*. \tag{6.8}$$

Clearly, $F_1 = 0$ and $F_2 = 0$ are not compatible, so supersymmetry must indeed be broken. If $m^2 > yk$ (which we assume from now on), then it is easy to show that the absolute minimum of the potential is at $\phi_2 = \phi_3 = 0$ with ϕ_1 undetermined, so $F_1 = k$ and $V = k^2$ at the minimum of the potential. The fact that ϕ_1 is undetermined is an example of a "flat direction" in the scalar potential; this is a common feature of supersymmetric models.[†]

If we presciently choose to expand V around $\phi_1 = 0$, the mass spectrum of the theory consists of 6 real scalars with tree-level squared masses

$$0,\ 0,\ m^2,\ m^2,\ m^2 - yk,\ m^2 + yk. \tag{6.9}$$

Meanwhile, there are 3 Weyl fermions with masses

$$0,\ m,\ m. \tag{6.10}$$

The non-degeneracy of scalars and fermions is a clear sign that supersymmetry has been spontaneously broken. The 0 eigenvalues in eqs. (6.9) and (6.10) correspond to the complex scalar ϕ_1 and its fermionic partner ψ_1. However, ϕ_1 and ψ_1 have different reasons for being massless. The masslessness of ϕ_1 corresponds to the existence of the flat direction, since any value of ϕ_1 gives the same energy at tree-level. This flat direction is an accidental feature of the classical scalar potential, and in this case it is removed ("lifted") by quantum corrections. This can be seen by computing the Coleman-Weinberg one-loop effective potential.[64] After some calculation, one finds the result that the global minimum is indeed fixed at $\phi_1 = \phi_2 = \phi_3 = 0$, with the complex scalar ϕ_1 receiving a small positive-definite (mass)^2 equal to

$$m_{\phi_1}^2 = \frac{1}{32\pi^2}\left[\left(\frac{ym^4}{k} + y^3 k\right)\ln\left(\frac{m^2+yk}{m^2-yk}\right) + 2y^2m^2\left(\ln[1 - \frac{y^2k^2}{m^4}] - 1\right)\right]. \tag{6.11}$$

[In the limit $yk \ll m^2$, this reduces to $m_{\phi_1}^2 = y^4k^2/(48\pi^2m^2)$.] In contrast, the Weyl fermion ψ_1 remains exactly massless because of a general feature of all models with spontaneously broken supersymmetry. To understand this, recall that the spontaneous breaking of any global symmetry always gives rise to a massless Nambu-Goldstone mode with the same quantum numbers as the broken symmetry generator. In the case of supersymmetry, the broken generator is the fermionic charge Q_α, so the Nambu-Goldstone particle must be a massless neutral Weyl fermion called the *goldstino*. In the O'Raifeartaigh model example, ψ_1 is the goldstino because it is the fermionic partner of the auxiliary field F_1 which got a VEV. (We will prove these statements in a more general context in section 6.2.)

The O'Raifeartaigh superpotential determines the mass scale of supersymmetry breaking $\sqrt{F_1}$ in terms of a dimensionful parameter k which is put in by hand. This

[†]More generally, "flat directions" are non-compact lines and surfaces in the space of scalar fields along which the scalar potential vanishes. The classical scalar potential of the MSSM would have many flat directions if supersymmetry were not broken.

is somewhat *ad hoc*, since $\sqrt{k}$ will have to be much less than $M_{\rm P}$ in order to give the right order of magnitude for the MSSM soft terms. We would like to have a mechanism which can instead generate such scales naturally. This can be done in models of dynamical supersymmetry breaking. In such theories, the small (compared to $M_{\rm P}$) mass scales associated with supersymmetry breaking arise by dimensional transmutation. In other words, they generally feature a new asymptotically-free non-Abelian gauge symmetry with a gauge coupling g which is perturbative at $M_{\rm P}$ and which gets strong in the infrared at some smaller scale $\Lambda \sim e^{-8\pi^2/|b|g_0^2} M_{\rm P}$, where g_0 is the running gauge coupling at $M_{\rm P}$ with beta function $-|b|g^3/16\pi^2$. Just as in QCD, it is perfectly natural for Λ to be many orders of magnitude below the Planck scale. Supersymmetry breaking may then be best described in terms of the effective dynamics of the strongly coupled theory. One possibility is that the auxiliary F field for a composite chiral supermultiplet (built out of the fundamental fields which transform under the new strongly-coupled gauge group) obtains a VEV. Constructing models which actually break supersymmetry in an acceptable way is a highly non-trivial business; for more information we refer the reader to Ref.[65]

The one thing that is now clear about spontaneous supersymmetry breaking (dynamical or not) is that it requires us to extend the MSSM. The ultimate supersymmetry-breaking order parameter cannot belong to any of the supermultiplets of the MSSM; a D-term VEV for $U(1)_Y$ does not lead to an acceptable spectrum, and there is no candidate gauge-singlet whose F-term could develop a VEV. Therefore one must ask what effects *are* responsible for spontaneous supersymmetry breaking, and how supersymmetry breakdown is "communicated" to the MSSM particles. It is very difficult to achieve the latter in a phenomenologically viable way working only with renormalizable interactions at tree-level. First, it is problematic to give masses to the MSSM gauginos, because supersymmetry does not allow (scalar)-(gaugino)-(gaugino) couplings which could turn into gaugino mass terms when the scalar gets a VEV. Second, at least some of the MSSM squarks and sleptons would have to be unacceptably light, and should have been discovered already. This can be understood in a general way from the existence of a sum rule which governs the tree-level squared masses of scalars and chiral fermions in theories with spontaneous supersymmetry breaking:

$$\mathrm{Tr}[M^2_{\text{real scalars}}] = 2\,\mathrm{Tr}[M^2_{\text{chiral fermions}}]. \tag{6.12}$$

If supersymmetry were not broken, then eq. (6.12) would follow immediately from the degeneracy of complex scalars [with two real scalar components, hence the factor of 2] and their Weyl fermion superpartners. However, eq. (6.12) still holds at tree-level when supersymmetry is broken spontaneously by F-terms and D-terms, as one can verify in general by explicitly computing the $(\text{mass})^2$ matrices for arbitrary values of the fields.[‡] One can easily see, for example, that with the O'Raifeartaigh spectrum of eqs. (6.9) and (6.10), the sum rule eq. (6.12) is indeed satisfied. This sum rule seems to be bad news for a phenomenologically viable model, because the masses of all of

[‡] This assumes only that the trace of the $U(1)$ charges over all chiral supermultiplets in the theory vanishes ($\mathrm{Tr}[T^a] = 0$). This holds for $U(1)_Y$ in the MSSM and more generally for any non-anomalous gauge symmetry.

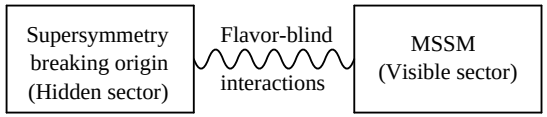

Figure 14: The presumed schematic structure for supersymmetry breaking.

the MSSM chiral fermions are already known to be small (except for the top quark and the higgsinos). Even if we could succeed in evading this, there is no reason why the resulting MSSM soft terms in this type of model should satisfy conditions like eqs. (5.14) or (5.15).

For these reasons, we expect that the MSSM soft terms arise indirectly or radiatively, rather than from tree-level renormalizable couplings to the supersymmetry-breaking order parameters. Supersymmetry breaking evidently occurs in a "hidden sector" of particles which have no (or only very small) direct couplings to the "visible sector" chiral supermultiplets of the MSSM. However, the two sectors do share some interactions which are responsible for mediating supersymmetry breaking from the hidden sector to the visible sector, where they appear as calculable soft terms. (See Fig. 14.) In this scenario, the tree-level sum rule eq. (6.12) need not hold for the visible sector fields, so that a phenomenologically viable superpartner mass spectrum is in principle achievable. As a bonus, if the mediating interactions are flavor-blind, then the soft terms appearing in the MSSM may automatically obey conditions like eqs. (5.14), (5.15) and (5.16).

There are two main competing proposals for what the mediating interactions might be. The first (and historically the more popular) is that they are gravitational. More precisely, they are associated with the new physics, including gravity, which enters at the Planck scale. In this *gravity-mediated supersymmetry breaking* scenario, if supersymmetry is broken in the hidden sector by a VEV $\langle F \rangle$, then the soft terms in the visible sector should be roughly of order

$$m_{\rm soft} \sim \frac{\langle F \rangle}{M_{\rm P}}, \tag{6.13}$$

by dimensional analysis. This is because we know that $m_{\rm soft}$ must vanish in the limit $\langle F \rangle \to 0$ where supersymmetry is unbroken, and also in the limit $M_{\rm P} \to \infty$ (corresponding to $G_{\rm Newton} \to 0$) in which gravity becomes irrelevant. For $m_{\rm soft}$ of order a few hundred GeV, one would therefore expect that the scale associated with the origin of supersymmetry breaking in the hidden sector should be roughly $\sqrt{\langle F \rangle} \sim 10^{10}$ or 10^{11} GeV. Another possibility is that the supersymmetry breaking order parameter is a gaugino condensate $\langle 0|\lambda^a\lambda^b|0\rangle = \delta^{ab}\Lambda^3 \neq 0$. If the composite field $\lambda^a\lambda^b$ is part of an auxiliary field F for some (perhaps composite) chiral superfield, then by dimensional analysis we expect supersymmetry breaking soft terms of order

$$m_{\rm soft} \sim \frac{\Lambda^3}{M_{\rm P}^2}, \tag{6.14}$$

with, effectively, $\langle F\rangle \sim \Lambda^3/M_{\rm P}$. In that case, the scale associated with dynamical supersymmetry breaking should be more like $\Lambda \sim 10^{13}$ GeV.

The second main possibility is that the flavor-blind mediating interactions for supersymmetry breaking are the ordinary electroweak and QCD gauge interactions. In this *gauge-mediated supersymmetry breaking* scenario, the MSSM soft terms arise from loop diagrams involving some *messenger* particles. The messengers couple to a supersymmetry-breaking VEV $\langle F\rangle$, and also have $SU(3)_C \times SU(2)_L \times U(1)_Y$ interactions which provide a link to the MSSM. Then, using dimensional analysis, one estimates for the MSSM soft terms

$$m_{\rm soft} \sim \frac{\alpha_a}{4\pi}\frac{\langle F\rangle}{M_{\rm mess}} \tag{6.15}$$

where the $\alpha_a/4\pi$ is a loop factor for Feynman diagrams involving gauge interactions, and $M_{\rm mess}$ is a characteristic scale of the masses of the messenger fields. So if $M_{\rm mess}$ and $\sqrt{\langle F\rangle}$ are roughly comparable, then the scale of supersymmetry breaking can be as low as about $\sqrt{\langle F\rangle} \sim 10^4$ or 10^5 GeV (much lower than in the gravity-mediated case!) to give $m_{\rm soft}$ of the right order of magnitude.

6.2 The goldstino and the gravitino

As explained in the previous section, the spontaneous breaking of global supersymmetry implies the existence of a massless Weyl fermion, the goldstino. In the particular case of the O'Raifeartaigh model, the goldstino was identified to be ψ_1. More generally, we might expect that in the case of F-term or D-term breaking, the goldstino is the fermionic component of the supermultiplet whose auxiliary field obtains a VEV.

Let us make this more precise by actually proving that the goldstino exists and, in the process, identifying it. This is actually rather easy. Consider a general supersymmetric model with both gauge and chiral supermultiplets as in section 3. The fermionic degrees of freedom consist of gauginos (λ^a) and chiral fermions (ψ_i). After some of the scalar fields in the theory obtain VEVs, the fermion mass matrix will have the form:

$$\mathbf{M}_{\rm fermion} = \begin{pmatrix} 0 & \sqrt{2}g_a(\langle\phi^*\rangle T^a)^i \\ \sqrt{2}g_a(\langle\phi^*\rangle T^a)^j & \langle W^{ij}\rangle \end{pmatrix} \tag{6.16}$$

in the (λ^a, ψ_i) basis. [The off-diagonal entries in this matrix come from the second line in eq. (3.72), and the lower right entry can be seen in eq. (3.46).] Now we simply note that $\mathbf{M}_{\rm fermion}$ annihilates the vector

$$\widetilde{G} = \begin{pmatrix} \langle D^a\rangle/\sqrt{2} \\ \langle F_i\rangle \end{pmatrix}. \tag{6.17}$$

The first row of $\mathbf{M}_{\rm fermion}$ annihilates $\widetilde{G}$ by virtue of the requirement eq. (3.73) that the superpotential is gauge invariant, and the second row annihilates $\widetilde{G}$ because of the condition $\langle \partial V/\partial\phi_i\rangle = 0$ which must be satisfied at the minimum of the scalar potential. Eq. (6.17) is proportional to the goldstino wavefunction; it is non-trivial if and

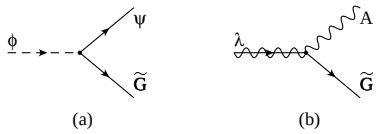

Figure 15: Goldstino/gravitino interactions with superpartner pairs (ϕ,ψ) and (λ^a, A^a).

only if at least one of the auxiliary fields has a VEV, breaking supersymmetry. So we have proven that if global supersymmetry is spontaneously broken, then the goldstino exists and has zero mass, and that its components among the various fermions in the theory are just proportional to the corresponding auxiliary field VEVs.

We can derive another very important property of the goldstino by considering the form of the conserved supercurrent eq. (3.76). Suppose for simplicity[§] that the non-vanishing auxiliary field VEV is $\langle F\rangle$ and that its goldstino superpartner is $\widetilde{G}$. Then the supercurrent conservation equation tells us that

$$0 = \partial_\mu J^\mu_\alpha = i\langle F\rangle(\sigma^\mu\partial_\mu\widetilde{G}^\dagger)_\alpha + \partial_\mu j^\mu_\alpha + \ldots \tag{6.18}$$

where j^μ_α is the part of the supercurrent which involves all of the other supermultiplets, and the ellipses represent other contributions of the goldstino supermultiplet to $\partial_\mu J^\mu_\alpha$ which we can ignore. [The first term in eq. (6.18) comes from the second term in eq. (3.76), using the equation of motion $F_i = -W_i^*$ for the goldstino's auxiliary field.] This equation of motion for the goldstino field allows us to write an effective lagrangian

$$\mathcal{L}_{\rm goldstino} = -i\widetilde{G}^\dagger\overline{\sigma}^\mu\partial_\mu\widetilde{G} - \frac{1}{\langle F\rangle}(\widetilde{G}\partial_\mu j^\mu + \text{c.c.}) \tag{6.19}$$

which describes the interactions of the goldstino with all of the other fermion-boson pairs.[66] In particular, since $j^\mu_\alpha = (\sigma^\nu\overline{\sigma}^\mu\psi_i)_\alpha\partial_\nu\phi^{*i} - (1/2\sqrt{2})\sigma^\nu\overline{\sigma}^\rho\sigma^\mu\lambda^{\dagger a}F^a_{\nu\rho} + \ldots$, there are goldstino-scalar-chiral fermion and goldstino-gaugino-gauge boson vertices as shown in Fig. 15. Since this derivation depends only on supercurrent conservation, eq. (6.19) holds independently of the details of how supersymmetry breaking is communicated from $\langle F\rangle$ to the MSSM sector fields (ϕ_i,ψ_i) and (λ^a, A^a). It may appear strange at first that the interaction terms in eq. (6.19) get larger as $\langle F\rangle$ goes to zero. However, the interaction term $\widetilde{G}\partial_\mu j^\mu$ contains two derivatives which turn out to always give a kinematic factor proportional to the $(\text{mass})^2$ difference of the superpartners when they are on-shell, i.e. $m^2_{\phi_i} - m^2_{\psi_i}$ and $m^2_\lambda - m^2_A$ for Figs. 15a and 15b respectively. These can be non-zero only by virtue of supersymmetry breaking, so they must also vanish as $\langle F\rangle \to 0$, and the interaction is well-defined in that limit. Nevertheless, for fixed values of $m^2_{\phi_i} - m^2_{\psi_i}$ and $m^2_\lambda - m^2_A$, the interaction term in eq. (6.19) can be phenomenologically important if $\langle F\rangle$ is not too large.[66,67,68,69]

[§]More generally, if supersymmetry is spontaneously broken by VEVs for several auxiliary fields F_i and D^a, then one should make the replacement $\langle F\rangle \to (\sum_i |\langle F_i\rangle|^2 + \frac{1}{2}\sum_a\langle D^a\rangle^2)^{1/2}$ everywhere in the following.

The above remarks apply to the breaking of global supersymmetry. However, when one takes into account gravity, supersymmetry must be a local symmetry. This means that the spinor parameter ϵ^α which first appeared in section 3.1 is no longer a constant, but can vary from point to point in spacetime. The resulting locally supersymmetric theory is called *supergravity*.[70,71] It necessarily unifies the spacetime symmetries of ordinary general relativity with local supersymmetry transformations. In supergravity, the spin-2 graviton has a spin-3/2 fermion superpartner called the gravitino, which we will denote $\widetilde{\Psi}^\alpha_\mu$. The gravitino has odd R-parity ($P_R = -1$), as can be seen from the definition eq. (5.10). It carries both a vector index (μ) and a spinor index (α), and transforms inhomogeneously under local supersymmetry transformations:

$$\delta\widetilde{\Psi}^\alpha_\mu = -\partial_\mu \epsilon^\alpha + \dots \tag{6.20}$$

Thus the gravitino should be thought of as the "gauge" particle of local supersymmetry transformations [compare eq. (3.52)]. As long as supersymmetry is unbroken, the graviton and the gravitino are both massless, each with two spin helicity states. Once supersymmetry is spontaneously broken, the gravitino acquires a mass by absorbing ("eating") the goldstino, which becomes its longitudinal (helicity $\pm 1/2$) components. This is called the *super-Higgs* mechanism. It is entirely analogous to the ordinary Higgs mechanism for gauge theories, by which the $W^\pm$ and Z^0 gauge bosons in the Standard Model gain mass by absorbing the Nambu-Goldstone bosons associated with the spontaneously broken electroweak gauge invariance. The counting works, because the massive spin-3/2 gravitino now has four helicity states, of which two were originally assigned to the would-be goldstino. The gravitino mass is traditionally called $m_{3/2}$, and in the case of F-term breaking can be estimated as[72]

$$m_{3/2} \sim \frac{\langle F \rangle}{M_{\rm P}}, \tag{6.21}$$

This follows simply from dimensional analysis, since $m_{3/2}$ must vanish in the limits that supersymmetry is restored ($\langle F\rangle \to 0$) and that gravity is turned off ($M_P \to \infty$). Equation (6.21) means that one has very different expectations for the mass of the gravitino in gravity-mediated and in gauge-mediated models, because they usually make very different predictions for $\langle F\rangle$.

In the gravity-mediated supersymmetry breaking case, the gravitino mass is comparable to the masses of the MSSM sparticles [compare eqs. (6.13) and (6.21)]. Therefore $m_{3/2}$ is expected to be at least 100 GeV or so. Its interactions will be of gravitational strength, so the gravitino will not play any role in collider physics, but it can be a very important consideration in cosmology.[73] If it is the LSP, then it is stable and its primordial density could easily exceed the critical density, causing the universe to become matter-dominated too early. Even if it is not the LSP, the gravitino can cause problems unless its density is diluted by inflation at late times, or it decays sufficiently rapidly.

In contrast, gauge-mediated supersymmetry breaking models predict that the gravitino is much lighter than the MSSM sparticles as long as $M_{\rm mess} \ll M_{\rm P}$. This can

be seen by comparing eqs. (6.15) and (6.21). The gravitino is almost certainly the LSP in this case, and all of the MSSM sparticles will eventually decay into final states that include it. Naively, one might expect that these decays are extremely slow. However, this is not necessarily true, because the gravitino inherits the *non*-gravitational interactions of the goldstino it has absorbed. This means that the gravitino, or more precisely its longitudinal (goldstino) components, can play an important role in collider physics experiments. The mass of the gravitino can generally be ignored for kinematic purposes, as can its transverse (helicity $\pm 3/2$) components which really do have only gravitational interactions. Therefore in collider phenomenology discussions one may interchangeably use the same symbol $\widetilde{G}$ for the goldstino and for the gravitino of which it is the longitudinal (helicity $\pm 1/2$) part. By using the effective lagrangian eq. (6.19), one can compute that the decay rate of any sparticle $\widetilde{X}$ into its Standard Model partner X plus a gravitino/goldstino $\widetilde{G}$ is given by

$$\Gamma(\widetilde{X} \to X\widetilde{G}) = \frac{m^5_{\widetilde{X}}}{16\pi\langle F\rangle^2}\left(1 - \frac{m_X^2}{m^2_{\widetilde{X}}}\right)^4. \tag{6.22}$$

This corresponds to either Fig. 15a or 15b, with $(\widetilde{X}, X) = (\phi, \psi)$ or (λ, A) respectively. One factor $(1 - m_X^2/m^2_{\widetilde{X}})^2$ came from the derivatives in the interaction term in eq. (6.19) evaluated for on-shell final states, and another such factor comes from the kinematic phase space integral with $m_{3/2} \ll m_{\widetilde{X}}, m_X$.

If the supermultiplet containing the goldstino and $\langle F\rangle$ has canonically-normalized kinetic terms, and one requires the tree-level vacuum energy to vanish, then the estimate eq. (6.21) may be sharpened to

$$m_{3/2} = \frac{\langle F\rangle}{\sqrt{3}M_{\rm P}}. \tag{6.23}$$

In that case, one can rewrite eq. (6.22) as

$$\Gamma(\widetilde{X} \to X\widetilde{G}) = \frac{m^5_{\widetilde{X}}}{48\pi M_{\rm P}^2 m^2_{3/2}}\left(1 - \frac{m_X^2}{m^2_{\widetilde{X}}}\right)^4, \tag{6.24}$$

and this is how the formula is sometimes presented by those who prefer to take eq. (6.23) seriously. Note that the decay width is larger for smaller $\langle F\rangle$, or equivalently for smaller $m_{3/2}$, if the other masses are fixed. If $\widetilde{X}$ is a mixture of superpartners of different Standard Model particles X, then eq. (6.22) should be multiplied by a suppression factor equal to the square of the cosine of the appropriate mixing angle. If $m_{\widetilde{X}}$ is of order 100 GeV or more, and $\sqrt{\langle F\rangle} \lesssim$ few$\times 10^6$ GeV [corresponding to $m_{3/2}$ less than roughly 1 keV according to eq. (6.23)], then the decay $\widetilde{X} \to X\widetilde{G}$ can occur quickly enough to be observed in a modern collider detector. This gives rise to some very interesting phenomenological signatures, which we will discuss further in sections 8.5 and 9.

We now turn to a slightly more systematic analysis of the way in which the MSSM soft terms arise, considering in turn the gravity-mediated and gauge-mediated scenarios.

The defining feature of these models is that the hidden sector of the theory communicates with our MSSM only (or dominantly) through gravitational-strength interactions. In an effective field theory format, this means that the supergravity lagrangian contains nonrenormalizable terms which communicate between the two sectors and which are suppressed by powers of the Planck mass, since the gravitational coupling is proportional to $1/M_{\rm P}$. These will include

$$\begin{aligned}\mathcal{L}_{\rm NR} &= -\frac{1}{M_{\rm P}}F_X\sum_a \frac{1}{2}f_a\lambda^a\lambda^a + \text{c.c.} \\ &\quad -\frac{1}{M_{\rm P}^2}F_X F_X^*\, k^i_j\phi_i\phi^{*j} \\ &\quad -\frac{1}{M_{\rm P}}F_X(\frac{1}{6}y'^{ijk}\phi_i\phi_j\phi_k + \frac{1}{2}\mu'^{ij}\phi_i\phi_j) + \text{c.c.} \end{aligned} \tag{6.25}$$

where F_X is the auxiliary field for a chiral supermultiplet X in the hidden sector, and ϕ_i and λ^a are the scalar and gaugino fields in the MSSM. By themselves, the terms in eq. (6.25) are not supersymmetric, but it is possible to show that they are part of a nonrenormalizable supersymmetric lagrangian (see Appendix) which contains other terms that we may ignore. Now if one assumes that $\langle F_X\rangle \sim 10^{10}$ or 10^{11} GeV, then $\mathcal{L}_{\rm NR}$ will give us nothing other than a lagrangian of the form $\mathcal{L}_{\rm soft}$ in eq. (4.1), with MSSM soft terms of order a few hundred GeV. [Note that terms of the form $\mathcal{L}_{\text{maybe soft}}$ in eq. (4.2) do not arise.]

The dimensionless parameters f_a, k^i_j, y'^{ijk} and μ'^{ij} in $\mathcal{L}_{\rm NR}$ are to be determined by the underlying theory. This is a difficult enterprise in general, but a dramatic simplification occurs if one assumes a "minimal" form for the normalization of kinetic terms and gauge interactions in the full, nonrenormalizable supergravity lagrangian (see Appendix). In that case, one finds that there is a common $f_a = f$ for the three gauginos; $k^i_j = k\delta^i_j$ is the same for all scalars; and the other couplings are proportional to the corresponding superpotential parameters, so that $y'^{ijk} = \alpha y^{ijk}$ and $\mu'^{ij} = \beta\mu^{ij}$ with universal dimensionless constants α and β. Then one finds that the soft terms in $\mathcal{L}^{\rm MSSM}_{\rm soft}$ can all be written in terms of just four parameters:

$$m_{1/2} = f\frac{\langle F_X\rangle}{M_{\rm P}}; \qquad m_0^2 = k\frac{|\langle F_X\rangle|^2}{M_{\rm P}^2}; \qquad A_0 = \alpha\frac{\langle F_X\rangle}{M_{\rm P}}; \qquad B_0 = \beta\frac{\langle F_X\rangle}{M_{\rm P}}. \tag{6.26}$$

In terms of these, one can write for the parameters appearing in eq. (5.11):

$$M_3 = M_2 = M_1 = m_{1/2}; \tag{6.27}$$

$$\mathbf{m_Q^2} = \mathbf{m_{\bar u}^2} = \mathbf{m_{\bar d}^2} = \mathbf{m_L^2} = \mathbf{m_{\bar e}^2} = m_0^2\,\mathbf{1}; \quad m_{H_u}^2 = m_{H_d}^2 = m_0^2; \tag{6.28}$$

$$\mathbf{a_u} = A_0\mathbf{y_u}; \qquad \mathbf{a_d} = A_0\mathbf{y_d}; \qquad \mathbf{a_e} = A_0\mathbf{y_e}; \tag{6.29}$$

$$b = B_0\mu. \tag{6.30}$$

It is a matter of some controversy whether the assumptions going into this parameterization are completely well-motivated on purely theoretical grounds,[¶] but from a phenomenological perspective they are clearly very nice. This framework successfully evades the most dangerous types of FCNC and CP-violation as discussed in section 5.4. In particular, eqs. (6.28) and (6.29) are just stronger versions of eqs. (5.14) and (5.15), respectively. If $m_{1/2}$, A_0 and B_0 all have the same complex phase, then eq. (5.16) will also be satisfied.

Equations (6.27)-(6.30) also have the virtue of being highly predictive. [Of course, eq. (6.30) is content-free unless one can relate B_0 to the other parameters in some non-trivial way.] As discussed in section 5.4, they should be applied as RG boundary conditions at the scale $M_{\rm P}$. The RG evolution of the soft parameters down to the electroweak scale will then allow us to predict the entire MSSM spectrum in terms of just five parameters $m_{1/2}$, m_0^2, A_0, B_0, and μ (plus the already-measured gauge and Yukawa couplings of the MSSM). In practice, the approximation is usually made of starting this RG running from the unification scale $M_U \approx 2\times 10^{16}$ GeV instead of $M_{\rm P}$. The reason for this is that the apparent unification of gauge couplings gives us a strong hint that we know something about how the RG equations are behaving up to M_U, but gives us little guidance about what to expect at scales between M_U and $M_{\rm P}$. The error made in neglecting these effects is proportional to a loop suppression factor times $\ln(M_{\rm P}/M_U)$ and can be partially absorbed into a redefinition of m_0^2, $m_{1/2}$, A_0 and B_0, but in some cases can lead to important effects.[74] The framework described in the above few paragraphs has been the subject of the bulk of phenomenological studies of supersymmetry. It is sometimes referred to as the *minimal supergravity* or *supergravity-inspired* scenario for the soft terms. A few examples of the many useful numerical RG studies of the MSSM spectrum which have been performed in this framework can be found in Ref.[75]

Particular models of gravity-mediated supersymmetry breaking can be even more predictive, relating some of the parameters $m_{1/2}$, m_0^2, A_0 and B_0 to each other and to the mass of the gravitino $m_{3/2}$. For example, three popular kinds of models for the soft terms are:

- Dilaton-dominated: [76] $m_0^2 = m_{3/2}^2$; $m_{1/2} = -A_0 = \sqrt{3}m_{3/2}$.
- Polonyi: [77] $m_0^2 = m_{3/2}^2$; $A_0 = (3-\sqrt{3})m_{3/2}$; $m_{1/2} = \mathcal{O}(m_{3/2})$.
- "No-scale": [78] $m_{1/2} \gg m_0, A_0, m_{3/2}$.

The dilaton-dominated scenario arises in a particular limit of superstring theory. While it appears to be highly predictive, it can easily be generalized in other limits.[79] The Polonyi model has the advantage of being the simplest possible model for supersymmetry breaking in the hidden sector, but it is rather *ad hoc* and does not seem to have a special place in grander schemes like superstrings. The "no-scale" limit may arise in a low-energy limit of superstrings in which the gravitino mass scale is undetermined at tree-level (hence the name). It implies that only the gaugino masses are appreciable at $M_{\rm P}$. As we will see in section 7.1, RG evolution feeds $m_{1/2}$ into the squark, slepton and Higgs (mass)^2 parameters with sufficient magnitude to give

[¶] The familiar flavor-blindness of gravitational interactions expressed in Einstein's equivalence principle does not, by itself, tell us anything about the form of eq. (6.25).

acceptable phenomenology at the electroweak scale. More recent versions of the no-scale scenario, however, also can give significant A_0 and m_0^2 at $M_{\rm P}$. In many cases B_0 can also be predicted in terms of the other parameters, but this is quite sensitive to model assumptions. For phenomenological studies, $m_{1/2}$, m_0^2, A_0 and B_0 are usually just taken to be convenient independent parameters of our ignorance of the supersymmetry breaking mechanism.

6.4 Gauge-mediated supersymmetry breaking models

A strong alternative to the scenario described in the previous section is provided by the gauge-mediated supersymmetry breaking proposal.[80,81] The basic idea is to introduce some new chiral supermultiplets, called messengers, which couple to the ultimate source of supersymmetry breaking, and which also couple indirectly to the (s)quarks and (s)leptons and Higgs(inos) of the MSSM through the ordinary $SU(3)_C \times SU(2)_L \times U(1)_Y$ gauge boson and gaugino interactions. In this way, the ordinary gauge interactions, rather than gravity, are responsible for the appearance of soft terms in the MSSM. There is still gravitational communication between the MSSM and the source of supersymmetry breaking, of course, but that effect is now relatively unimportant compared to the gauge interaction effects.

In the simplest such model, the messenger fields are a set of chiral supermultiplets q, $\overline{q}$, ℓ, $\overline{\ell}$ which transform under $SU(3)_C \times SU(2)_L \times U(1)_Y$ as

$$q \sim (\mathbf{3}, \mathbf{1}, -\frac{1}{3}); \qquad \overline{q} \sim (\overline{\mathbf{3}}, \mathbf{1}, \frac{1}{3}); \qquad \ell \sim (\mathbf{1}, \mathbf{2}, \frac{1}{2}); \qquad \overline{\ell} \sim (\mathbf{1}, \mathbf{2}, -\frac{1}{2}). \tag{6.31}$$

These supermultiplets contain messenger quarks $\psi_q, \psi_{\overline{q}}$ and scalar quarks $q, \overline{q}$ and messenger leptons $\psi_\ell, \psi_{\overline{\ell}}$ and scalar leptons $\ell, \overline{\ell}$. All of these particles must get very large masses so as not to have been discovered already. They manage to do so by coupling to a gauge-singlet chiral supermultiplet S through a superpotential:

$$W_{\rm mess} = y_2 S \ell \overline{\ell} + y_3 S q \overline{q}. \tag{6.32}$$

The scalar component of S and its auxiliary (F-term) component are each supposed to acquire VEVs, denoted $\langle S \rangle$ and $\langle F_S \rangle$ respectively. This can be accomplished either by putting S into an O'Raifeartaigh-type model,[80] or by a dynamical mechanism.[81] Exactly how this happens is a very interesting and important question. Here, we will simply parameterize our ignorance of the precise mechanism of supersymmetry breaking by asserting that S participates in another part of the superpotential, call it $W_{\rm breaking}$, which provides for supersymmetry breakdown.

Let us now consider the mass spectrum of the messenger fermions and bosons. The messenger part of the superpotential now effectively becomes $W_{\rm mess} = y_2 \langle S \rangle \ell \overline{\ell} + y_3 \langle S \rangle q \overline{q}$. So, the fermionic messenger fields pair up to get mass terms:

$$\mathcal{L} \;=\; -(y_2 \langle S \rangle \psi_\ell \psi_{\overline{\ell}} + y_3 \langle S \rangle \psi_q \psi_{\overline{q}} + \text{c.c.}) \tag{6.33}$$

as in eq. (3.48). Meanwhile, their scalar messenger partners $\ell, \overline{\ell}$ and $q\overline{q}$ have a scalar potential given by (neglecting D-term contributions, which do not affect the following

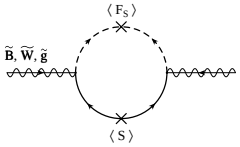

Figure 16: Contributions to the MSSM gaugino masses in gauge-mediated supersymmetry breaking models arise from one-loop graphs involving virtual messenger particles.

discussion):

$$V \;=\; \left|\frac{\delta W_{\rm mess}}{\delta \ell}\right|^2 + \left|\frac{\delta W_{\rm mess}}{\delta \overline{\ell}}\right|^2 + \left|\frac{\delta W_{\rm mess}}{\delta q}\right|^2 + \left|\frac{\delta W_{\rm mess}}{\delta \overline{q}}\right|^2 + \left|\frac{\delta W_{\rm mess}}{\delta S} + \frac{\delta W_{\rm breaking}}{\delta S}\right|^2 \tag{6.34}$$

as in eq. (3.47). Now, using the supposition that

$$\langle \delta W_{\rm breaking}/\delta S \rangle = -\langle F_S^* \rangle \tag{6.35}$$

(with $\langle \delta W_{\rm mess}/\delta S \rangle = 0$), and replacing S and F_S by their VEVs, one finds quadratic mass terms in the potential for the messenger scalar leptons:

$$\begin{aligned} V \;=\; & |y_2 \langle S \rangle|^2 \left(|\ell|^2 + |\overline{\ell}|^2\right) + |y_3 \langle S \rangle|^2 \left(|q|^2 + |\overline{q}|^2\right) \\ & - \left(y_2 \langle F_S \rangle \ell \overline{\ell} + y_3 \langle F_S \rangle q \overline{q} + \text{c.c.}\right) \\ & + \text{quartic terms}. \end{aligned} \tag{6.36}$$

The first line in eq. (6.36) represents supersymmetric mass terms that go along with eq. (6.33), while the second line consists of soft supersymmetry-breaking masses. The complex scalar messengers $\ell, \overline{\ell}$ thus obtain a $(\text{mass})^2$ matrix equal to:

$$\begin{pmatrix} |y_2 \langle S \rangle|^2 & -y_2^* \langle F_S^* \rangle \\ -y_2 \langle F_S \rangle & |y_2 \langle S \rangle|^2 \end{pmatrix} \tag{6.37}$$

with squared mass eigenvalues $|y_2 \langle S \rangle|^2 \pm |y_2 \langle F_S \rangle|$. In just the same way, the scalars $q, \overline{q}$ get squared masses $|y_3 \langle S \rangle|^2 \pm |y_3 \langle F_S \rangle|$.

So far, we have found that the effect of supersymmetry breaking is to split each messenger supermultiplet pair apart:

$$\ell, \overline{\ell}: \qquad m^2_{\rm fermions} = |y_2 \langle S \rangle|^2, \qquad m^2_{\rm scalars} = |y_2 \langle S \rangle|^2 \pm |y_2 \langle F_S \rangle|; \tag{6.38}$$

$$q, \overline{q}: \qquad m^2_{\rm fermions} = |y_3 \langle S \rangle|^2, \qquad m^2_{\rm scalars} = |y_3 \langle S \rangle|^2 \pm |y_3 \langle F_S \rangle|. \tag{6.39}$$

The supersymmetry violation apparent in this messenger spectrum for $\langle F_S \rangle \neq 0$ is communicated to the MSSM sparticles through radiative quantum corrections. The MSSM gauginos obtain masses from the 1-loop graph shown in Fig. 16. The scalar and fermion lines in the loop are messenger fields. Recall that the interaction vertices in Fig. 16 are of gauge coupling strength even though they do not involve gauge

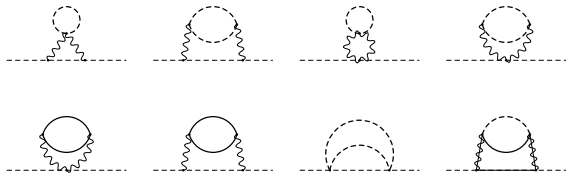

Figure 17: Contributions to MSSM scalar squared masses in gauge-mediated supersymmetry breaking models arise in leading order from these two-loop Feynman graphs.

bosons; compare Fig. 5g. In this way, gauge-mediation provides that $q, \overline{q}$ messenger loops give masses to the gluino and the bino, and $\ell, \overline{\ell}$ messenger loops give masses to the wino and bino fields. By computing the 1-loop diagrams one finds[81] that the resulting MSSM gaugino masses are given by

$$M_a = \frac{\alpha_a}{4\pi}\Lambda, \qquad (a = 1, 2, 3), \tag{6.40}$$

(in the normalization discussed in section 5.4) where we have introduced a mass parameter

$$\Lambda \equiv \langle F_S \rangle / \langle S \rangle \,. \tag{6.41}$$

(Note that if $\langle F_S \rangle$ were 0, then $\Lambda = 0$ and the messenger scalars would be degenerate with their fermionic superpartners and there would be no contribution to the MSSM gaugino masses.) In contrast, the corresponding MSSM gauge bosons cannot get a corresponding mass shift, since they are protected by gauge invariance. So supersymmetry breaking has been successfully communicated to the MSSM ("visible sector"). To a good approximation, eq. (6.40) holds for the running gaugino masses at an RG scale Q_0 corresponding to the average characteristic mass of the heavy messenger particles, roughly of order $M_{\rm mess} \sim y_i \langle S \rangle$. The running mass parameters can then be RG-evolved down to the electroweak scale to predict the physical masses to be measured by future experiments.

The scalars of the MSSM do not get any radiative corrections to their masses at one-loop order. The leading contribution to their masses comes from the two-loop graphs shown in Fig. 17, with the messenger fermions (heavy solid lines) and messenger scalars (heavy dashed lines) and ordinary gauge bosons and gauginos running around the loops. By computing these graphs, one finds that each MSSM scalar ϕ gets a (mass)^2 given by:

$$m_\phi^2 = 2\Lambda^2 \left[\left(\frac{\alpha_3}{4\pi}\right)^2 C_3^\phi + \left(\frac{\alpha_2}{4\pi}\right)^2 C_2^\phi + \left(\frac{\alpha_1}{4\pi}\right)^2 C_1^\phi \right]. \tag{6.42}$$

Here C_a^ϕ are the quadratic Casimir group theory invariants for the scalar ϕ for each gauge group. They are defined by $C_a^\phi \delta_i^j = (T^a T^a)_i{}^j$ where the T^a are the group

generators which act on the scalar ϕ. Explicitly, they are:

$$C_3^\phi = \begin{cases} 4/3 & \text{for } \phi = \widetilde{Q}_i, \widetilde{\overline{u}}_i, \widetilde{\overline{d}}_i; \\ 0 & \text{for } \phi = \widetilde{L}_i, \widetilde{\overline{e}}_i, H_u, H_d \end{cases} \tag{6.43}$$

$$C_2^\phi = \begin{cases} 3/4 & \text{for } \phi = \widetilde{Q}_i, \widetilde{L}_i, H_u, H_d; \\ 0 & \text{for } \phi = \widetilde{\overline{u}}_i, \widetilde{\overline{d}}_i, \widetilde{\overline{e}}_i \end{cases} \tag{6.44}$$

$$C_1^\phi = 3Y_\phi^2/5 \quad \text{for each } \phi \text{ with weak hypercharge } Y_\phi. \tag{6.45}$$

The squared masses in eq. (6.42) are positive (fortunately!).

The terms $\mathbf{a_u}$, $\mathbf{a_d}$, $\mathbf{a_e}$ arise first at two-loop order, and are suppressed by an extra factor of $\alpha_a/(4\pi)$ compared to the gaugino masses. So, to a very good approximation one has, at the messenger scale,

$$\mathbf{a_u} = \mathbf{a_d} = \mathbf{a_e} = 0, \tag{6.46}$$

a significantly stronger condition than eq. (5.15). Again, eqs. (6.42) and (6.46) should be applied at an RG scale equal to the average mass of the messenger fields running in the loops. However, after evolving the RG equations down to the electroweak scale, non-zero $\mathbf{a_u}$, $\mathbf{a_d}$ and $\mathbf{a_e}$ are generated proportional to the corresponding Yukawa matrices and the non-zero gaugino masses, as we will see in section 7.1. These will only be large for the third family squarks and sleptons, in the approximation of eq. (5.2). The parameter b may also be taken to vanish near the messenger scale, but this is quite model-dependent, and in any case b will be non-zero when it is RG-evolved to the electroweak scale. In practice, b is determined by the requirement of correct electroweak symmetry breaking, as discussed below in section 7.2.

Because the gaugino masses arise at *one*-loop order and the scalar (mass)^2 contributions appear at *two*-loop order, both eq. (6.40) and (6.42) correspond to the estimate eq. (6.15) for $m_{\rm soft}$, with $M_{\rm mess} \sim y_i \langle S \rangle$. Equations (6.40) and (6.42) hold in the limit of small $\langle F_S \rangle / y_i \langle S \rangle^2$, corresponding to mass splittings within each messenger supermultiplet that are small compared to the overall messenger mass scale. The subleading corrections in an expansion in $\langle F_S \rangle / y_i \langle S \rangle^2$ turn out[82] to be quite small unless there are very large hierarchies in the messenger sector.

The model we have described so far is often called the minimal model of gauge-mediated supersymmetry breaking. Let us now generalize it to a more complicated messenger sector. Suppose that $q, \overline{q}$ and $\ell, \overline{\ell}$ are replaced by a collection of messengers $\Phi_i, \overline{\Phi}_i$ with a superpotential

$$W_{\rm mess} = \sum_i y_i S \Phi_i \overline{\Phi}_i. \tag{6.47}$$

The bar means that the chiral superfields $\overline{\Phi}_i$ transform as the complex conjugate representations of the Φ_i chiral superfields. Together they are said to form a "vector-like" (real) representation of the Standard Model gauge group. As before, the fermionic

components of each pair Φ_i and $\overline{\Phi}_i$ pair up to get squared masses $y_i\langle S\rangle$ and their scalar partners mix to get squared masses $|y_i\langle S\rangle|^2 \pm |y_i\langle F_S\rangle|$. The MSSM gaugino mass parameters induced are now

$$M_a = \frac{\alpha_a}{4\pi}\Lambda \sum_i n_a(i) \qquad (a = 1, 2, 3) \tag{6.48}$$

where $n_a(i)$ is the Dynkin index for each $\Phi_i + \overline{\Phi}_i$, in a normalization where $n_3 = 1$ for a $\mathbf{3} + \overline{\mathbf{3}}$ of $SU(3)_C$ and $n_2 = 1$ for a pair of doublets of $SU(2)_L$. For $U(1)_Y$, one has $n_1 = 6Y^2/5$ for each messenger pair with weak hypercharges $\pm Y$. In computing n_1 one must remember to add up the contributions for each component of an $SU(3)_C$ or $SU(2)_L$ multiplet. So, for example, $(n_1, n_2, n_3) = (2/5, 0, 1)$ for $q+\overline{q}$ and $(n_1, n_2, n_3) = (3/5, 1, 0)$ for $\ell + \overline{\ell}$. Thus the total is $\sum_i (n_1, n_2, n_3) = (1, 1, 1)$ for the minimal model, so that eq. (6.48) is in agreement with eq. (6.40). On general group-theoretic grounds, n_2 and n_3 must be integers, and n_1 is always an integer multiple of $1/5$ if fractional electric charges are confined.

The MSSM scalar masses in this generalized gauge-mediation framework are now:

$$m_\phi^2 = 2\Lambda^2 \left[\left(\frac{\alpha_3}{4\pi}\right)^2 C_3^\phi \sum_i n_3(i) + \left(\frac{\alpha_2}{4\pi}\right)^2 C_2^\phi \sum_i n_2(i) + \left(\frac{\alpha_1}{4\pi}\right)^2 C_1^\phi \sum_i n_1(i) \right]. \tag{6.49}$$

In writing eqs. (6.48) and (6.49) as simple sums, we have implicitly assumed that the messengers are all approximately equal in mass, with

$$M_{\rm mess} \approx y_i\langle S\rangle. \tag{6.50}$$

This is a good approximation if the y_i are not too different from each other, because the dependence of the MSSM mass spectrum on the y_i is only logarithmic (due to RG running) for fixed Λ. However, if large hierarchies in the messenger masses are present, then the additive contributions to the gaugino and scalar masses from each individual messenger multiplet i should really instead be incorporated at the mass scale of that messenger multiplet. Then RG evolution is used to run these various contributions down to the electroweak or TeV scale; the individual messenger contributions to scalar and gaugino masses as indicated above can be thought of as threshold corrections to this RG running.

Messengers with masses far below the GUT scale will affect the running of gauge couplings and might therefore be expected to ruin the apparent unification shown in Fig. 13. However, if the messengers come in complete multiplets of the $SU(5)$ global symmetry[‖] that contains the Standard Model gauge group and are not very different in mass, then approximate unification of gauge couplings will still occur when they are extrapolated up to the same scale M_U (but with a larger unified value for the

[‖] This $SU(5)$ symmetry may or may not be promoted to a local gauge symmetry at the GUT scale. For our present purposes, it is used simply as a classification scheme, since the global $SU(5)$ symmetry is only approximate below the GUT scale at the messenger mass scale where gauge mediation takes place.

gauge couplings at that scale). For this reason, a popular class of models is obtained by taking the messengers to consist of N_5 copies of the $\mathbf{5} + \overline{\mathbf{5}}$ of $SU(5)$, resulting in

$$N_5 = \sum_i n_1(i) = \sum_i n_2(i) = \sum_i n_3(i)\ . \tag{6.51}$$

In terms of this integer parameter N_5, eqs. (6.48) and (6.49) reduce to

$$M_a = \frac{\alpha_a}{4\pi}\Lambda N_5 \tag{6.52}$$

$$m_\phi^2 = 2\Lambda^2 N_5 \sum_{a=1}^{3} C_a^\phi \left(\frac{\alpha_a}{4\pi}\right)^2, \tag{6.53}$$

since now there are N_5 copies of the minimal messenger sector particles running around the loops. For example, the minimal model in eq. (6.31) corresponds to $N_5 = 1$. A single copy of $\mathbf{10} + \overline{\mathbf{10}}$ of $SU(5)$ has Dynkin indices $\sum_i n_a(i) = 3$, and so can be substituted for 3 copies of $\mathbf{5} + \overline{\mathbf{5}}$. (Other combinations of messenger multiplets can also preserve the apparent unification of gauge couplings.) Note that the gaugino masses scale like N_5, while the scalar masses scale like $\sqrt{N_5}$. This means that sleptons and squarks will tend to be relatively lighter for larger values of N_5 in non-minimal models. However, if N_5 is too large, then the running gauge couplings will diverge before they can unify at M_U. For messenger masses of order 10^6 GeV or less, for example, one needs $N_5 \leq 4$.

There are many other possible generalizations of the basic gauge-mediation scenario as described above. An important general expectation in these models is that the strongly-interacting sparticles (squarks, gluino) should be heavier than weakly-interacting sparticles (sleptons, bino, winos, higgsinos) simply because of the hierarchy of gauge couplings $\alpha_3 > \alpha_2 > \alpha_1$. The common feature which makes all of these models very attractive is that the masses of the squarks and sleptons depend only on their gauge quantum numbers, leading automatically to the degeneracy of squark and slepton masses needed for suppression of FCNC effects. But the most distinctive phenomenological prediction of gauge-mediated models may be the fact that the gravitino is the LSP. This can have crucial consequences for both cosmology and collider physics, as we will discuss further in sections 8.5 and 9.

7 The mass spectrum of the MSSM

In this section, we will study the sparticle and Higgs mass spectrum of the MSSM. We will pay special attention to the general classes of models which fit into the minimal supergravity eqs. (6.27)-(6.29) or gauge-mediated eqs. (6.40)-(6.46) boundary conditions for the soft terms. As we have already discussed in section 5.4, the renormalization group (RG) equations are a crucial tool in determining the lagrangian at the electroweak scale, given a set of boundary conditions on the theory at the (much higher) input scale. Therefore, we will begin by looking at the RG equations for the parameters of the model, in section 7.1. Of course, the boundary conditions on soft parameters are quite model-dependent even within the minimal supergravity

and gauge-mediated frameworks, but there are some important general lessons to be learned from the form of the RG equations. Once the RG equations have been used to determine the effective lagrangian at the electroweak scale, one can use the results of the earlier sections to predict the mass spectrum, mixing angles, and interactions of all of the new particles in the model. In section 7.2 we will discuss electroweak symmetry breaking and the Higgs scalars. Sections 7.3, 7.4, 7.5 are devoted to the sparticle masses and mixings. Finally in section 7.6 we will summarize some of the general features and expectations for the MSSM spectrum.

7.1 Renormalization Group Equations

In order to translate a set of predictions at the input scale into physically meaningful quantities which describe physics at the electroweak scale, it is necessary to evolve the gauge couplings, superpotential parameters, and soft terms using the RG equations. As a technical aside, we note that when computing RG effects and other radiative corrections in supersymmetry, it is important to choose regularization and renormalization schemes that do not violate supersymmetry. The most popular regularization method for discussing radiative corrections within the Standard Model is dimensional regularization (DREG), in which the number of spacetime dimensions is continued to $d = 4 - 2\epsilon$. Unfortunately, DREG violates supersymmetry explicitly because it introduces a mismatch between the numbers of gauge boson degrees of freedom and the gaugino degrees of freedom off-shell. This mismatch is only 2ϵ, but can be multiplied by factors up to $1/\epsilon^n$ in an n-loop calculation. In DREG, supersymmetric relations between dimensionless coupling constants ("supersymmetric Ward identities") are therefore disrespected by radiative corrections involving the finite parts of one-loop graphs and by the divergent parts of two-loop graphs. Instead, one may use the slightly different scheme known as regularization by dimensional reduction, or DRED, which does respect supersymmetry.[83] In the DRED method, all momentum integrals are still performed in $d = 4 - 2\epsilon$ dimensions, but the vector index μ on the gauge boson fields A^a_μ now runs over all 4 dimensions. Running couplings are then renormalized using DRED with modified minimal subtraction ($\overline{\rm DR}$) rather than the usual DREG with modified minimal subtraction ($\overline{\rm MS}$). In particular, the boundary conditions at the input scale should be applied in the supersymmetry-preserving $\overline{\rm DR}$ scheme. (See Ref.[84] for an alternative supersymmetric scheme.) One loop β-functions are always the same in the two schemes, but it is important to realize that the $\overline{\rm MS}$ scheme does violate supersymmetry, so that $\overline{\rm DR}$ is preferred[†] from that point of view. (It is also possible to work consistently within the $\overline{\rm MS}$ scheme, as long as one is careful to correctly translate all $\overline{\rm DR}$ couplings and masses into their $\overline{\rm MS}$ counterparts.[88,89])

The MSSM RG equations in the $\overline{\rm DR}$ scheme are given in Refs.[90–93]; they are now known for the gauge couplings and superpotential parameters up to 3-loop order, and for the soft parameters at 2-loop order. However, for many purposes including pedagogical ones it suffices to work in the 1-loop approximation. Here, we will also use the approximation that only the third family Yukawa couplings are significant; see eq. (5.2). Then the superpotential parameters run with scale according to:

$$\frac{d}{dt}y_t = \frac{y_t}{16\pi^2}\Big[6|y_t|^2 + |y_b|^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\Big]; \tag{7.1}$$

$$\frac{d}{dt}y_b = \frac{y_b}{16\pi^2}\Big[6|y_b|^2 + |y_t|^2 + |y_\tau|^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\Big]; \tag{7.2}$$

$$\frac{d}{dt}y_\tau = \frac{y_\tau}{16\pi^2}\Big[4|y_\tau|^2 + 3|y_b|^2 - 3g_2^2 - \frac{9}{5}g_1^2\Big]; \tag{7.3}$$

$$\frac{d}{dt}\mu = \frac{\mu}{16\pi^2}\Big[3|y_t|^2 + 3|y_b|^2 + |y_\tau|^2 - 3g_2^2 - \frac{3}{5}g_1^2\Big]. \tag{7.4}$$

The one-loop RG equations for the gauge couplings g_1, g_2, g_3 have already been listed in eq. (5.17). Note that the β-functions (the quantities on the right side of each equation) for each supersymmetric parameter are proportional to the parameter itself. This is actually a consequence of a general and powerful result known as the *supersymmetric nonrenormalization theorem*.[94] This theorem implies that the logarithmically divergent contributions to a given process can always be written in the form of a wave-function renormalization, without any vertex renormalization.[‡] It is true for any supersymmetric theory, not just the MSSM, and holds to all orders in perturbation theory. It can be proved most easily using superfield techniques. In particular, it means that once we have a theory which can explain why μ is of order 10^2 or 10^3 GeV at tree-level, we do not have to worry about μ being infected (made very large) by radiative corrections involving the masses of some very heavy unknown particles; all such RG corrections to μ will be directly proportional to μ itself.

The one-loop RG equations for the three gaugino mass parameters in the MSSM are determined by the same quantities $b_a^{\rm MSSM}$ which appear in the gauge coupling RG eqs. (5.17):

$$\frac{d}{dt}M_a = \frac{1}{8\pi^2}b_a g_a^2 M_a \qquad (b_a = 33/5, 1, -3) \tag{7.5}$$

for $a = 1, 2, 3$. It is therefore easy to show that the three ratios M_a/g_a^2 are each constant (RG-scale independent) up to small two-loop corrections. In minimal supergravity models, we can therefore write

$$M_a(Q) = \frac{g_a^2(Q)}{g_a^2(Q_0)}m_{1/2} \qquad (a = 1, 2, 3) \tag{7.6}$$

at any RG scale $Q < Q_0$, where Q_0 is the input scale which is presumably nearly equal to M_P. Since the gauge couplings are observed to unify at $M_U \sim 0.01 M_P$, one

[†] Even the DRED scheme may not provide a supersymmetric regulator, because of ambiguities which appear at five-loop order at the latest.[85] Fortunately, this does not seem to cause any practical difficulties.[86] See also Ref.[87] for a promising proposal which avoids doing violence to the number of spacetime dimensions.

[‡] Actually, there *is* vertex renormalization in the field theory in which auxiliary fields have been integrated out, but the sum of divergent contributions for a given process always has the form of wave-function renormalization. See Ref.[23] for a discussion of this point.

expects[§] that $g_1^2(Q_0) \approx g_2^2(Q_0) \approx g_3^2(Q_0)$. Therefore, one finds that

$$\frac{M_1}{g_1^2} = \frac{M_2}{g_2^2} = \frac{M_3}{g_3^2} \tag{7.7}$$

at any RG scale, up to small two-loop effects and possibly larger threshold effects near M_U and M_P. The common value in eq. (7.7) is also equal to $m_{1/2}/g_U^2$ in minimal supergravity models, where g_U is the unified gauge coupling at the input scale where $m_{1/2}$ is the common gaugino mass. Interestingly, eq. (7.7) is *also* the solution to the one-loop RG equations in the case of the gauge-mediated boundary conditions eq. (6.40) applied at the messenger mass scale. This is true even though there is no such thing as a unified gaugino mass $m_{1/2}$ in the gauge-mediated case, because of the fact that the gaugino masses are proportional to the g_a^2 times a constant. So eq. (7.7) is theoretically well-motivated (but certainly not inevitable) in both frameworks. The prediction eq. (7.7) is particularly useful since the gauge couplings g_1^2, g_2^2, and g_3^2 are already quite well known at the electroweak scale from experiment. Therefore they can be extrapolated up to at least M_U, assuming that the apparent unification of gauge couplings is not a fake. The gaugino mass parameters feed into the RG equations for all of the other soft terms, as we will see.

Next we consider the 1-loop RG equations for the analytic soft parameters $\mathbf{a_u}$, $\mathbf{a_d}$, $\mathbf{a_e}$. In models obeying eq. (5.15), these matrices start off proportional to the corresponding Yukawa couplings at the input scale, and the RG evolution respects this property. With the approximation of eq. (5.2), one can therefore also write, at any RG scale,

$$\mathbf{a_u} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a_t \end{pmatrix}; \qquad \mathbf{a_d} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a_b \end{pmatrix}; \qquad \mathbf{a_e} \approx \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & a_\tau \end{pmatrix}, \tag{7.8}$$

which defines[¶] running parameters a_t, a_b, and a_τ. The RG equations for these parameters and b are given by

$$\begin{aligned} 16\pi^2 \frac{d}{dt} a_t &= a_t\Big[18|y_t|^2 + |y_b|^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{13}{15}g_1^2\Big] \\ &\quad + 2a_b y_b^* y_t + y_t\Big[\frac{32}{3}g_3^2 M_3 + 6g_2^2 M_2 + \frac{26}{15}g_1^2 M_1\Big]; \qquad (7.9) \\ 16\pi^2 \frac{d}{dt} a_b &= a_b\Big[18|y_b|^2 + |y_t|^2 + |y_\tau|^2 - \frac{16}{3}g_3^2 - 3g_2^2 - \frac{7}{15}g_1^2\Big] \\ &\quad + 2a_t y_t^* y_b + 2a_\tau y_\tau^* y_b + y_b\Big[\frac{32}{3}g_3^2 M_3 + 6g_2^2 M_2 + \frac{14}{15}g_1^2 M_1\Big]; \qquad (7.10) \\ 16\pi^2 \frac{d}{dt} a_\tau &= a_\tau\Big[12|y_\tau|^2 + 3|y_b|^2 - 3g_2^2 - \frac{9}{5}g_1^2\Big] \\ &\quad + 6a_b y_b^* y_\tau + y_\tau\Big[6g_2^2 M_2 + \frac{18}{5}g_1^2 M_1\Big]; \qquad (7.11) \\ 16\pi^2 \frac{d}{dt} b &= b\Big[3|y_t|^2 + 3|y_b|^2 + |y_\tau|^2 - 3g_2^2 - \frac{3}{5}g_1^2\Big] \\ &\quad + \mu\Big[6a_t y_t^* + 6a_b y_b^* + 2a_\tau y_\tau^* + 6g_2^2 M_2 + \frac{6}{5}g_1^2 M_1\Big] \qquad (7.12) \end{aligned}$$

in this approximation. The β-function for each of these soft parameters is *not* proportional to the parameter itself; this makes sense because couplings which violate supersymmetry are not protected by the supersymmetric nonrenormalization theorem. In particular, even if A_0 and B_0 appearing in eqs. (6.29) and (6.30) vanish at the input scale, the RG corrections proportional to gaugino masses appearing in eqs. (7.9)-(7.12) ensure that a_t, a_b, a_τ and b will still be non-zero at the electroweak scale.

Next let us consider the RG equations for the scalar masses in the MSSM. In the approximation of eqs. (5.2) and (7.8), the squarks and sleptons of the first two families have only gauge interactions. This means that if the scalar masses satisfy a boundary condition like eq. (5.14) at an input RG scale, then when renormalized to any other RG scale, they will still be almost diagonal, with the approximate form

$$\mathbf{m_Q^2} \approx \begin{pmatrix} m_{Q_1}^2 & 0 & 0 \\ 0 & m_{Q_1}^2 & 0 \\ 0 & 0 & m_{Q_3}^2 \end{pmatrix}; \qquad \mathbf{m_{\overline{u}}^2} \approx \begin{pmatrix} m_{\overline{u}_1}^2 & 0 & 0 \\ 0 & m_{\overline{u}_1}^2 & 0 \\ 0 & 0 & m_{\overline{u}_3}^2 \end{pmatrix}; \tag{7.13}$$

etc. The first and second family squarks and sleptons with given gauge quantum numbers remain very nearly degenerate, but the third family squarks and sleptons feel the effects of the larger Yukawa couplings and so get renormalized differently. The one-loop RG equations for the first and second family squark and slepton squared masses can be written as[‖]

$$16\pi^2 \frac{d}{dt} m_\phi^2 = -\sum_{a=1,2,3} 8 g_a^2 C_a^\phi |M_a|^2 \tag{7.14}$$

for each scalar ϕ, where the $\sum_a$ is over the three gauge groups $U(1)_Y$, $SU(2)_L$ and $SU(3)_C$; M_a are the corresponding running gaugino mass parameters which are known from eq. (7.7); and the constants C_a^ϕ are the same quadratic Casimir invariants which appeared in eqs. (6.43)-(6.45). An important feature of eq. (7.14) is that the right-hand sides are strictly negative, so that the scalar (mass)^2 parameters *grow* as they

[§] In a GUT model, it is automatic that the gauge couplings and gaugino masses are unified at all scales $Q > M_U$ and in particular at $Q \approx M_P$, because in the unified theory the gauginos all live in the same representation of the unified gauge group. In many superstring models, this is also known to be a good approximation.

[¶] We must warn the reader that rescaled soft parameters $A_t = a_t/y_t$, $A_b = a_b/y_b$, and $A_\tau = a_\tau/y_\tau$ are commonly used in the literature. We do not follow this notation, because it cannot be generalized beyond the approximation of eqs. (5.2), (7.8) without introducing horrible complications such as non-polynomial RG equations, and because a_t, a_b and a_τ are the couplings that actually appear in the lagrangian anyway.

[‖] There are also terms in the scalar (mass)^2 RG equations which are proportional to $\mathrm{Tr}[Ym^2]$ (the sum of the weak hypercharge times the soft (mass)^2 for all scalars in the theory). However, these contributions vanish in both the cases of minimal supergravity and gauge-mediated boundary conditions for the soft terms, as one can see by explicitly calculating $\mathrm{Tr}[Ym^2]$ in each case. If $\mathrm{Tr}[Ym^2]$ is zero at the input scale, then it will remain zero under RG evolution. Therefore we neglect such terms in our discussion, although they can have an important effect in more general situations.

are RG-evolved from the input scale down to the electroweak scale. Even if the scalars have zero or very small masses at the input scale, as in the "no-scale" boundary condition limit $m_0^2 = 0$, they will obtain large positive squared masses at the electroweak scale, thanks to the effects of the gaugino masses.

The RG equations for the (mass)^2 parameters of the Higgs scalars and third family squarks and sleptons get the same gauge contributions as in eq. (7.14), but they also have contributions due to the large Yukawa ($y_{t,b,\tau}$) and soft ($a_{t,b,\tau}$) couplings. At one-loop order, these only appear in three combinations:

$$X_t = 2|y_t|^2(m_{H_u}^2 + m_{Q_3}^2 + m_{\overline{u}_3}^2) + 2|a_t|^2, \tag{7.15}$$
$$X_b = 2|y_b|^2(m_{H_d}^2 + m_{Q_3}^2 + m_{\overline{d}_3}^2) + 2|a_b|^2, \tag{7.16}$$
$$X_\tau = 2|y_\tau|^2(m_{H_d}^2 + m_{L_3}^2 + m_{\overline{e}_3}^2) + 2|a_\tau|^2. \tag{7.17}$$

In terms of these quantities, the RG equations for the soft Higgs (mass)^2 parameters $m_{H_u}^2$ and $m_{H_d}^2$ are

$$16\pi^2 \frac{d}{dt} m_{H_u}^2 = 3X_t - 6g_2^2|M_2|^2 - \frac{6}{5}g_1^2|M_1|^2, \tag{7.18}$$
$$16\pi^2 \frac{d}{dt} m_{H_d}^2 = 3X_b + X_\tau - 6g_2^2|M_2|^2 - \frac{6}{5}g_1^2|M_1|^2. \tag{7.19}$$

Note that X_t, X_b, and X_τ are positive, so their effect is always to *decrease* the Higgs masses as one evolves the RG equations downward from the input scale to the electroweak scale. Since y_t is the largest of the Yukawa couplings because of the experimental fact that the top quark is heavy, X_t is typically expected to be larger than X_b and X_τ. This can cause the RG-evolved $m_{H_u}^2$ to run negative near the electroweak scale, helping to destabilize the point $H_u = 0$ and so provoking a Higgs VEV which is just what we want.** Thus a large top Yukawa coupling favors the breakdown of the electroweak symmetry breaking because it induces negative radiative corrections to the Higgs (mass)^2.

The third family squark and slepton (mass)^2 parameters also get contributions which depend on X_t, X_b and X_τ. Their RG equations are given by

$$16\pi^2 \frac{d}{dt} m_{Q_3}^2 = X_t + X_b - \frac{32}{3}g_3^2|M_3|^2 - 6g_2^2|M_2|^2 - \frac{2}{15}g_1^2|M_1|^2 \tag{7.20}$$
$$16\pi^2 \frac{d}{dt} m_{\overline{u}_3}^2 = 2X_t - \frac{32}{3}g_3^2|M_3|^2 - \frac{32}{15}g_1^2|M_1|^2 \tag{7.21}$$
$$16\pi^2 \frac{d}{dt} m_{\overline{d}_3}^2 = 2X_b - \frac{32}{3}g_3^2|M_3|^2 - \frac{8}{15}g_1^2|M_1|^2 \tag{7.22}$$
$$16\pi^2 \frac{d}{dt} m_{L_3}^2 = X_\tau - 6g_2^2|M_2|^2 - \frac{3}{5}g_1^2|M_1|^2 \tag{7.23}$$
$$16\pi^2 \frac{d}{dt} m_{\overline{e}_3}^2 = 2X_\tau - \frac{24}{5}g_1^2|M_1|^2. \tag{7.24}$$

**One should think of "$m_{H_u}^2$" as a parameter unto itself, and not as the square of some mythical real number m_{H_u}. Thus there is nothing strange about having $m_{H_u}^2 < 0$. However, strictly speaking $m_{H_u}^2 < 0$ is neither necessary nor sufficient for electroweak symmetry breaking; see section 7.2.

In eqs. (7.18)-(7.24), the terms proportional to $|M_3|^2$, $|M_2|^2$ and $|M_1|^2$ are just the same ones as in eq. (7.14). Note that the terms proportional to X_t appear with smaller numerical coefficients in the $m_{Q_3}^2$ and $m_{\overline{u}_3}^2$ RG equations than they did for the Higgs scalars, and they do not appear at all in the $m_{\overline{d}_3}^2$, $m_{L_3}^2$ and $m_{\overline{e}_3}^2$ RG equations. Furthermore, the third-family squark (mass)^2 get a large positive contribution proportional to $|M_3|^2$ from the RG evolution, which the Higgs scalars do not get. These facts make it easy to understand why the Higgs scalars in the MSSM can get VEVs, but the squarks and sleptons, having large positive (mass)^2, do not. An examination of the RG equations (7.9)-(7.12), (7.14), and (7.18)-(7.24) reveals that if the gaugino mass parameters M_1, M_2, and M_3 are non-zero at the input scale, then all of the other soft terms will be generated. This is why the "no-scale" limit with $m_{1/2} \gg m_0, A_0, B_0$ can be phenomenologically viable even though the squarks and sleptons are massless at tree-level. On the other hand, if the gaugino masses were to vanish at tree-level, then they would not get any contributions to their masses at one-loop order; in that case M_1, M_2, and M_3 would be extremely small.

Now that we have reviewed the effects of RG evolution from the input scale down to the electroweak or TeV scale, we are ready to work out the expected features of the MSSM spectrum in some detail. We will begin with the Higgs sector in the next section.

7.2 Electroweak symmetry breaking and the Higgs bosons

In the MSSM, the description of electroweak symmetry breaking is slightly complicated by the fact that there are two complex Higgs doublets $H_u = (H_u^+, H_u^0)$ and $H_d = (H_d^0, H_d^-)$ rather than just one in the ordinary Standard Model. The classical scalar potential for the Higgs scalar fields in the MSSM is given by

$$\begin{aligned} V &= (|\mu|^2 + m_{H_u}^2)(|H_u^0|^2 + |H_u^+|^2) + (|\mu|^2 + m_{H_d}^2)(|H_d^0|^2 + |H_d^-|^2) \\ &\quad + b\,(H_u^+ H_d^- - H_u^0 H_d^0) + \text{c.c.} \\ &\quad + \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2)^2 \\ &\quad + \frac{1}{2}g^2|H_u^+ H_d^{0*} + H_u^0 H_d^{-*}|^2. \end{aligned} \tag{7.25}$$

The terms proportional to $|\mu|^2$ come from F-terms [see the first term on the right-hand side of eq. (5.5)]. The terms proportional to $m_{H_u}^2$, $m_{H_d}^2$ and b are nothing but a rewriting of the last three terms of eq. (5.11). Finally, the terms proportional to g^2 and g'^2 are the D-term contributions which may be derived from the general formula eq. (3.75), after some rearranging. The full scalar potential of the theory will also include many terms involving the squark and slepton fields that we can ignore here, since they do not get VEVs because they have large positive (mass)^2.

We now have to demand that the minimum of this potential should break electroweak symmetry down to electromagnetism $SU(2)_L \times U(1)_Y \to U(1)_{\rm EM}$, in accord with experiment. We can use the freedom to make gauge transformations to simplify this analysis. First, the freedom to make $SU(2)_L$ gauge transformations allows us

to rotate away a possible VEV for one of the weak isospin components of one of the scalar fields; so without loss of generality we can take $H_u^+ = 0$ at the minimum of the potential. Then one finds that a minimum of the potential satisfying $\partial V/\partial H_u^+ = 0$ must also have $H_d^- = 0$. This is good, because it means that at the minimum of the potential electromagnetism is necessarily unbroken, since the charged components of the Higgs scalars cannot get VEVs. So after setting $H_u^+ = H_d^- = 0$ we are left to consider the scalar potential

$$\begin{aligned} V &= (|\mu|^2 + m_{H_u}^2)|H_u^0|^2 + (|\mu|^2 + m_{H_d}^2)|H_d^0|^2 - (b\, H_u^0 H_d^0 + \text{c.c.}) \\ &\quad + \frac{1}{8}(g^2 + g'^2)(|H_u^0|^2 - |H_d^0|^2)^2. \end{aligned} \tag{7.26}$$

The only term in this potential which depends on the phases of the fields is the b-term. Therefore a redefinition of the phases of H_u and H_d can absorb any phase in b, so we can take b to be real and positive. Then it is clear that a minimum of the potential V requires that $H_u^0 H_d^0$ is also real and positive, so $\langle H_u^0 \rangle$ and $\langle H_d^0 \rangle$ must have opposite phases. We can therefore use a $U(1)_Y$ gauge transformation to make them both be real and positive without loss of generality, since H_u and H_d have opposite weak hypercharges ($\pm 1/2$). It follows that CP cannot be spontaneously broken by the Higgs scalar potential, since all of the VEVs and couplings can be simultaneously chosen to be real. This means that the Higgs scalar mass eigenstates can be assigned well-defined eigenvalues of CP.

Note that the b-term always favors electroweak symmetry breaking. The combination of the b term and the terms $m_{H_u}^2$ and $m_{H_d}^2$ can allow for one linear combination of H_u^0 and H_d^0 to have a negative $(\text{mass})^2$ near $H_u^0 = H_d^0 = 0$. This requires that

$$b^2 > (|\mu|^2 + m_{H_u}^2)(|\mu|^2 + m_{H_d}^2). \tag{7.27}$$

If this inequality is not satisfied, then $H_u^0 = H_d^0 = 0$ will be a stable minimum of the potential, and electroweak symmetry breaking will not occur. A negative value for $|\mu|^2 + m_{H_u}^2$ will help eq. (7.27) to be satisfied, but it is not necessary. Furthermore, even if $m_{H_u}^2 < 0$, there may be no electroweak symmetry breaking if $|\mu|$ is too large or if b is too small. Still, the large negative contributions to $m_{H_u}^2$ from the RG equation (7.18) discussed in the previous section are an important factor in ensuring that electroweak symmetry breaking can occur in models with minimal supergravity or gauge-mediated boundary conditions for the soft terms.

In order for the MSSM scalar potential to be viable, it is not enough that the point $H_u^0 = H_d^0 = 0$ is destabilized by a negative $(\text{mass})^2$ direction; we must also make sure that the potential is bounded from below for arbitrarily large values of the scalar fields, so that V will really have a minimum. (Recall from the discussion in sections 3.2 and 3.4 that scalar potentials in purely supersymmetric theories are automatically positive and so clearly bounded from below. But, now that we have introduced supersymmetry breaking, we must be careful.) The scalar quartic interactions in V will stabilize the potential for almost all arbitrarily large values of H_u^0 and H_d^0. However, there are special directions in field space with $|H_u^0| = |H_d^0|$, along which the quartic contributions to V [the second line in eq. (7.26)] are identically zero. Such

directions in field space are called D-flat directions, because along them the part of the scalar potential coming from D-terms vanishes. In order for the potential to be bounded from below, we need the quadratic part of the scalar potential to be positive along the D-flat directions. This requirement amounts to

$$2b < 2|\mu|^2 + m_{H_u}^2 + m_{H_d}^2. \tag{7.28}$$

Interestingly, if $m_{H_u}^2 = m_{H_d}^2$, the constraints eqs. (7.27) and (7.28) cannot both be satisfied. In models derived from the minimal supergravity or gauge-mediated boundary conditions, $m_{H_u}^2 = m_{H_d}^2$ holds at tree-level at the input scale, but the X_t contribution to the RG equation for $m_{H_u}^2$ naturally pushes it to negative or small values $m_{H_u}^2 < m_{H_d}^2$ at the electroweak scale, as we saw in section 7.1. Unless this effect is large, the parameter space in which the electroweak symmetry is broken would be quite small. So in these models electroweak symmetry breaking is actually driven purely by quantum corrections; this mechanism is therefore known as *radiative electroweak symmetry breaking*. The realization that this works most naturally with a large top-quark Yukawa coupling provides additional motivation for these models.[90,95]

Having established the conditions necessary for H_u^0 and H_d^0 to get non-zero VEVs, we can now require that they are compatible with the observed phenomenology of electroweak symmetry breaking $SU(2)_L \times U(1)_Y \to U(1)_{\rm EM}$. Let us write $\langle H_u^0 \rangle = v_u$ and $\langle H_d^0 \rangle = v_d$ for the VEVs at the minimum of the potential. These VEVs can be connected to the known mass of the Z^0 boson and the electroweak gauge couplings:

$$v_u^2 + v_d^2 = v^2 = 2m_Z^2/(g^2 + g'^2) \approx (174\,\text{GeV})^2. \tag{7.29}$$

The ratio of the two VEVs is traditionally written as

$$\tan\beta \equiv v_u/v_d. \tag{7.30}$$

The value of $\tan\beta$ is not fixed by present experiments, but it depends on the lagrangian parameters of the MSSM in a calculable way. Since $v_u = v\sin\beta$ and $v_d = v\cos\beta$ were taken to be real and positive, we have $0 < \beta < \pi/2$, a requirement that will be sharpened below. Now one can write down the conditions $\partial V/\partial H_u^0 = \partial V/\partial H_d^0 = 0$ under which the potential eq. (7.26) will have a minimum satisfying eqs. (7.29) and (7.30):

$$|\mu|^2 + m_{H_d}^2 = b\tan\beta - (m_Z^2/2)\cos 2\beta; \tag{7.31}$$

$$|\mu|^2 + m_{H_u}^2 = b\cot\beta + (m_Z^2/2)\cos 2\beta. \tag{7.32}$$

It is easy to check that these equations indeed satisfy the necessary conditions eqs. (7.27) and (7.28). They allow us to eliminate two of the lagrangian parameters b and $|\mu|$ in favor of $\tan\beta$, but do not determine the phase of μ.

As an aside, we note that eqs. (7.31) and (7.32) highlight the "μ problem" already mentioned in section 5.1. If we view $|\mu|^2$, b, $m_{H_u}^2$ and $m_{H_d}^2$ as input parameters, and m_Z^2 and $\tan\beta$ as output parameters obtained by solving these two equations, then without miraculous cancellations we expect that all of the input parameters ought

to be within an order of magnitude or two of m_Z^2. However, in the MSSM, μ is a supersymmetry-respecting parameter appearing in the superpotential, while b, $m_{H_u}^2$, $m_{H_d}^2$ are supersymmetry-breaking parameters. This has lead to a widespread belief that the MSSM must be extended at very high energies to include a mechanism which relates the effective value of μ to the supersymmetry-breaking mechanism in some way; see section 10.2 and Refs.[41,42,43] for examples.

The Higgs scalar fields in the MSSM consist of two complex $SU(2)_L$-doublet, or eight real, scalar degrees of freedom. When the electroweak symmetry is broken, three of them are the would-be Nambu-Goldstone bosons G^0, $G^\pm$ which become the longitudinal modes of the Z^0 and $W^\pm$ massive vector bosons. The remaining five Higgs scalar mass eigenstates consist of one CP-odd neutral scalar A^0, a charge $+1$ scalar H^+ and its conjugate charge -1 scalar H^-, and two CP-even neutral scalars h^0 and H^0. In terms of the original gauge-eigenstate fields, the mass eigenstates and would-be Nambu-Goldstone bosons are given by

$$\begin{pmatrix} G^0 \\ A^0 \end{pmatrix} = \sqrt{2} \begin{pmatrix} \sin\beta & -\cos\beta \\ \cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} \mathrm{Im}[H_u^0] \\ \mathrm{Im}[H_d^0] \end{pmatrix}, \tag{7.33}$$

$$\begin{pmatrix} G^+ \\ H^+ \end{pmatrix} = \begin{pmatrix} \sin\beta & -\cos\beta \\ \cos\beta & \sin\beta \end{pmatrix} \begin{pmatrix} H_u^+ \\ H_d^{-*} \end{pmatrix}, \tag{7.34}$$

with $G^- = G^{+*}$ and $H^- = H^{+*}$, and

$$\begin{pmatrix} h^0 \\ H^0 \end{pmatrix} = \sqrt{2} \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} \mathrm{Re}[H_u^0] - v_u \\ \mathrm{Re}[H_d^0] - v_d \end{pmatrix}. \tag{7.35}$$

which defines a mixing angle α. The tree-level masses of these fields can be found by expanding the scalar potential around the minimum. One obtains

$$m_{A^0}^2 = 2b/\sin 2\beta \tag{7.36}$$

$$m_{H^\pm}^2 = m_{A^0}^2 + m_W^2 \tag{7.37}$$

$$m_{h^0,H^0}^2 = \frac{1}{2}\Big(m_{A^0}^2 + m_Z^2 \mp \sqrt{(m_{A^0}^2 + m_Z^2)^2 - 4m_Z^2 m_{A^0}^2 \cos^2 2\beta}\Big). \tag{7.38}$$

In terms of these masses, the mixing angle α appearing in eq. (7.35) is determined at tree-level by

$$\frac{\sin 2\alpha}{\sin 2\beta} = -\frac{m_{A^0}^2 + m_Z^2}{m_{H^0}^2 - m_{h^0}^2}; \qquad \frac{\cos 2\alpha}{\cos 2\beta} = -\frac{m_{A^0}^2 - m_Z^2}{m_{H^0}^2 - m_{h^0}^2}. \tag{7.39}$$

The Feynman rules for couplings of the mass eigenstate Higgs scalars to the Standard Model quarks and leptons and the electroweak vector bosons, as well as to the various sparticles, have been worked out in detail in Ref.[96,97]

The masses of A^0, H^0 and $H^\pm$ can in principle be arbitrarily large since they all grow with $b/\sin 2\beta$. In contrast, the mass of h^0 is bounded from above. It is not hard to show from eq. (7.38) that

$$m_{h^0} < |\cos 2\beta| m_Z \tag{7.40}$$

at tree-level.[98] If this inequality were robust, it would guarantee that the lightest Higgs boson of the MSSM would be kinematically accessible to LEP2, with large regions of parameter space already ruled out. However, the tree-level mass formulas given above for the Higgs mass eigenstates are subject to quite significant quantum corrections which are especially important to take into account in the case of h^0. The largest such contributions typically come from top-stop loop corrections to the terms in the scalar potential. In the limit of stop squark masses $m_{\tilde{t}_1}$, $m_{\tilde{t}_2}$ much greater than the top quark mass m_t, one finds a one-loop radiative correction to eq. (7.38):

$$\Delta(m_{h^0}^2) = \frac{3}{4\pi^2} v^2 y_t^4 \sin^4\beta \ \ln\left(\frac{m_{\tilde{t}_1} m_{\tilde{t}_2}}{m_t^2}\right). \tag{7.41}$$

Including this and other corrections,[99,100] one can obtain only a considerably weaker, but still very interesting, bound

$$m_{h^0} \lesssim 130\,\mathrm{GeV} \tag{7.42}$$

in the MSSM. This assumes that all of the sparticles that can contribute to $\Delta(m_{h^0}^2)$ in loops have masses that do not exceed 1 TeV. By adding extra supermultiplets to the MSSM, this bound can be made even weaker. However, assuming that none of the MSSM sparticles have masses exceeding 1 TeV and that all of the couplings in the theory remain perturbative up to the unification scale, one still finds [101]

$$m_{h^0} \lesssim 150\,\mathrm{GeV}. \tag{7.43}$$

This bound is also weakened if, for example, the top squarks are heavier than 1 TeV, but the upper bound rises only logarithmically with the soft masses, as can be seen from eq. (7.41). Thus it is a fairly robust prediction of supersymmetry at the electroweak scale that at least one of the Higgs scalar bosons must be light.

An interesting limit occurs when $m_{A^0} \gg m_Z$. In that case, m_{h^0} can saturate the upper bound just mentioned with $m_{h^0} \approx m_Z|\cos 2\beta|$ at tree-level, but subject to large positive quantum corrections. The particles A^0, H^0, and $H^\pm$ are much heavier and nearly degenerate, forming an isospin doublet which decouples from sufficiently low-energy experiments. The angle α is fixed to be approximately $\beta - \pi/2$. In this limit, h^0 has the same couplings to quarks and leptons and electroweak gauge bosons as would the physical Higgs boson of the ordinary Standard Model without supersymmetry. Indeed, model-building experiences have shown that it is quite common for h^0 to behave in a way nearly indistinguishable from a Standard Model-like Higgs boson, even if m_{A^0} is not too huge. On the other hand, it is important to keep in mind that the couplings of h^0 might turn out to deviate in important ways from those of a Standard Model Higgs boson. For a given set of model parameters, it is very important to take into account the complete set of one-loop corrections and even the dominant two-loop effects in a leading logarithm approximation in order to get accurate predictions for the Higgs masses and mixing angles.[99,100]

In the MSSM, the masses and CKM mixing angles of the quarks and leptons are determined by the Yukawa couplings of the superpotential and the parameter

$\tan\beta$. This is because the top, charm and up quarks get masses proportional to $v_u = v\sin\beta$ and the bottom, strange, and down quarks and the charge leptons get masses proportional to $v_d = v\cos\beta$. Therefore one finds at tree-level

$$y_t = \frac{g m_t}{\sqrt{2} m_W \sin\beta}; \qquad y_b = \frac{g m_b}{\sqrt{2} m_W \cos\beta}; \qquad y_\tau = \frac{g m_\tau}{\sqrt{2} m_W \cos\beta}. \tag{7.44}$$

These relations hold for the running masses of t, b, τ rather than the physical pole masses which are significantly larger.[102] Including those corrections, one can relate the Yukawa couplings to $\tan\beta$ and the known fermion masses and CKM mixing angles. It is now clear why we have not neglected y_b and y_τ, even though $m_b, m_\tau \ll m_t$. To a first approximation, $y_b/y_t = (m_b/m_t)\tan\beta$ and $y_\tau/y_t = (m_\tau/m_t)\tan\beta$, so that y_b and y_τ cannot be neglected if $\tan\beta$ is much larger than 1. In fact, there are good theoretical motivations for considering models with large $\tan\beta$. For example, models based on the GUT gauge group $SO(10)$ (or certain of its subgroups) can unify the running top, bottom and tau Yukawa couplings at the unification scale; this requires $\tan\beta$ to be very roughly of order m_t/m_b.[103,104]

Note that if one tries to make $\sin\beta$ too small, y_t will become nonperturbatively large. Requiring that y_t does not blow up above the electroweak scale, one finds that $\tan\beta \gtrsim 1.2$ or so, depending on the mass of the top quark, the QCD coupling, and other fine details. In principle, one can also determine a lower bound on $\cos\beta$ and thus an upper bound on $\tan\beta$ by requiring that y_b and y_τ are not nonperturbatively large. This gives a rough upper bound of $\tan\beta \lesssim 65$. However, this is complicated slightly by the fact that the bottom quark mass gets significant one-loop corrections in the large $\tan\beta$ limit.[104] One can obtain a slightly stronger upper bound on $\tan\beta$ in models where $m^2_{H_u} = m^2_{H_d}$ at the input scale, by requiring that y_b does not significantly exceed y_t. [Otherwise, X_b would be larger than X_t in eqs. (7.18) and (7.19), so one would find $m^2_{H_d} < m^2_{H_u}$ at the electroweak scale, and the minimum of the potential would have to be at $\langle H_d^0\rangle > \langle H_u^0\rangle$ which would be a contradiction with the supposition that $\tan\beta$ is large.] In the following, we will see that the parameter $\tan\beta$ has an important effect on the masses and mixings of the MSSM sparticles.

7.3 Neutralinos and charginos

The higgsinos and electroweak gauginos mix with each other because of the effects of electroweak symmetry breaking. The neutral higgsinos ($\widetilde{H}_u^0$ and $\widetilde{H}_d^0$) and the neutral gauginos ($\widetilde{B}$, $\widetilde{W}^0$) combine to form four neutral mass eigenstates called *neutralinos*. The charged higgsinos ($\widetilde{H}_u^+$ and $\widetilde{H}_d^-$) and winos ($\widetilde{W}^+$ and $\widetilde{W}^-$) mix to form two mass eigenstates with charge ± 1 called *charginos*. We will denote [††] the neutralino and chargino mass eigenstates by $\widetilde{N}_i$ $(i = 1,2,3,4)$ and $\widetilde{C}_i^\pm$ $(i = 1,2)$. By convention, these are labelled in ascending order, so that $m_{\widetilde{N}_1} < m_{\widetilde{N}_2} < m_{\widetilde{N}_3} < m_{\widetilde{N}_4}$ and $m_{\widetilde{C}_1} < m_{\widetilde{C}_2}$. The lightest neutralino, $\widetilde{N}_1$, is usually assumed to be the LSP, unless there is a lighter gravitino or unless R-parity is not conserved, because it is the only MSSM particle

[††] Other common notations use $\widetilde{\chi}_i^0$ or $\widetilde{Z}_i$ for neutralinos, and $\widetilde{\chi}_i^\pm$ or $\widetilde{W}_i^\pm$ for charginos.

which can make a good cold dark matter candidate. In this subsection, we will describe the mass spectrum and mixing of the neutralinos and charginos in the MSSM.

In the gauge-eigenstate basis $\psi^0 = (\widetilde{B}, \widetilde{W}^0, \widetilde{H}_d^0, \widetilde{H}_u^0)$, the neutralino mass terms in the lagrangian are

$$\mathcal{L} \supset -\frac{1}{2}(\psi^0)^T \mathbf{M}_{\widetilde{N}} \psi^0 + \text{c.c.} \tag{7.45}$$

where

$$\mathbf{M}_{\widetilde{N}} = \begin{pmatrix} M_1 & 0 & -c_\beta\, s_W\, m_Z & s_\beta\, s_W\, m_Z \\ 0 & M_2 & c_\beta\, c_W\, m_Z & -s_\beta\, c_W\, m_Z \\ -c_\beta\, s_W\, m_Z & c_\beta\, c_W\, m_Z & 0 & -\mu \\ s_\beta\, s_W\, m_Z & -s_\beta\, c_W\, m_Z & -\mu & 0 \end{pmatrix}. \tag{7.46}$$

Here we have introduced abbreviations $s_\beta = \sin\beta$, $c_\beta = \cos\beta$, $s_W = \sin\theta_W$, and $c_W = \cos\theta_W$. The entries M_1 and M_2 in this matrix come directly from the MSSM soft Lagrangian [see eq. (5.11)] while the entries $-\mu$ are the supersymmetric higgsino mass terms [see eq. (5.4)]. The terms proportional to m_Z are the result of Higgs-higgsino-gaugino couplings [see eq. (3.72) and Fig. 5g], with the Higgs scalars getting their VEVs [eqs. (7.29),(7.30)]. The mass matrix $\mathbf{M}_{\widetilde{N}}$ can be diagonalized by a unitary matrix $\mathbf{N}$ with $\widetilde{N}_i = \mathbf{N}_{ij}\psi_j^0$, so that

$$\mathbf{M}_{\widetilde{N}}^{\text{diag}} = \mathbf{N}^* \mathbf{M}_{\widetilde{N}} \mathbf{N}^{-1} \tag{7.47}$$

has positive real entries $m_{\widetilde{N}_1}$, $m_{\widetilde{N}_2}$, $m_{\widetilde{N}_3}$, $m_{\widetilde{N}_4}$ on the diagonal. These are the absolute values of the eigenvalues of $\mathbf{M}_{\widetilde{N}}$, or equivalently the square roots of the eigenvalues of $\mathbf{M}_{\widetilde{N}}^\dagger \mathbf{M}_{\widetilde{N}}$. The indices (i,j) on $\mathbf{N}_{ij}$ are (mass, gauge) eigenstate labels. The mass eigenvalues and the mixing matrix $\mathbf{N}_{ij}$ can be given in closed form in terms of the parameters M_1, M_2, μ and $\tan\beta$, but the results are very complicated and not very illuminating.

In general, the parameters M_1, M_2, and μ can have arbitrary complex phases. In the broad class of minimal supergravity or gauge-mediated models satisfying the gaugino unification conditions eq. (6.27) or (6.40), M_2 and M_1 will have the same complex phase which is preserved by RG evolution eq. (7.5). In that case, a redefinition of the phases of $\widetilde{B}$ and $\widetilde{W}$ allows us to make M_1 and M_2 both real and positive. The phase of μ is then really a physical parameter which cannot be rotated away. [We have already used up the freedom to redefine the phases of the Higgs fields, since we have picked b and $\langle H_u^0\rangle$ and $\langle H_d^0\rangle$ to be real and positive, to guarantee that the off-diagonal entries in eq. (7.46) proportional to m_Z are real.] However, if μ is not real, then there can be potentially disastrous CP-violating effects in low-energy physics, including electric dipole moments for both the electron and the neutron. Therefore, it is usual (although not mandatory because of the possibility of nontrivial cancellations) to assume that μ is real in the same set of phase conventions which make M_1, M_2, b, $\langle H_u^0\rangle$ and $\langle H_d^0\rangle$ real and positive. The sign of μ is still undetermined by this constraint.

In models which satisfy eq. (7.7), one has the nice prediction

$$M_1 \approx \frac{5}{3}\tan^2\theta_W\, M_2 \approx 0.5 M_2 \tag{7.48}$$

at the electroweak scale. If so, then the neutralino masses and mixing angles depend on only three unknown parameters. This assumption is sufficiently theoretically compelling that it has been made in almost all phenomenological studies; nevertheless it should be recognized as an assumption, to be tested someday by experiment.

Specializing further, there is an interesting and not unlikely limit in which electroweak symmetry breaking effects can be viewed as a small perturbation on the neutralino mass matrix. If

$$m_Z \ll |\mu \pm M_1|, |\mu \pm M_2| \tag{7.49}$$

then the neutralino mass eigenstates are very nearly $\widetilde{N}_1 \approx \tilde{B}$; $\widetilde{N}_2 \approx \widetilde{W}^0$; $\widetilde{N}_3, \widetilde{N}_4 \approx (\widetilde{H}_u^0 \pm \widetilde{H}_d^0)/\sqrt{2}$, with mass eigenvalues:

$$m_{\widetilde{N}_1} = M_1 - \frac{m_Z^2 s_W^2 (M_1 + \mu \sin 2\beta)}{\mu^2 - M_1^2} + \ldots \tag{7.50}$$

$$m_{\widetilde{N}_2} = M_2 - \frac{m_W^2 (M_2 + \mu \sin 2\beta)}{\mu^2 - M_2^2} + \ldots \tag{7.51}$$

$$m_{\widetilde{N}_3}, m_{\widetilde{N}_4} = |\mu| + \frac{m_Z^2(1 - \epsilon \sin 2\beta)(|\mu| + M_1 c_W^2 + M_2 s_W^2)}{2(|\mu| + M_1)(|\mu| + M_2)} + \ldots, \tag{7.52}$$

$$|\mu| + \frac{m_Z^2(1 + \epsilon \sin 2\beta)(|\mu| - M_1 c_W^2 - M_2 s_W^2)}{2(|\mu| - M_1)(|\mu| - M_2)} + \ldots \tag{7.53}$$

where we have assumed μ is real with sign $\epsilon = \pm 1$. The labeling of the mass eigenstates $\widetilde{N}_1$ and $\widetilde{N}_2$ assumes $M_1 < M_2 < |\mu|$; otherwise the subscripts may need to be rearranged. It turns out that a "bino-like" LSP $\widetilde{N}_1$ can very easily have the right cosmological abundance to make a good dark matter candidate, so the large $|\mu|$ limit may be preferred from that point of view. In addition, this limit tends to emerge from minimal supergravity boundary conditions on the soft parameters, which often require $|\mu|$ to be larger than M_1 and M_2 in order to get correct electroweak symmetry breaking.

The chargino spectrum can be analyzed in a similar way. In the gauge-eigenstate basis $\psi^\pm = (\widetilde{W}^+, \widetilde{H}_u^+, \widetilde{W}^-, \widetilde{H}_d^-)$, the chargino mass terms in the lagrangian are

$$\mathcal{L} \supset -\frac{1}{2}(\psi^\pm)^T \mathbf{M}_{\widetilde{C}} \psi^\pm + \text{c.c.} \tag{7.54}$$

where, in 2×2 block form,

$$\mathbf{M}_{\widetilde{C}} = \begin{pmatrix} \mathbf{0} & \mathbf{X}^T \\ \mathbf{X} & \mathbf{0} \end{pmatrix}; \qquad \mathbf{X} = \begin{pmatrix} M_2 & \sqrt{2} s_\beta\, m_W \\ \sqrt{2} c_\beta\, m_W & \mu \end{pmatrix}. \tag{7.55}$$

The mass eigenstates are related to the gauge eigenstates by two unitary 2×2 matrices $\mathbf{U}$ and $\mathbf{V}$ according to

$$\begin{pmatrix} \widetilde{C}_1^+ \\ \widetilde{C}_2^+ \end{pmatrix} = \mathbf{V} \begin{pmatrix} \widetilde{W}^+ \\ \widetilde{H}_u^+ \end{pmatrix}; \qquad \begin{pmatrix} \widetilde{C}_1^- \\ \widetilde{C}_2^- \end{pmatrix} = \mathbf{U} \begin{pmatrix} \widetilde{W}^- \\ \widetilde{H}_d^- \end{pmatrix}. \tag{7.56}$$

Note that there are different mixing matrices for the positively charged states and for the negatively charged states. They are to be chosen so that

$$\mathbf{U}^* \mathbf{X} \mathbf{V}^{-1} = \begin{pmatrix} m_{\widetilde{C}_1} & 0 \\ 0 & m_{\widetilde{C}_2} \end{pmatrix}. \tag{7.57}$$

Because these are only 2×2 matrices, it is not hard to solve for the masses explicitly:

$$\begin{aligned} m^2_{\widetilde{C}_1}, m^2_{\widetilde{C}_2} &= \frac{1}{2}\Big[(|M_2|^2 + |\mu|^2 + 2m_W^2) \\ &\quad \mp \sqrt{(|M_2|^2 + |\mu|^2 + 2m_W^2)^2 - 4|\mu M_2 - m_W^2 \sin 2\beta|^2}\Big]. \end{aligned} \tag{7.58}$$

It should be noted that these are the (doubly degenerate) eigenvalues of the 4×4 matrix $\mathbf{M}_{\widetilde{C}}^\dagger \mathbf{M}_{\widetilde{C}}$, or equivalently the eigenvalues of $\mathbf{X}^\dagger \mathbf{X}$, but they are *not* the squares of the eigenvalues of $\mathbf{X}$. In the limit of eq. (7.49) with real M_2 and μ, one finds that the charginos mass eigenstates consist of a wino-like $\widetilde{C}_1^\pm$ and and a higgsino-like $\widetilde{C}_2^\pm$, with masses

$$m_{\widetilde{C}_1} = M_2 - \frac{m_W^2 (M_2 + \mu \sin 2\beta)}{\mu^2 - M_2^2} + \ldots \tag{7.59}$$

$$m_{\widetilde{C}_2} = |\mu| + \frac{m_W^2 (|\mu| + \epsilon M_2 \sin 2\beta)}{\mu^2 - M_2^2} + \ldots. \tag{7.60}$$

Here again the labeling assumes $M_2 < |\mu|$, and ϵ is the sign of μ. Amusingly, the lighter chargino $\widetilde{C}_1$ is nearly degenerate with the second lightest neutralino $\widetilde{N}_2$ in this limit, but this is not an exact result. Their higgsino-like colleagues $\widetilde{N}_3$, $\widetilde{N}_4$ and $\widetilde{C}_2$ have masses of order $|\mu|$. The case of $M_1 \approx 0.5 M_2 \ll |\mu|$ is not uncommonly found in viable models following from the boundary conditions in section 6, and it has been elevated to the status of a benchmark scenario in many phenomenological studies. However it cannot be overemphasized that such expectations are not mandatory.

In practice, the masses and mixing angles for the neutralinos and charginos are best computed numerically. The corresponding Feynman rules may be inferred in terms of $\mathbf{N}$, $\mathbf{U}$ and $\mathbf{V}$ from the MSSM lagrangian as discussed above; they are collected in Refs.[19,96]

7.4 The gluino

The gluino is a color octet fermion, so it cannot mix with any other particle in the MSSM, even if R-parity is violated. In this regard, it is unique among all of the MSSM sparticles. In the models following from minimal supergravity or gauge-mediated boundary conditions, the gluino mass parameter M_3 is related to the bino and wino mass parameters M_1 and M_2 by eq. (7.7)

$$M_3 = \frac{\alpha_S}{\alpha}\sin^2\theta_W\, M_2 = \frac{3}{5}\frac{\alpha_S}{\alpha}\cos^2\theta_W\, M_1 \tag{7.61}$$

at any RG scale, up to small two-loop corrections. If we use values $\alpha_S = 0.118$, $\alpha = 1/128$, $\sin^2\theta_W = 0.23$, then one finds the rough prediction

$$M_3 : M_2 : M_1 \approx 7 : 2 : 1 \tag{7.62}$$

at the electroweak scale. In particular, we suspect that the gluino should be much heavier than the lighter neutralinos and charginos.

For more precise estimates, one must take into account the fact that the parameter M_3 is really a running mass which has an implicit dependence on the RG scale Q. Because the gluino is a strongly interacting particle, M_3 runs rather quickly with Q [see eq. (7.5)]. A more useful quantity physically is the RG scale-independent mass $m_{\widetilde{g}}$ at which the renormalized gluino propagator has a pole. Including one-loop corrections to the gluino propagator due to gluon exchange and quark-squark loops, one finds that the pole mass is given in terms of the running mass in the $\overline{\rm DR}$ scheme by [88]

$$m_{\widetilde{g}} = M_3(Q)\Big(1 + \frac{\alpha_S}{4\pi}[15 + 6\,\ln(Q/M_3) + \sum A_{\widetilde{q}}]\Big) \tag{7.63}$$

where

$$A_{\widetilde{q}} = \int_0^1 dx\; x\,\ln[x\,m_{\widetilde{q}}^2/M_3^2 + (1-x)m_q^2/M_3^2 - x(1-x)]. \tag{7.64}$$

The sum in eq. (7.63) is over all 12 squark-quark supermultiplets, and we have neglected small effects due to squark mixing. It is easy to check that requiring $m_{\widetilde{g}}$ to be independent of Q in eq. (7.63) reproduces the one-loop RG equation for $M_3(Q)$ in eq. (7.5). The correction terms proportional to α_S in eq. (7.63) can be quite significant, so that $m_{\widetilde{g}}/M_3(M_3)$ can exceed unity by 25% or more. The reasons for this are that the gluino is strongly interacting, with a large group theory factor [the 15 in eq. (7.63)] due to its color octet nature, and that it couples to all the squark-quark pairs. Of course, there are similar corrections which relate the running masses of all the other MSSM particles to their physical masses. These have been systematically evaluated at one-loop order in Ref.[105] They are more complicated in form and usually numerically smaller than for the gluino, but in some cases they could be quite important in future efforts to connect a given candidate model for the soft terms to experimentally measured masses and mixing angles of the MSSM particles.

7.5 The squark and slepton mass spectrum

In principle, any scalars with the same electric charge, R-parity, and color quantum numbers can mix with each other. This means that with completely arbitrary soft terms, the mass eigenstates of the squarks and sleptons of the MSSM should be obtained by diagonalizing three 6×6 (mass)^2 matrices for up-type squarks ($\widetilde{u}_L$, $\widetilde{c}_L$, $\widetilde{t}_L$, $\widetilde{u}_R$, $\widetilde{c}_R$, $\widetilde{t}_R$), down-type squarks ($\widetilde{d}_L$, $\widetilde{s}_L$, $\widetilde{b}_L$, $\widetilde{d}_R$, $\widetilde{s}_R$, $\widetilde{b}_R$), and charged sleptons ($\widetilde{e}_L$, $\widetilde{\mu}_L$, $\widetilde{\tau}_L$, $\widetilde{e}_R$, $\widetilde{\mu}_R$, $\widetilde{\tau}_R$), and one 3×3 matrix for sneutrinos ($\widetilde{\nu}_e$, $\widetilde{\nu}_\mu$, $\widetilde{\nu}_\tau$). Fortunately, the general hypothesis of flavor-blind soft parameters eqs. (5.14) and (5.15) predicts that most of these mixing angles are very small. The third-family squarks and sleptons can have very different masses compared to their first- and second-family counterparts, because of the effects of large Yukawa (y_t, y_b, y_τ) and soft (a_t, a_b, a_τ) couplings in the RG equations (7.20)-(7.24). Furthermore, they can have substantial mixing in pairs ($\widetilde{t}_L$, $\widetilde{t}_R$), ($\widetilde{b}_L$, $\widetilde{b}_R$) and ($\widetilde{\tau}_L$, $\widetilde{\tau}_R$). In contrast, the first- and second-family squarks and

sleptons have negligible Yukawa couplings, so they end up in 7 very nearly degenerate, unmixed pairs ($\widetilde{e}_R, \widetilde{\mu}_R$), ($\widetilde{\nu}_e, \widetilde{\nu}_\mu$), ($\widetilde{e}_L, \widetilde{\mu}_L$), ($\widetilde{u}_R, \widetilde{c}_R$), ($\widetilde{d}_R, \widetilde{s}_R$), ($\widetilde{u}_L, \widetilde{c}_L$), ($\widetilde{d}_L, \widetilde{s}_L$). As we have already discussed in section 5.4, this avoids the problem of disastrously large virtual sparticle contributions to FCNC processes.

Let us first consider the spectrum of first- and second-family squarks and sleptons. In models fitting into both of the broad categories of minimal supergravity [eq. (6.28)] or gauge-mediated [eq. (6.42)] boundary conditions, their running masses can be conveniently parameterized in the following way:

$$m_{Q_1}^2 = m_{Q_2}^2 = m_0^2 + K_3 + K_2 + \frac{1}{36}K_1, \tag{7.65}$$

$$m_{\bar{u}_1}^2 = m_{\bar{u}_2}^2 = m_0^2 + K_3 \qquad + \frac{4}{9}K_1, \tag{7.66}$$

$$m_{\bar{d}_1}^2 = m_{\bar{d}_2}^2 = m_0^2 + K_3 \qquad + \frac{1}{9}K_1, \tag{7.67}$$

$$m_{L_1}^2 = m_{L_2}^2 = m_0^2 \qquad + K_2 + \frac{1}{4}K_1, \tag{7.68}$$

$$m_{\bar{e}_1}^2 = m_{\bar{e}_2}^2 = m_0^2 \qquad + K_1. \tag{7.69}$$

In minimal supergravity models, m_0^2 is the common scalar (mass)^2 which appears in eq. (6.28). It can be 0 in the "no-scale" limit, but it could also be the dominant source of the scalar masses. The contributions K_3, K_2 and K_1 are due to the RG running proportional to the gaugino masses; see eq. (7.14). They are strictly positive. A key point is that the same K_3, K_2 and K_1 appear everywhere in eqs. (7.65)-(7.69), since all of the chiral supermultiplets couple to the same gauginos with the same gauge couplings. The different coefficients in front of K_1 just correspond to the various values of weak hypercharge squared for each scalar. The quantities K_1, K_2, K_3 depend on the RG scale Q at which they are evaluated. Explicitly, they are found by solving eq. (7.14):

$$K_a(Q) = \left\{ \begin{matrix} 3/5 \\ 3/4 \\ 4/3 \end{matrix} \right\} \times \frac{1}{2\pi^2} \int_{\ln Q}^{\ln Q_0} dt\; g_a^2(t)\,|M_a(t)|^2 \qquad (a = 1, 2, 3). \tag{7.70}$$

Here Q_0 is the input RG scale at which the boundary condition eq. (6.28) is applied, and Q should be taken to be evaluated near the squark and slepton mass under consideration, presumably less than about 1 TeV or so. The values of the running parameters $g_a(Q)$ and $M_a(Q)$ can be found using eqs. (5.17) and (7.7). If the input scale is approximated by the apparent scale of gauge coupling unification $Q_0 = M_U \approx 2 \times 10^{16}$ GeV, one finds that numerically

$$K_1 \approx 0.15 m_{1/2}^2; \qquad K_2 \approx 0.5 m_{1/2}^2; \qquad K_3 \approx (4.5\,\text{to}\,6.5) m_{1/2}^2. \tag{7.71}$$

for Q near 1 TeV. Here $m_{1/2}$ is the common gaugino mass parameter at the unification scale. Note that $K_3 \gg K_2 \gg K_1$; this is a direct consequence of the relative sizes of the gauge couplings g_3, g_2, and g_1. The large uncertainty in K_3 is due in part

to the experimental uncertainty in the QCD coupling constant, and in part to the uncertainty in where to choose Q, since K_3 runs rather quickly below 1 TeV. If the gauge couplings and gaugino masses are unified between M_U and M_P, as would occur in a GUT model, then the effect of RG running for $M_U < Q < M_P$ can be absorbed into a redefinition of m_0^2. Otherwise, it adds a further uncertainty which is roughly proportional to $\ln(M_P/M_U)$, compared to the larger contributions in eq. (7.70) which go roughly like $\ln(M_U/1\text{ TeV})$.

In gauge-mediated models, the same parameterization eqs. (7.65)-(7.69) holds, but m_0^2 is always 0. At the input scale Q_0, each MSSM scalar gets contributions to its $(\text{mass})^2$ which depend only on its gauge interactions, as in eq. (6.42). It is not hard to see that in general these contribute in exactly the same pattern as K_1, K_2, and K_3 in eq. (7.65)-(7.69). The subsequent evolution of the scalar squared masses down to the electroweak scale again just yields more contributions to the K_1, K_2, and K_3 parameters. It is somewhat more difficult to give meaningful numerical estimates for these parameters in gauge-mediated models than in the minimal supergravity models, because of uncertainties in the messenger mass scale(s) and in the multiplicities of the messenger fields. However, in the gauge-mediated case one quite generally expects that the numerical values of the ratios K_3/K_2, K_3/K_1 and K_2/K_1 should be even larger than in eq. (7.71). There are two reasons for this. First, the running squark squared masses start off larger than slepton squared masses already at the input scale in gauge-mediated models, rather than having a common value m_0^2. Furthermore, in the gauge-mediated case, the input scale Q_0 is typically much lower than M_P or M_U, so that the RG evolution gives relatively more weight to smaller RG scales where the hierarchies $g_3 > g_2 > g_1$ and $M_3 > M_2 > M_1$ are already in effect.

In general, one therefore expects that the squarks should be considerably heavier than the sleptons, with the effect being more pronounced in gauge-mediated supersymmetry breaking models than in minimal supergravity models. For any specific choice of model, this effect can be easily quantified with an RG analysis. The hierarchy $m_{\text{squark}} > m_{\text{slepton}}$ tends to hold even in models which do not really fit into any of the categories outlined in section 6, because the RG contributions to squark masses from the gluino are always present and usually quite large, since QCD has a larger gauge coupling than the electroweak interactions.

There is also a "hyperfine" splitting in the squark and slepton mass spectrum produced by electroweak symmetry breaking. Each squark and slepton ϕ will get a contribution Δ_ϕ to its $(\text{mass})^2$, coming from the $SU(2)_L$ and $U(1)_Y$ D-term quartic interactions [see the last term in eq. (3.75)] of the form $(\text{squark})^2(\text{Higgs})^2$ and $(\text{slepton})^2(\text{Higgs})^2$, when the neutral Higgs scalars H_u^0 and H_d^0 get VEVs. They are model-independent for a given value of $\tan\beta$, and are given by

$$\Delta_\phi = (T_3^\phi - Q_{\text{EM}}^\phi \sin^2\theta_W)\cos 2\beta\, m_Z^2, \tag{7.72}$$

where T_3^ϕ and Q_{EM}^ϕ are the third component of weak isospin and the electric charge of the chiral supermultiplet to which ϕ belongs. [For example, $\Delta_u = (\frac{1}{2} - \frac{2}{3}\sin^2\theta_W)\cos 2\beta\, m_Z^2$ and $\Delta_{\overline{u}} = (\frac{2}{3}\sin^2\theta_W)\cos 2\beta\, m_Z^2$]. These D-term contributions are typically smaller than the m_0^2 and K_1, K_2, K_3 contributions, but should not be neglected. They split

apart the components of the $SU(2)_L$-doublet sleptons and squarks $L_1 = (\widetilde{\nu}_e, \widetilde{e}_L)$, etc. Including them, the first-family squark and slepton masses are now given by:

$$m^2_{\widetilde{d}_L} = m_0^2 + K_3 + K_2 + \frac{1}{36}K_1 + \Delta_d, \tag{7.73}$$

$$m^2_{\widetilde{u}_L} = m_0^2 + K_3 + K_2 + \frac{1}{36}K_1 + \Delta_u, \tag{7.74}$$

$$m^2_{\widetilde{u}_R} = m_0^2 + K_3 \qquad + \frac{4}{9}K_1 + \Delta_{\overline{u}}, \tag{7.75}$$

$$m^2_{\widetilde{d}_R} = m_0^2 + K_3 \qquad + \frac{1}{9}K_1 + \Delta_{\overline{d}}, \tag{7.76}$$

$$m^2_{\widetilde{e}_L} = m_0^2 \qquad + K_2 + \frac{1}{4}K_1 + \Delta_e, \tag{7.77}$$

$$m^2_{\widetilde{\nu}} = m_0^2 \qquad + K_2 + \frac{1}{4}K_1 + \Delta_\nu, \tag{7.78}$$

$$m^2_{\widetilde{e}_R} = m_0^2 \qquad\qquad + K_1 + \Delta_{\overline{e}}, \tag{7.79}$$

with identical formulas for the second-family squarks and sleptons. The mass splittings for the left-handed squarks and sleptons are governed by model-independent sum rules

$$m^2_{\widetilde{e}_L} - m^2_{\widetilde{\nu}_e} = m^2_{\widetilde{d}_L} - m^2_{\widetilde{u}_L} = -\cos 2\beta\, m_W^2. \tag{7.80}$$

Since $\cos 2\beta < 0$ in the allowed range $\tan\beta > 1$, it follows that $m_{\widetilde{e}_L} > m_{\widetilde{\nu}_e}$ and $m_{\widetilde{d}_L} > m_{\widetilde{u}_L}$, with the magnitude of the splittings constrained by electroweak symmetry breaking.

Let us next consider the masses of the top squarks, for which there are several non-negligible contributions. First, there are $(\text{mass})^2$ terms for $\widetilde{t}_L^*\widetilde{t}_L$ and $\widetilde{t}_R^*\widetilde{t}_R$ which are just equal to $m^2_{Q_3} + \Delta_u$ and $m^2_{\overline{u}_3} + \Delta_{\overline{u}}$, respectively, just as for the first- and second-family squarks. Second, there are contributions equal to m_t^2 for each of $\widetilde{t}_L^*\widetilde{t}_L$ and $\widetilde{t}_R^*\widetilde{t}_R$. These come from F-terms in the scalar potential of the form $y_t^2 H_u^{0*} H_u^0 \widetilde{t}_L^*\widetilde{t}_L$ and $y_t^2 H_u^{0*} H_u^0 \widetilde{t}_R^*\widetilde{t}_R$ (see Figs. 8b and 8c), with the Higgs fields replaced by their VEVs. These contributions are of course present for all of the squarks and sleptons, but they are much too small to worry about except in the case of the top squarks. Third, there are contributions to the scalar potential from F-terms of the form $-\mu y_t \widetilde{\overline{t}}\widetilde{t} H_d^{0*} + \text{c.c.}$; see eqs. (5.6) and Fig. 10a. These become $-\mu v y_t \cos\beta\, \widetilde{t}_R^*\widetilde{t}_L + \text{c.c.}$ when H_d^0 is replaced by its VEV. Finally, there are contributions to the scalar potential from the soft $(\text{scalar})^3$ couplings $a_t \widetilde{\overline{t}}\widetilde{Q}_3 H_u^0 + \text{c.c.}$ [see the first term of the second line of eq. (5.11) and eq. (7.8)], which become $a_t v \sin\beta\, \widetilde{t}_L\widetilde{t}_R^* + \text{c.c.}$ when H_u^0 is replaced by its VEV. Putting these all together, we have a $(\text{mass})^2$ matrix for the top squarks, which in the gauge-eigenstate basis $(\widetilde{t}_L, \widetilde{t}_R)$ is given by

$$-\mathcal{L} \supset (\,\widetilde{t}_L^* \quad \widetilde{t}_R^*\,)\, \mathbf{m}^{\mathbf{2}}_{\widetilde{\mathbf{t}}} \begin{pmatrix} \widetilde{t}_L \\ \widetilde{t}_R \end{pmatrix} \tag{7.81}$$

where

$$\mathbf{m}^{\mathbf{2}}_{\widetilde{\mathbf{t}}} = \begin{pmatrix} m^2_{Q_3} + m_t^2 + \Delta_u & v(a_t \sin\beta - \mu y_t \cos\beta) \\ v(a_t \sin\beta - \mu y_t \cos\beta) & m^2_{\overline{u}_3} + m_t^2 + \Delta_{\overline{u}} \end{pmatrix}. \tag{7.82}$$

This matrix can be diagonalized to give mass eigenstates

$$\begin{pmatrix} \tilde{t}_1 \\ \tilde{t}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_{\tilde{t}} & \sin\theta_{\tilde{t}} \\ -\sin\theta_{\tilde{t}} & \cos\theta_{\tilde{t}} \end{pmatrix} \begin{pmatrix} \tilde{t}_L \\ \tilde{t}_R \end{pmatrix} \tag{7.83}$$

with $m^2_{\tilde{t}_1} < m^2_{\tilde{t}_2}$ being the eigenvalues of eq. (7.82) and $0 \le \theta_{\tilde{t}} \le \pi$. Because of the large RG effects proportional to X_t in eq. (7.20) and eq. (7.21), at the electroweak scale one finds that $m^2_{\overline{u}_3} < m^2_{Q_3}$, and both of these quantities are usually significantly smaller than the squark squared masses for the first two families. The diagonal terms m_t^2 in eq. (7.82) tend to mitigate this effect somewhat, but the off-diagonal entries will typically induce a significant mixing which always reduces the lighter top-squark (mass)^2 eigenvalue. For this reason, it is often found in models that $\tilde{t}_1$ is the lightest squark of all.

A very similar analysis can be performed for the bottom squarks and charged tau sleptons, which in their respective gauge-eigenstate bases $(\tilde{b}_L, \tilde{b}_R)$ and $(\tilde{\tau}_L, \tilde{\tau}_R)$ have (mass)^2 matrices:

$$\mathbf{m}^2_{\tilde{\mathbf{b}}} = \begin{pmatrix} m^2_{Q_3} + \Delta_d & v(a_b \cos\beta - \mu y_b \sin\beta) \\ v(a_b \cos\beta - \mu y_b \sin\beta) & m^2_{\overline{d}_3} + \Delta_{\overline{d}} \end{pmatrix}; \tag{7.84}$$

$$\mathbf{m}^2_{\tilde{\tau}} = \begin{pmatrix} m^2_{L_3} + \Delta_e & v(a_\tau \cos\beta - \mu y_\tau \sin\beta) \\ v(a_\tau \cos\beta - \mu y_\tau \sin\beta) & m^2_{\overline{e}_3} + \Delta_{\overline{e}} \end{pmatrix}. \tag{7.85}$$

These can be diagonalized to give mass eigenstates $\tilde{b}_1, \tilde{b}_2$ and $\tilde{\tau}_1, \tilde{\tau}_2$ in exact analogy with eq. (7.83).

The magnitude and importance of mixing in the sbottom and stau sectors depends on how large $\tan\beta$ is. If $\tan\beta$ is not too large (in practice, this usually means less than about 10 or so, depending on the situation under study), the sbottoms and staus do not get a very large effect from the mixing terms and the RG effects due to X_b and X_τ, because $y_b, y_\tau \ll y_t$ from eq. (7.44). In that case the mass eigenstates are very nearly the same as the gauge eigenstates $\tilde{b}_L$, $\tilde{b}_R$, $\tilde{\tau}_L$ and $\tilde{\tau}_R$. The latter three, and $\tilde{\nu}_\tau$, will be nearly degenerate with their first- and second-family counterparts with the same $SU(3)_C \times SU(2)_L \times U(1)_Y$ quantum numbers. However, even in the case of small $\tan\beta$, $\tilde{b}_L$ will feel the effects of the large top Yukawa coupling because it is part of the doublet $\tilde{Q}_3$ which contains $\tilde{t}_L$. In particular, from eq. (7.20) we see that X_t acts to decrease $m^2_{\tilde{Q}_3}$ as it is RG-evolved down from the input scale to the electroweak scale. Therefore the mass of $\tilde{b}_L$ can be significantly less than the masses of $\tilde{d}_L$ and $\tilde{s}_L$.

For larger values of $\tan\beta$, the mixing in eqs. (7.84) and (7.85) can be quite significant, because y_b, y_τ and a_b, a_τ are non-negligible. Just as in the case of the top squarks, the lighter sbottom and stau mass eigenstates (denoted $\tilde{b}_1$ and $\tilde{\tau}_1$) can be significantly lighter than their first- and second-family counterparts. Furthermore, $\tilde{\nu}_\tau$ can be significantly lighter than the nearly degenerate $\tilde{\nu}_e$, $\tilde{\nu}_\mu$.

The requirement that the third-family squarks and sleptons should all have positive (mass)^2 implies limits on the sizes of $a_t \sin\beta - \mu y_t \cos\beta$, $a_b \cos\beta - \mu y_b \sin\beta$, and $a_\tau \cos\beta - \mu y_\tau \sin\beta$. If they are too large, the smaller eigenvalue of eq. (7.82), (7.84)

Table 3: Undiscovered particles in the Minimal Supersymmetric Standard Model

Names	Spin	P_R	Mass Eigenstates	Gauge Eigenstates
Higgs bosons	0	+1	h^0 H^0 A^0 $H^\pm$	H_u^0 H_d^0 H_u^+ H_d^-
squarks	0	−1	$\tilde{u}_L$ $\tilde{u}_R$ $\tilde{d}_L$ $\tilde{d}_R$	"
			$\tilde{s}_L$ $\tilde{s}_R$ $\tilde{c}_L$ $\tilde{c}_R$	"
			$\tilde{t}_1$ $\tilde{t}_2$ $\tilde{b}_1$ $\tilde{b}_2$	$\tilde{t}_L$ $\tilde{t}_R$ $\tilde{b}_L$ $\tilde{b}_R$
sleptons	0	−1	$\tilde{e}_L$ $\tilde{e}_R$ $\tilde{\nu}_e$	"
			$\tilde{\mu}_L$ $\tilde{\mu}_R$ $\tilde{\nu}_\mu$	"
			$\tilde{\tau}_1$ $\tilde{\tau}_2$ $\tilde{\nu}_\tau$	$\tilde{\tau}_L$ $\tilde{\tau}_R$ $\tilde{\nu}_\tau$
neutralinos	1/2	−1	$\widetilde{N}_1$ $\widetilde{N}_2$ $\widetilde{N}_3$ $\widetilde{N}_4$	$\tilde{B}^0$ $\widetilde{W}^0$ $\widetilde{H}_u^0$ $\widetilde{H}_d^0$
charginos	1/2	−1	$\widetilde{C}_1^\pm$ $\widetilde{C}_2^\pm$	$\widetilde{W}^\pm$ $\widetilde{H}_u^+$ $\widetilde{H}_d^-$
gluino	1/2	−1	$\tilde{g}$	"
gravitino/ goldstino	3/2	−1	$\tilde{G}$	"

or (7.85) will be driven negative, implying that a squark or charged slepton gets a VEV, breaking $SU(3)_C$ or electromagnetism. Since this is clearly unacceptable, one can put bounds on the (scalar)^3 couplings, or equivalently on the parameter A_0 in minimal supergravity models. Even if all of the (mass)^2 eigenvalues are positive, the presence of large (scalar)^3 couplings can yield global minima of the scalar potential with non-zero squark and/or charged slepton VEVs which are disconnected from the vacuum which conserves $SU(3)_C$ and electromagnetism.[106] However, it is not always clear whether the non-existence of such disconnected global minima should really be taken as a constraint, because the tunneling rate from our "good" vacuum to the "bad" vacua can easily be much longer than the age of the universe.[107]

7.6 Summary: the MSSM sparticle spectrum

In the MSSM there are 32 distinct masses corresponding to undiscovered particles, not including the gravitino. In this section we have explained how the masses and mixing angles for these particles can be computed, given an underlying model for the soft terms at some input scale. Assuming only that the mixing of first- and second-family squarks and sleptons is negligible, the mass eigenstates of the MSSM are listed in Table 3. A complete set of Feynman rules for the interactions of these particles with each other and with the Standard Model quarks, leptons, and gauge bosons can be found in Refs.[19,96] Specific models for the soft terms typically predict the masses and the mixing angles angles for the MSSM in terms of far fewer parameters. For example, in the minimal supergravity models, one has only the parameters m_0^2, $m_{1/2}$,

A_0, μ, and b which are not already measured by experiment. On the other hand, in gauge-mediated supersymmetry breaking models, the free parameters include at least the scale Λ, the typical messenger mass scale $M_{\rm mess}$, the integer number N_5 of copies of the minimal messengers, the goldstino decay constant $\langle F \rangle$, and the Higgs mass parameters μ and b. After RG evolving the soft terms down to the electroweak scale, one can impose that the scalar potential gives correct electroweak symmetry breaking. This allows us to trade $|\mu|$ and b (or B_0) for one parameter $\tan\beta$, as in eqs. (7.31)-(7.32). So, to a reasonable approximation, the entire mass spectrum in minimal supergravity models is determined by only five unknown parameters: m_0^2, $m_{1/2}$, A_0, $\tan\beta$, and $\mathrm{Arg}(\mu)$, while in the simplest gauge-mediated supersymmetry breaking models one can pick parameters Λ, $M_{\rm mess}$, N_5, $\langle F \rangle$, $\tan\beta$, and $\mathrm{Arg}(\mu)$. Both frameworks are highly predictive. Of course, it is easy to imagine that the essential physics of supersymmetry breaking is not captured by either of these two scenarios in their minimal forms.

While it would be a mistake to underestimate the uncertainties in the MSSM mass and mixing spectrum, it is also useful to keep in mind some general lessons that recur in various different scenarios. Indeed, there has emerged a sort of folklore concerning likely features of the MSSM spectrum, which is partly based on theoretical bias and partly on the constraints inherent in any supersymmetric theory. We remark on these features mainly because they represent the prevailing prejudice among supersymmetry theorists, which is certainly a useful thing for the reader to know even if he or she wisely decides to remain skeptical. For example, it is perhaps not unlikely that:

- The LSP is the lightest neutralino $\widetilde{N}_1$, unless the gravitino is lighter or R-parity is not conserved. If $\mu > M_1, M_2$, then $\widetilde{N}_1$ is likely to be bino-like, with a mass roughly 0.5 times the masses of $\widetilde{N}_2$ and $\widetilde{C}_1$. In the opposite case $\mu < M_1, M_2$, then $\widetilde{N}_1$ has a large higgsino content and $\widetilde{N}_2$ and $\widetilde{C}_1$ are not much heavier.

- The gluino will be much heavier than the lighter neutralinos and charginos. This is certainly true in the case of the "standard" gaugino mass relation eq. (7.7); more generally, the running gluino mass parameter grows relatively quickly as it is RG-evolved into the infrared because the QCD coupling is larger than the electroweak gauge couplings. So even if there are big corrections to the gaugino mass boundary conditions eqs. (6.27) or (6.40), the gluino mass parameter M_3 is likely to come out larger than M_1 and M_2.

- The squarks of the first and second families are nearly degenerate and much heavier than the sleptons. This is because each squark mass gets the same large positive-definite radiative corrections from loops involving the gluino. The left-handed squarks $\widetilde{u}_L$, $\widetilde{d}_L$, $\widetilde{s}_L$ and $\widetilde{c}_L$ are likely to be heavier than their right-handed counterparts $\widetilde{u}_R$, $\widetilde{d}_R$, $\widetilde{s}_R$ and $\widetilde{c}_R$, because of the effect of K_2 in eqs. (7.73)-(7.79).

- The squarks of the first two families cannot be lighter than about 0.8 times the mass of the gluino in minimal supergravity models, and about 0.6 times the mass of the gluino in the simplest gauge-mediated models as discussed in section 6.4 if the number of messenger squark pairs is $N_5 \leq 4$. In the minimal

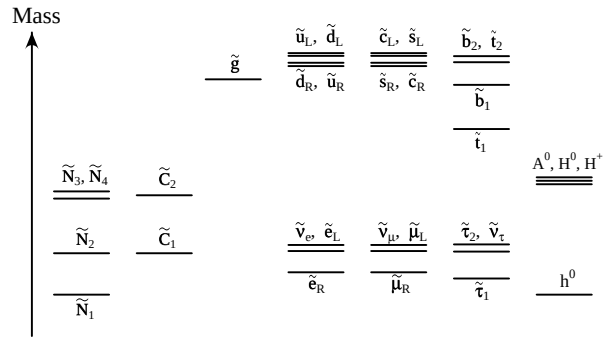

Figure 18: A schematic sample spectrum for the undiscovered particles in the MSSM. This spectrum is presented for entertainment purposes only. No warranty, expressed or implied, guarantees that this spectrum looks anything like the real world.

supergravity case this is because the gluino mass feeds into the squark masses through RG evolution; in the gauge-mediated case it is because the gluino and squark masses are tied together by eqs. (6.40) and (6.42) [multiplied by N_5, as explained at the end of section 6.4].

- The lighter stop $\tilde{t}_1$ and the lighter sbottom $\tilde{b}_1$ are probably the lightest squarks. This is because stop and sbottom mixing effects and the effects of X_t and X_b in eqs. (7.20)-(7.22) both tend to decrease the lighter stop and sbottom masses.

- The lightest charged slepton is probably a stau $\tilde{\tau}_1$. The mass difference $m_{\tilde{e}_R} - m_{\tilde{\tau}_1}$ is likely to be significant if $\tan\beta$ is large, because of the effects of a large tau Yukawa coupling. For smaller $\tan\beta$, $\tilde{\tau}_1$ is predominantly $\tilde{\tau}_R$ and it is not so much lighter than $\tilde{e}_R$, $\tilde{\mu}_R$.

- The left-handed charged sleptons $\tilde{e}_L$ and $\tilde{\mu}_L$ are likely to be heavier than their right-handed counterparts $\tilde{e}_R$ and $\tilde{\mu}_R$. This is because of the effect of K_2 in eq. (7.77). (Note also that $\Delta_e - \Delta_{\overline{e}}$ is positive but very small because of the numerical accident $\sin^2\theta_W \approx 1/4$.)

- The lightest neutral Higgs boson h^0 should be lighter than about 150 GeV, and may be much lighter than the other Higgs scalar mass eigenstates A^0, $H^\pm$, H^0.

In Figure 18 we show a qualitative sketch of a sample MSSM mass spectrum which illustrates these features. Variations in the model parameters can have important and predictable effects. For example, taking larger (smaller) m_0^2 in minimal supergravity models will tend to move the entire spectrum of squarks, sleptons and the Higgs scalars A^0, $H^\pm$, H^0 higher (lower) compared to the neutralinos, charginos and gluino; taking larger values of $\tan\beta$ with other model parameters held fixed will usually tend to lower $\tilde{b}_1$ and $\tilde{\tau}_1$ masses compared to those of the other sparticles, etc. The important

point is that by measuring the masses and mixing angles of the MSSM particles we will be able to gain a great deal of information which can rule out or bolster evidence for competing proposals for the origin of supersymmetry breaking. Testing the various possible organizing principles will provide the high-energy physicists of the next millennium with an exciting challenge.

8 Sparticle decays

In this section we will give a brief qualitative overview of the decay patterns of sparticles in the MSSM, assuming that R-parity is exactly conserved. We will consider in turn the possible decays of neutralinos, charginos, sleptons, squarks, and the gluino. If, as is most often assumed, the lightest neutralino $\widetilde{N}_1$ is the LSP, then all decay chains will end up containing it in the final state. In section 8.5 we consider the alternative possibility that the gravitino/goldstino $\widetilde{G}$ is the LSP.

8.1 Decays of neutralinos and charginos

Let us first consider the possible two-body decays. Each neutralino and chargino contains at least a small admixture of the electroweak gauginos $\widetilde{B}$, $\widetilde{W}^0$ or $\widetilde{W}^\pm$, as we saw in section 7.3. So $\widetilde{N}_i$ and $\widetilde{C}_i$ inherit couplings of weak interaction strength to (scalar, fermion) pairs, as shown in Fig. 9b,c. If sleptons or squarks are sufficiently light, a neutralino or chargino can therefore decay into lepton+slepton or quark+squark. (We will often not distinguish between particle and antiparticle names and labels in this section.) Since sleptons are probably lighter than squarks, the lepton+slepton final states are more likely to be open. A neutralino or chargino may also decay into any lighter neutralino or chargino plus a Higgs scalar or an electroweak gauge boson, because they inherit the gaugino-higgsino-Higgs (see Fig. 9b,c) and $SU(2)_L$ gaugino-gaugino-vector boson (see Fig. 5c) couplings of their components. So, the possible two-body decay modes for neutralinos and charginos in the MSSM are:

$$\widetilde{N}_i \to Z\widetilde{N}_j,\ \ W\widetilde{C}_j,\ \ h^0\widetilde{N}_j,\ \ \ell\widetilde{\ell},\ \ \nu\widetilde{\nu},\ \ [A^0\widetilde{N}_j,\ \ H^0\widetilde{N}_j,\ \ H^\pm\widetilde{C}_j^\mp,\ \ q\widetilde{q}]; \tag{8.1}$$

$$\widetilde{C}_i \to W\widetilde{N}_j,\ \ Z\widetilde{C}_1,\ \ h^0\widetilde{C}_1,\ \ \ell\widetilde{\nu},\ \ \nu\widetilde{\ell},\ \ [A^0\widetilde{C}_1,\ \ H^0\widetilde{C}_1,\ \ H^\pm\widetilde{N}_j,\ \ q\widetilde{q}'], \tag{8.2}$$

using a generic notation ν, ℓ, q for neutrinos, charged leptons, and quarks. The final states in brackets are the more kinematically-implausible ones. (Since h^0 is required to be light, it is the most likely of the Higgs scalars to appear in these decays.) For the heavier neutralinos and chargino ($\widetilde{N}_3$, $\widetilde{N}_4$ and $\widetilde{C}_2$), one or more of the decays in eqs. (8.1) and (8.2) is likely to be kinematically allowed. However, it may be that all of these two-body modes are kinematically forbidden for a given chargino or neutralino, especially in the case of $\widetilde{C}_1$ and $\widetilde{N}_2$ decays. If so, then one has three-body decays

$$\widetilde{N}_i \to ff\widetilde{N}_j,\quad \widetilde{N}_i \to ff'\widetilde{C}_j,\quad \widetilde{C}_i \to ff'\widetilde{N}_j,\quad \text{and}\quad \widetilde{C}_2 \to ff'\widetilde{C}_1, \tag{8.3}$$

through the same (but now off-shell) gauge bosons, Higgs scalars, sleptons, and squarks that appeared in the two-body decays eqs. (8.1) and (8.2). Here f is generic

notation for a lepton or quark, with f and f' belonging to the same $SU(2)_L$ multiplet. The chargino and neutralino decay widths into the various final states can be found in Ref.[108,109] The decays

$$\widetilde{C}_1^\pm \to \ell^\pm\nu\widetilde{N}_1, \qquad \widetilde{N}_2 \to \ell^+\ell^-\widetilde{N}_1 \tag{8.4}$$

can be particularly important for phenomenology, because the leptons in the final state often will result in clean signals. In certain regions of parameter space, the above decays can be suppressed by kinematics or by coupling, and one-loop decays (notably $\widetilde{N}_2 \to \gamma\widetilde{N}_1$) might play an important role.[110]

8.2 Slepton decays

Sleptons have two-body decays into a lepton and a chargino or neutralino, because of the gaugino admixture of the latter, as can be seen directly from the couplings in Figs. 9b,c. The two-body decays

$$\widetilde{\ell} \to \ell\widetilde{N}_i,\ \ \widetilde{\ell} \to \nu\widetilde{C}_i,\ \ \widetilde{\nu} \to \nu\widetilde{N}_i,\ \ \widetilde{\nu} \to \ell\widetilde{C}_i \tag{8.5}$$

are therefore of weak interaction strength. In particular, the direct decays

$$\widetilde{\ell} \to \ell\widetilde{N}_1 \ \text{ and } \ \widetilde{\nu} \to \nu\widetilde{N}_1 \tag{8.6}$$

are (essentially[†]) always kinematically allowed if $\widetilde{N}_1$ is the LSP. However, if the sleptons are sufficiently heavy, then the two-body decays to charginos and heavier neutralinos can be important, especially

$$\widetilde{\ell} \to \nu\widetilde{C}_1,\ \ \widetilde{\ell} \to \ell\widetilde{N}_2,\ \ \text{and}\ \ \widetilde{\nu} \to \ell\widetilde{C}_1. \tag{8.7}$$

The right-handed sleptons do not have a coupling to the $SU(2)_L$ gauginos, so they typically prefer the direct decay $\widetilde{\ell}_R \to \ell\widetilde{N}_1$, if $\widetilde{N}_1$ is bino-like. In contrast, the left-handed sleptons may prefer to decay as in eq. (8.7) rather than the direct decays to the LSP as in eq. (8.6), if the former is kinematically open and if $\widetilde{C}_1$ and $\widetilde{N}_2$ are mostly wino. This is because the slepton-lepton-wino interactions in Fig. 9b are proportional to the $SU(2)_L$ gauge coupling g, whereas the slepton-lepton-bino interactions in Fig. 9c are proportional to the much smaller $U(1)_Y$ coupling g'. General results for these decay widths can be found in Ref. [109]

8.3 Squark decays

If the decay $\widetilde{q} \to q\widetilde{g}$ is kinematically allowed, it will always dominate, because the quark-squark-gluino vertex in Fig. 9a has QCD strength. Otherwise, the squarks can decay into a quark plus neutralino or chargino: $\widetilde{q} \to q\widetilde{N}_i$ or $q'\widetilde{C}_i$. The direct decay to the LSP $\widetilde{q} \to q\widetilde{N}_1$ is always kinematically favored, and for right-handed squarks it can dominate because $\widetilde{N}_1$ is mostly bino. However, the left-handed squarks may

[†] An exception occurs if the mass difference $m_{\widetilde{\tau}_1} - m_{\widetilde{N}_1}$ is less than m_τ.

strongly prefer to decay into heavier charginos or neutralinos instead, for example $\tilde{q} \to q\widetilde{N}_2$ or $q'\widetilde{C}_1$, because the relevant squark-quark-wino couplings are much bigger than the squark-quark-bino couplings. Squark decays to higgsino-like charginos and neutralinos are less important, except in the cases of stops and sbottoms which have sizeable Yukawa couplings. The gluino, chargino or neutralino resulting from the squark decay will in turn decay, and so on, until a final state containing $\widetilde{N}_1$ is reached. This can result in very numerous and complicated decay chain possibilities called cascade decays.[111] Special attention must be payed to the top squark, because it is possible that the decays $\tilde{t}_1 \to t\tilde{g}$ and $\tilde{t}_1 \to t\widetilde{N}_1$ are both kinematically forbidden. If so, then the stop may decay only into charginos, by $\tilde{t}_1 \to b\widetilde{C}_1$. If even this decay is kinematically closed, then the stop has only the flavor-suppressed decay to a charm quark: $\tilde{t}_1 \to c\widetilde{N}_1$. This decay can be very slow,[112] so that the lightest stop can be quasi-stable on the time scale relevant for collider physics, and can hadronize and form bound states inside the detector.

8.4 Gluino decays

The decay of the gluino can only proceed through an on-shell or a virtual squark. If two-body decays $\tilde{g} \to q\tilde{q}$ are open, they will dominate, again because the relevant gluino-quark-squark coupling in Fig. 9a has QCD strength. Since the top and bottom squarks can easily be much lighter than all of the other squarks, it is quite possible that $\tilde{g} \to t\tilde{t}_1$ and/or $\tilde{g} \to b\tilde{b}_1$ are the only available two-body decay mode(s) for the gluino, in which case they will dominate over all others. If instead all of the squarks are heavier than the gluino, the gluino will decay only through off-shell squarks, so $\tilde{g} \to qq'\widetilde{N}_i$ and $qq'\widetilde{C}_i$. The squarks, neutralinos and charginos in these final states will then decay as discussed above, so there can be very many competing gluino decay chains. These cascade decays can have final-state branching fractions that are individually small and quite sensitive to the parameters of the model.

8.5 Decays to the gravitino/goldstino

Most phenomenological studies of supersymmetry assume explicitly or implicitly that the lightest neutralino is the LSP. This is typically the case in gravity-mediated models for the soft terms. However, in gauge-mediated models (and in "no-scale" models), the LSP is instead the gravitino. As we saw in section 6.2, a very light gravitino may be relevant for collider phenomenology, because it contains as its longitudinal component the goldstino, which has a non-gravitational coupling to all sparticle-particle pairs $(\widetilde{X}, X)$. The decay rate found in eq. (6.22) for $\widetilde{X} \to X\widetilde{G}$ is usually not fast enough to compete with the other decays of sparticles $\widetilde{X}$ as mentioned above, *except* in the case that $\widetilde{X}$ is the next-to-lightest supersymmetric particle (NLSP). Since the NLSP has no competing decays, it should always decay into its superpartner and the LSP gravitino.

In principle, any of the MSSM superpartners could be the NLSP in models with a light goldstino, but most models with gauge-mediation of supersymmetry breaking have either a neutralino or a charged lepton playing this role. The argument for

this can be seen immediately from eqs. (6.48) and (6.49); since $\alpha_1 < \alpha_2, \alpha_3$, those superpartners which have only $U(1)_Y$ interactions will tend to get the smallest masses. The gauge-eigenstate sparticles with this property are the bino and the right-handed sleptons $\tilde{e}_R$, $\tilde{\mu}_R$, $\tilde{\tau}_R$, so the appropriate corresponding mass eigenstates should be plausible candidates for the NLSP.

First suppose that $\widetilde{N}_1$ is the NLSP in light goldstino models. Since $\widetilde{N}_1$ contains an admixture of the photino (the linear combination of bino and neutral wino whose superpartner is the photon), from eq. (6.22) it should then decay into photon + goldstino/gravitino with a width given by

$$\Gamma(\widetilde{N}_1 \to \gamma\widetilde{G}) \;=\; 2 \times 10^{-3}\, \kappa_{1\gamma} \left(\frac{m_{\widetilde{N}_1}}{100\ \mathrm{GeV}}\right)^5 \left(\frac{\sqrt{\langle F\rangle}}{100\ \mathrm{TeV}}\right)^{-4} \ \mathrm{eV}. \tag{8.8}$$

Here $\kappa_{1\gamma} \equiv |\mathbf{N}_{11}\cos\theta_W + \mathbf{N}_{12}\sin\theta_W|^2$ is the "photino content" of $\widetilde{N}_1$, in terms of the neutralino mixing matrix $\mathbf{N}_{ij}$ defined by eq. (7.47). We have normalized $m_{\widetilde{N}_1}$ and $\sqrt{\langle F\rangle}$ to (very roughly) minimum expected values in gauge-mediated models. This width is much smaller than for a typical flavor-unsuppressed weak interaction decay, but it is still large enough to allow $\widetilde{N}_1$ to decay before it has left a collider detector, if $\sqrt{\langle F\rangle}$ is less than a few thousand TeV in gauge-mediated models, or equivalently if $m_{3/2}$ is less than a keV or so when eq. (6.21) holds. In fact, from eq. (8.8), the mean decay length of an $\widetilde{N}_1$ with energy E in the lab frame is

$$d = 9.9 \times 10^{-3}\, \frac{1}{\kappa_{1\gamma}}\, (E^2/m^2_{\widetilde{N}_1} - 1)^{1/2} \left(\frac{m_{\widetilde{N}_1}}{100\ \mathrm{GeV}}\right)^{-5} \left(\frac{\sqrt{\langle F\rangle}}{100\ \mathrm{TeV}}\right)^{4} \ \mathrm{cm}, \tag{8.9}$$

which could be anywhere from sub-micron to multi-kilometer depending on the scale of supersymmetry breaking $\sqrt{\langle F\rangle}$. (In other models with a gravitino LSP which are not described by F-term breaking of global supersymmetry, including certain "no-scale" models,[113] the same formulas may be applied with $\langle F\rangle \to \sqrt{3} m_{3/2} M_{\rm P}$.)

Of course, $\widetilde{N}_1$ is not a pure photino, but contains also admixtures of the superpartner of the Z boson and the neutral Higgs scalars. So, one can also have[68] $\widetilde{N}_1 \to Z\widetilde{G}$, $h^0\widetilde{G}$, $A^0\widetilde{G}$, or $H^0\widetilde{G}$, with decay widths given in Ref.[69] Of these decays, the last two are unlikely to be kinematically allowed, and only the $\widetilde{N}_1 \to \gamma\widetilde{G}$ mode is guaranteed to be kinematically allowed for a gravitino LSP. Furthermore, even if they are open, the decays $\widetilde{N}_1 \to Z\widetilde{G}$ and $\widetilde{N}_1 \to h^0\widetilde{G}$ are subject to strong kinematic suppressions proportional to $(1 - m_Z^2/m^2_{\widetilde{N}_1})^4$ and $(1 - m^2_{h^0}/m^2_{\widetilde{N}_1})^4$, respectively, in view of eq. (6.22). Still, these decays may play an important role in phenomenology if $\sqrt{\langle F\rangle}$ is not too large, $\widetilde{N}_1$ has a sizeable zino or higgsino content, and $m_{\widetilde{N}_1}$ is significantly greater than m_Z or m_{h^0}.

A charged slepton makes another likely candidate for the NLSP. Actually, it is important to note that more than one slepton can act effectively as the NLSP, even though one of them is slightly lighter, if they are sufficiently degenerate in mass so

that each has no kinematically allowed decays except to the goldstino. In GMSB models, the squared masses obtained by $\widetilde{e}_R$, $\widetilde{\mu}_R$ and $\widetilde{\tau}_R$ are equal because of the flavor-blindness of the gauge couplings. However, this is not the whole story, because one must take into account mixing with $\widetilde{e}_L$, $\widetilde{\mu}_L$, and $\widetilde{\tau}_L$ and renormalization group running. These effects are very small for $\widetilde{e}_R$ and $\widetilde{\mu}_R$ because of the tiny electron and muon Yukawa couplings, so we can quite generally treat them as degenerate, unmixed mass eigenstates. In contrast, $\widetilde{\tau}_R$ usually has a quite significant mixing with $\widetilde{\tau}_L$, proportional to the tau Yukawa coupling. This means that the lighter stau mass eigenstate $\widetilde{\tau}_1$ is pushed lower in mass than $\widetilde{e}_R$ or $\widetilde{\mu}_R$, by an amount that depends most strongly on $\tan\beta$. If $\tan\beta$ is not too large then the stau mixing effect leaves the slepton mass eigenstates $\widetilde{e}_R$, $\widetilde{\mu}_R$, and $\widetilde{\tau}_1$ degenerate to within less than $m_\tau \approx 1.8$ GeV, so they act effectively as co-NLSPs. In particular, this means that even though the stau is slightly lighter, the three-body slepton decays $\widetilde{e}_R \to e\tau^\pm\widetilde{\tau}_1^\mp$ and $\widetilde{\mu}_R \to \mu\tau^\pm\widetilde{\tau}_1^\mp$ are not kinematically allowed; the only allowed decays for the three lightest sleptons are $\widetilde{e}_R \to e\widetilde{G}$ and $\widetilde{\mu}_R \to \mu\widetilde{G}$ and $\widetilde{\tau}_1 \to \tau\widetilde{G}$. This situation is called the "slepton co-NLSP" scenario.

For larger values of $\tan\beta$, the lighter stau eigenstate $\widetilde{\tau}_1$ is more than 1.8 GeV lighter than $\widetilde{e}_R$ and $\widetilde{\mu}_R$ and $\widetilde{N}_1$. This means that the decays $\widetilde{N}_1 \to \tau\widetilde{\tau}_1$ and $\widetilde{e}_R \to e\tau\widetilde{\tau}_1$ and $\widetilde{\mu}_R \to \mu\tau\widetilde{\tau}_1$ are open. Then $\widetilde{\tau}_1$ is the sole NLSP, with all other MSSM supersymmetric particles having kinematically allowed decays into it. This is called the "stau NLSP" scenario.

In any case, a slepton NLSP can decay like $\widetilde{\ell} \to \ell\widetilde{G}$ according to eq. (6.22), with a width and decay length just given by eqs. (8.8) and (8.9) with the replacements $\kappa_{1\gamma} \to 1$ and $m_{\widetilde{N}_1} \to m_{\widetilde{\ell}}$. So, just as for the neutralino NLSP case, the decay $\widetilde{\ell} \to \ell\widetilde{G}$ can be either fast or very slow, depending on the scale of supersymmetry breaking.

If $\sqrt{\langle F\rangle}$ is larger than roughly 10^3 TeV (or the gravitino is heavier than a keV or so), then the NLSP is so long-lived that it will usually escape a typical collider detector. If $\widetilde{N}_1$ is the NLSP, then, it might as well be the LSP from the point of view of collider physics. However, the decay of $\widetilde{N}_1$ into the gravitino is obviously still crucial for cosmology, since an unstable $\widetilde{N}_1$ is clearly not a good dark matter candidate while the gravitino LSP conceivably could be. On the other hand, if the NLSP is a long-lived charged slepton, then one can see its tracks (or possibly decay kinks) inside a collider detector.[68] The presence of a massive charged NLSP can be established by measuring its anomalously high ionization rate or its time-of-flight in the detector.

9 Experimental signals for supersymmetry

So far, the experimental study of supersymmetry has unfortunately been confined to setting limits. As we have already remarked in section 5.4, there can be indirect signals for supersymmetry from processes that are rare or forbidden in the Standard Model but can have contributions from loops involving virtual sparticles. These include $\mu \to e\gamma$, $b \to s\gamma$, neutral meson mixing, electric dipole moments for the neutron and the electron, etc. There are also virtual sparticle effects on Standard Model predictions

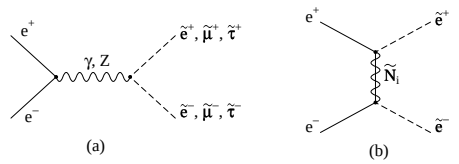

Figure 19: Diagrams contributing to slepton pair-production at e^+e^- colliders.

like R_b (the fraction of $b\overline{b}$ pairs in hadronic Z decays).[114] Extensions of the MSSM (GUT and otherwise) can quite easily predict proton decay and neutron-antineutron oscillations at low but observable rates, even if R-parity is exactly conserved. However, it would be quite difficult to ascribe a positive result for any of these processes to supersymmetry in an unambiguous way. There is no substitute for the direct detection of sparticles. In this section we will give an incomplete and entirely qualitative review of some of the possible signals for direct detection of supersymmetry. The reader is encouraged to consult Refs.[27,34,115] for recent reviews which cover the subject more systematically.

9.1 Signals at e^+e^- colliders

At e^+e^- colliders, sparticles (other than the gluino) can be pair-produced through tree-level processes:

$$e^+e^- \to \widetilde{C}_i^+\widetilde{C}_j^-, \quad \widetilde{N}_i\widetilde{N}_j, \quad \widetilde{\ell}\widetilde{\ell}, \quad \widetilde{\nu}\overline{\widetilde{\nu}}, \quad \widetilde{q}\overline{\widetilde{q}}. \tag{9.1}$$

with cross-sections determined just by the electroweak gauge couplings and the sparticle mixings. All of the processes in eq. (9.1) get contributions from the s-channel exchange of the Z boson and (for charged sparticle pairs) of the photon. In the cases of $\widetilde{C}_i^+\widetilde{C}_j^-$, $\widetilde{N}_i\widetilde{N}_j$, $\widetilde{e}_R\widetilde{e}_R$, $\widetilde{e}_L\widetilde{e}_L$ and $\widetilde{\nu}_e\widetilde{\nu}_e$ production, there are also t-channel contributions from the exchanges of a virtual sneutrino, selectron, neutralino, neutralino and chargino, respectively. The t-channel contributions are quite significant if the exchanged sparticle is not too heavy, and interference between the s- and t-channel contributions can be either destructive or constructive. For example, the production of wino-like $\widetilde{C}_1^+\widetilde{C}_1^-$ pairs typically suffers a destructive interference between the s-channel graphs with γ, Z exchange and the t-channel graphs with $\widetilde{\nu}_e$ exchange, if the sneutrinos are not too heavy. In the case of sleptons, the pair-production of smuons and staus proceeds only through the s-channel diagrams of Fig. 19a, while selectron production also has a contribution from the t-channel exchanges of the neutralinos, as shown in Fig. 19b. [We have drawn the neutralino line as if it were a pure gaugino, since the gaugino components of $\widetilde{N}_i$ are responsible for the coupling to electron-selectron.] For this reason, selectron production may be significantly larger than smuon or stau production at e^+e^- colliders. The important interactions for sparticle production processes are always of electroweak interaction strength, namely the ones shown in Figs. 9b,c and the ordinary gauge interactions. The cross sections are too complicated to be listed here, but can be found in Ref.[109]

The pair-produced sparticles will decay as discussed in section 8. If the LSP is the lightest neutralino, it will always escape the detector because it has no strong or electromagnetic interactions. Therefore every event will have two LSPs leaving the detector, so there will be at least $2m_{\widetilde{N}_1}$ of missing energy ($\not{E}$). For example, in the case of $\widetilde{C}_1^+\widetilde{C}_1^-$ production, the possible signals include a pair of acollinear leptons plus $\not{E}$, one lepton and a pair of jets plus $\not{E}$, and multiple jets plus $\not{E}$. The relative importance of these signals depends on the branching fraction of the chargino into the competing channels $\widetilde{C}_1 \to \ell\nu\widetilde{N}_1$ and $qq'\widetilde{N}_1$. In the case of slepton pair-production, the signal should be two energetic, acollinear, same-flavor leptons plus $\not{E}$. It is not difficult to construct the other possible signatures for sparticle pairs, which can become quite complicated for the heavier charginos, neutralinos and squarks.

At the CERN LEP e^+e^- collider, one has a reasonable possibility of seeing neutralino, chargino, charged slepton, sneutrino, or top-squark pairs. In the LEP1 runs at $\sqrt{s} = m_Z$, the measurement of the invisible decay width of the Z boson placed a lower bound on sneutrino masses of about 40 GeV, even though they can decay completely invisibly like $\widetilde{\nu} \to \nu\widetilde{N}_1$. Similarly, the contribution of $\widetilde{N}_1\widetilde{N}_1$ to the invisible width of the Z rules out a significant region of parameter space, with a lower bound on $m_{\widetilde{N}_1}$ which unfortunately depends strongly on the other parameters. Model-independent lower bounds have been set on the charged sparticle masses of roughly $m_Z/2$. At this writing, LEP2 upgrades at $\sqrt{s}$ = 130-140, 161, 172, 183, 189 GeV and beyond are continuing to raise the lower bounds on the lightest "visible" sparticles. It is worth noting that in all future e^+e^- collider searches, there will be a large background for the acollinear leptons plus $\not{E}$ and the lepton plus jets plus $\not{E}$ signals from W^+W^- production with one or both of the W bosons decaying leptonically. However, these and other Standard Model backgrounds can be kept under control with clever cuts. It should also be mentioned that LEP2 is conducting a promising search for the lightest Higgs boson of supersymmetry through $e^+e^- \to h^0Z$ or perhaps $e^+e^- \to h^0A^0$. However, observation of the Higgs at LEP2 would be only a powerful clue that we are on the right track in pursuing supersymmetry, and not a proof. Conversely, the non-observation of h^0 at LEP2 should not be construed as evidence against supersymmetry, in view of eqs. (7.42) and (7.43).

At a future linear e^+e^- collider with $\sqrt{s}$ = a few hundred GeV to 1.5 TeV, the processes in eq. (9.1) should be probed close to the kinematic limit, given sufficient integrated luminosity.[116] In the case of $\widetilde{\nu}\overline{\widetilde{\nu}}$ production, this assumes that some of the decays are visible, rather than just $\widetilde{\nu} \to \nu\widetilde{N}_1$. In the cases of the heavier sparticles, the cascade decays mentioned in the previous section will provide a rich set of signals to study. By making use of polarized beams and the relatively clean e^+e^- collider environment, one can disentangle the sparticle spectrum. For example, measuring the maximum and minimum energy endpoints of the leptons produced in $e^+e^- \to \widetilde{\ell}_R\overline{\widetilde{\ell}}_R$ with $\widetilde{\ell}_R \to \ell\widetilde{N}_1$, one can precisely determine both $m_{\widetilde{\ell}_R}$ and $m_{\widetilde{N}_1}$. By varying the polarization of the electron beam, one can control the W^+W^- background and simultaneously check that the contributions to the sparticle production cross-sections have the correct magnitude and vary in the right way. This will allow one to check the spin and the "handedness" of the produced squarks and sleptons. Similar precision

studies of chargino and neutralino production can also be performed. In general, a high-energy linear lepton collider will provide an excellent way of testing supersymmetric relations. It is also worth noting that searches for $e^+e^- \to h^0Z$, h^0A^0, H^0Z, H^0A^0 and H^+H^- should be able to definitively test the Higgs sector of the MSSM.

If the gravitino is the LSP as in gauge-mediated models, then one must take into account the possibilities mentioned in section 8.5. If the lightest neutralino is the NLSP and the decay $\widetilde{N}_1 \to \gamma\widetilde{G}$ occurs within the detector, then even the process $e^+e^- \to \widetilde{N}_1\widetilde{N}_1$ leads to a dramatic signal of two energetic photons plus missing energy.[67,68,69] There are significant backgrounds to the $\gamma\gamma\not{E}$ signal, but they are easily removed by cuts. Each of the other sparticle pair-production modes eq. (9.1) will lead to the same signals as in the neutralino LSP case, but now with two additional energetic photons which should make the experimentalists' tasks quite easy. If the decay length for $\widetilde{N}_1 \to \gamma\widetilde{G}$ is much larger than the size of a detector, then the signals revert back to those found in the neutralino LSP scenario. In an intermediate regime for the $\widetilde{N}_1 \to \gamma\widetilde{G}$ decay length, one may see events with one or both photons displaced from the event vertex by a macroscopic distance.

If the NLSP is a charged slepton $\widetilde{\ell}$, then $e^+e^- \to \widetilde{\ell}^+\widetilde{\ell}^-$ followed by prompt decays $\widetilde{\ell} \to \ell\widetilde{G}$ will yield two energetic same-flavor leptons in every event, and with a different energy distribution than the acollinear leptons that would follow from either $\widetilde{C}_1^+\widetilde{C}_1^-$ or $\widetilde{\ell}^+\widetilde{\ell}^-$ production in the neutralino LSP scenario. The W^+W^- background can be a problem here, but can be defeated with angular cuts at LEP2 or polarized beams at future e^+e^- colliders. Pair-production of non-NLSP sparticles will yield unmistakable signals which are the same as those found in the neutralino NLSP case but with two additional energetic leptons (not necessarily of the same flavor). A perhaps even more exciting possibility is that the NLSP is a slepton which decays very slowly.[68] If the slepton NLSP is so long-lived that it decays outside the detector, then slepton pair-production will lead to events featuring a pair of charged particle tracks with a high ionization rate which betrays their very large mass. If the sleptons decay within the detector, then one can look for kinks in the charged particle tracks, or a macroscopic impact parameter. The pair-production of any of the other heavy charged sparticles will also yield heavy charged particle tracks or decay kinks, plus leptons and/or jets, but no $\not{E}$ unless the decay chains happen to include neutrinos. It may also be possible to identify the presence of a heavy charged NLSP by measuring its anomalously long time-of-flight through the detector.

9.2 Signals at hadron colliders

At hadron colliders, the most important channels for sparticle production are typically expected to be

$$\widetilde{C}_i^+\widetilde{C}_j^-, \quad \widetilde{N}_i\widetilde{C}_j^\pm, \quad \widetilde{N}_i\widetilde{N}_j, \quad \text{and} \tag{9.2}$$

$$\widetilde{g}\widetilde{g}, \quad \widetilde{g}\widetilde{q}, \quad \widetilde{q}\overline{\widetilde{q}}. \tag{9.3}$$

At the Fermilab Tevatron $p\overline{p}$ collider with $\sqrt{s} = 2$ TeV, the chargino and neutralino production processes (through valence quark annihilation into virtual weak bosons)

tend to have the larger cross-sections, unless the squarks or gluino are rather light (less than 300 GeV or so). In a typical scenario where $\widetilde{C}_1$ and $\widetilde{N}_2$ are mostly $SU(2)_L$ gauginos and $\widetilde{N}_1$ is mostly bino, the largest production cross-sections in eq. (9.2) belong to the $\widetilde{C}_1\widetilde{C}_1$ and $\widetilde{N}_2\widetilde{C}_1$ channels, because they have significant couplings to W and γ, Z bosons, respectively. At the future CERN LHC pp collider with $\sqrt{s} \sim$ 14 TeV, the situation is typically reversed, with production of gluinos and squarks by gluon fusion and gluon-quark fusion usually dominating, unless the gluino and squarks are heavier than 1 TeV or so. At both colliders, one can also have associated production of a chargino or neutralino together with a squark or gluino, but the cross-sections for such processes are probably significantly lower than for the ones in eqs. (9.2) and (9.3). Slepton pair production may be rather small at the Tevatron, but might be observable there or at the LHC.[117] Cross-sections for sparticle production at hadron colliders can be found in Ref.[118]

The decays of the produced sparticles result in final states with two neutralino LSPs which escape the detector. The LSPs again carry away at least $2m_{\widetilde{N}_1}$ of missing energy, but at hadron colliders only the component of the missing energy which is manifest in momenta transverse to the colliding beams (denoted $\not{E}_T$) is observable. Therefore in general the observable signals for supersymmetry at hadron colliders are n leptons + m jets + $\not{E}_T$, where either n or m might be 0. There are important Standard Model backgrounds to many of these signals, especially from processes involving production of W and Z bosons which can decay to neutrinos, yielding $\not{E}_T$. Therefore it is important to identify specific signals for which the backgrounds can be reduced. Of course, this depends on which sparticles are being produced and how they are decaying. For example, the "classic" $\not{E}_T$ signal for supersymmetry at hadron colliders is events with jets and $\not{E}_T$ but no energetic isolated leptons. The latter requirement reduces backgrounds from Standard Model processes with leptonic W decays, and is obviously most effective if the relevant sparticle decays have sizeable branching fractions into channels with no leptons in the final state.

Another type of signal arises if the gluino decays with a significant branching fraction to hadrons plus a chargino, which can subsequently decay into a final state with a charged lepton, a neutrino, and $\widetilde{N}_1$. Since the gluino doesn't know anything about electric charge, the single charged lepton produced from each gluino decay can have either sign with equal probability. This means that gluino pair production will often lead to events with two leptons with the same charge (but possibly different flavors) plus jets and $\not{E}_T$. This signal can also arise from $\widetilde{q}\widetilde{q}$ and $\widetilde{q}\widetilde{g}$ production, e.g. if the squarks decay like $\widetilde{q} \to q\widetilde{g}$. This same-sign dilepton signal [119] has small physics backgrounds from the Standard Model both at the Tevatron and the LHC. The reason is that the largest background sources for isolated lepton pairs, namely W^+W^-, Drell-Yan and $t\bar{t}$ production, can only yield opposite-charge dileptons.

Despite the backgrounds just mentioned, opposite-charge dilepton signals, e.g. from slepton pair production with subsequent decays $\widetilde{\ell} \to \ell\widetilde{N}_1$, can give an observable signal especially at the LHC.

Another useful possibility is the trilepton signal,[120] which features three leptons plus $\not{E}_T$ and possibly jets. This can come about from $\widetilde{C}_1\widetilde{N}_2$ production followed by

the decays indicated in eq. (8.4), in which case one expects little hadronic activity in the event. It could also come from $\widetilde{g}\widetilde{g}$, $\widetilde{q}\widetilde{g}$, or $\widetilde{q}\widetilde{q}$ production, with one of the gluinos or squarks decaying through a $\widetilde{C}_1$ and the other through a $\widetilde{N}_2$. In that case, there will be jets from the decays, in addition to the three leptons and $\not{E}_T$. These signatures rely on the $\widetilde{N}_2$ having a significant branching fraction for the three-body decay to leptons in eq. (8.4). For this reason, the two-body decay modes in eq. (8.1) are sometimes called "spoiler" modes, since if they are kinematically allowed they can dominate, spoiling the trilepton signal. This is because if the $\widetilde{N}_2$ decay is through an on-shell h^0, then the final state will very likely include bottom-quark jets rather than isolated leptons, while if the decay is through an on-shell Z, then there can still be two leptons but there are Standard Model backgrounds with unfortunately similar kinematics from processes involving $Z \to \ell^+\ell^-$. Either way, the trilepton signal can be spoiled, but other leptons + jets + $\not{E}_T$ signals may be observable above Standard Model backgrounds, especially if bottom quark jets can be tagged with high efficiency.

The single lepton plus jets plus $\not{E}_T$ signal[121] has large Standard Model backgrounds from processes with $W \to \ell\nu$. However, it also can have a large rate from various superpartner production modes, and may still give the best signal at the LHC. One should also be aware of very interesting signals which can arise for particular ranges of parameters. For example, in a scenario studied in Ref.[115], the only two-body decay channel for the gluino is $\widetilde{g} \to b\tilde{b}_1$, with subsequent decays $\tilde{b}_1 \to b\widetilde{N}_2$ and $\widetilde{N}_2 \to \ell^+\ell^-\widetilde{N}_1$ or $\widetilde{N}_2 \to qq\widetilde{N}_1$. In that case, gluino pair production gives a spectacular signal of four bottom jets plus up to four leptons plus $\not{E}_T$. In general, production of relatively light $\tilde{t}_1$ and $\tilde{b}_1$ can give hadron collider signals rich in bottom jets, either through direct production or cascade decays.

If the gravitino is the LSP, these signals can be significantly modified. If the NLSP is a neutralino with a prompt decay $\widetilde{N}_1 \to \gamma\widetilde{G}$, then one expects events with two energetic, isolated photons plus $\not{E}_T$ from the escaping gravitinos, rather than just $\not{E}_T$. So at a hadron collider the signal is $\gamma\gamma + X + \not{E}_T$ where X is any collection of leptons plus jets. The Standard Model backgrounds relevant for such events are quite small. If the $\widetilde{N}_1$ decay length is long enough, then it may be measurable because the photons will not point back to the event vertex. If the $\widetilde{N}_1$ decay is outside of the detector, then one just has the usual leptons + jets + $\not{E}_T$ signals as discussed above in the neutralino LSP scenario.

In the case that the NLSP is a charged slepton, then the decay $\widetilde{\ell} \to \ell\widetilde{G}$ can provide two extra leptons in each event, compared to the signals with a neutralino LSP. If the $\tilde{\tau}_1$ is sufficiently lighter than the other charged sleptons $\tilde{e}_R$, $\tilde{\mu}_R$, and so is effectively the sole NLSP, then events will always have a pair of taus. If the slepton NLSP is long-lived, one can look for events with a pair of very heavy charged particle tracks or a long time-of-flight in the detector. Since slepton pair-production usually has a much smaller cross-section than the processes in eq. (9.2) and (9.3), this will typically be accompanied by leptons and/or jets from the same event vertex, which may be of crucial help in identifying candidate events. It is also quite possible that the decay length of $\widetilde{\ell} \to \ell\widetilde{G}$ is measurable within the detector, seen as a macroscopic kink in the charged particle track.

9.3 Dark matter detection

One of the major successes of supersymmetry with exact R-parity conservation is that an electrically neutral LSP can be a good candidate for the dark matter. There are three obvious candidates: the gravitino, the lightest sneutrino, and the lightest neutralino. If the gravitino is the LSP, as in gauge-mediated models, then relic gravitinos left over from the early universe would be essentially impossible to detect even if they can be arranged to have the right cosmological density today. The possibility of a sneutrino LSP making up the dark matter with a cosmologically interesting density has now been ruled out by direct searches.[122] The most attractive prospects for direct detection of supersymmetric dark matter, therefore, are based on the idea that the lightest neutralino $\widetilde{N}_1$ is the LSP, as happens quite naturally in the minimal supergravity models.

In the early universe, sparticles existed in thermal equilibrium with the ordinary Standard Model particles. As the universe cooled and expanded, the sparticles could no longer be produced and they all annihilated or decayed into $\widetilde{N}_1$. The remaining $\widetilde{N}_1$ can annihilate through processes $\widetilde{N}_1\widetilde{N}_1 \to f\overline{f}$ with t-channel exchange of squarks and sleptons or the s-channel exchange of Higgs scalars or a Z boson. Depending on the mass of $\widetilde{N}_1$, other processes like $\widetilde{N}_1\widetilde{N}_1 \to W^+W^-$, ZZ, Zh^0, h^0h^0 or even $W^\pm H^\mp$, ZA^0, h^0A^0, h^0H^0, H^0A^0, H^0H^0, A^0A^0, or H^+H^- could also have been important. Eventually, as the density of LSPs decreased, the annihilation rate became very small, and the $\widetilde{N}_1$ relic density is determined by this small rate and the subsequent dilution due to the expansion of the universe.

It is a remarkable coincidence that the predicted density of a bino-like (or perhaps higgsino-like) neutralino LSP obtained by doing these calculations carefully can be in the right range to make up a significant fraction of the critical density of the universe, and perhaps to explain the rotation curves of galaxies.[123] (A wino-like $\widetilde{N}_1$ would have only a tiny relic density, but given the gaugino mass hierarchy eq. (7.48), there is little motivation for such a thing anyway.) It is also necessary to require that the density of surviving LSPs not be too large, so that the universe could have reached its present size and age of at least 10^{10} years. This tends to put an upper limit on the LSP mass, but unfortunately it is difficult to make a general, parameter-independent bound out of this because if the masses are arranged just right, the LSP may happen to annihilate very efficiently through a resonance. If neutralino LSPs really make up the cold dark matter, then their mass density in our neighborhood ought to be at least about 0.1 GeV/cm^3 in order to explain the rotation curves of galaxies. In principle, they should be detectable through their weak interactions with ordinary matter, or by their ongoing annihilations.

The direct detection of $\widetilde{N}_1$ depends on their elastic scattering off of heavy nuclei in a detector. At a fundamental level, $\widetilde{N}_1$ can scatter off of a quark by virtual exchange of squarks, a Z boson, or Higgs scalars, or can scatter off of gluons through one-loop diagrams. The energy transferred to the nucleus in these collisions is typically of order tens of keV. However, there are important backgrounds from radioactivity and cosmic rays. The optimal detector material (e.g. germanium, silicon, or niobium) depends on the details of the $\widetilde{N}_1$-nucleus interaction. Present detectors are still not sensitive to

most regions of parameter space, but there is hope that this can change in the future.

Another, more indirect, way to detect neutralino LSPs is through ongoing annihilations. This can occur in regions of space where the density is greatly enhanced compared to our own neighborhood. This can occur if the LSPs lose energy by repeated scattering off of nuclei, eventually becoming concentrated inside massive astronomical bodies like the Earth or the Sun. In this case the annihilation of neutralino pairs into neutrinos is the most important process, since no other particles can escape from the center of the object where the annihilation is going on. In particular, muon neutrinos and antineutrinos from $\widetilde{N}_1\widetilde{N}_1 \to \nu_\mu\overline{\nu}_\mu$ will travel large distances, finally undergoing a charged-current interaction leading to energetic muons pointing back to the center of the Earth or Sun. There are also interesting possible signatures from neutralino LSP annihilation in the galactic halo which might produce detectable quantities of high-energy photons, positrons, and antiprotons.[123]

10 Some miscellaneous variations

In this section we will briefly consider a few variations on the simple picture of the MSSM that has been outlined above. First, we will consider the possibility of R-parity violation in section 10.1. Another obvious way to extend the MSSM is to introduce new chiral supermultiplets, corresponding to scalars and fermions that are all sufficiently heavy to have avoided discovery so far. In general, this requires that the new chiral supermultiplets must form a real representation of the Standard Model gauge group. The simplest such possibility is that the new particles live in just one gauge-singlet chiral supermultiplet; this possibility is discussed in section 10.2. One can also extend the MSSM by introducing new gauge interactions which are spontaneously broken at very high energies. The possibilities here include GUT models like $SU(5)$ and $SO(10)$ and E_6 which unify the Standard Model gauge interactions, with important implications for rare processes like proton decay. Superstring models also quite generically imply that the Standard Model gauge group should be extended at high energies. There is a vast literature on these possibilities, but we will concentrate instead on the implications of just adding a single additional abelian factor to the gauge group, in section 10.3.

10.1 Models with R-parity violation.

So far we have assumed that R-parity (or equivalently matter parity) is an exact symmetry of the MSSM. This assumption precludes renormalizable proton decay and predicts that the LSP should be stable, but despite these virtues R-parity is not inevitable. Because of the threat of proton decay, we expect that if R-parity is violated, then in the renormalizable lagrangian either B-violating or L-violating couplings are allowed, but not both, as explained in section 5.2.

One proposal is that matter parity can be replaced by an alternative discrete symmetry which still manages to forbid proton decay at the level of the renormalizable lagrangian. The possibilities have been cataloged in Ref.[124], where it was found that

provided no new particles are to be added to the MSSM, that the discrete symmetry is family-independent, and that it can be defined at the level of the superpotential, there is only one other candidate discrete symmetry besides matter parity. That other possibility is a Z_3 discrete symmetry [124] which was originally called "baryon parity", but is more appropriately referred to as "baryon triality". The baryon triality of any particle with baryon number B and weak hypercharge Y is defined to be

$$Z_3^{\rm B} = \exp\left(\frac{2\pi i}{3}[{\rm B} - 2Y]\right). \qquad (10.1)$$

It is easy to check that this is always a cube root of unity for the MSSM particles, since B$-2Y$ is always an integer. The symmetry principle to be enforced is that the product of the baryon trialities of the particles in any term in the lagrangian (or superpotential) must be 1. This symmetry conserves baryon number at the renormalizable level while allowing lepton number violation; in other words, it allows the superpotential terms in eq. (5.7) but forbids those in eq. (5.8). In fact, baryon triality conservation has the remarkable property that it absolutely forbids proton decay.[125] The reason for this is simply that baryon triality requires that B can only be violated in multiples of 3 units (even in nonrenormalizable interactions), while any kind of proton decay would have to violate B by 1 unit. So it is eminently falsifiable. Similarly, baryon triality conservation predicts that experimental searches for neutron-antineutron oscillations will be negative, since they would violate baryon number by 2 units. However, baryon triality conservation does allow the LSP to decay. If one adds some new chiral supermultiplets to the MSSM (corresponding to particles which are presumably very heavy), one can concoct a variety of new candidate discrete symmetries besides matter parity and baryon triality. Some of these will allow B violation in the superpotential, while forbidding the lepton number violating superpotential terms in eq. (5.7).

Another idea is that matter parity is an exact symmetry of the underlying superpotential, but it is spontaneously broken by the VEV of a scalar with $P_R = -1$. One possibility is that an MSSM sneutrino gets a VEV,[126] since sneutrinos are scalars carrying L=1. However, there are strong bounds [127] on $SU(2)_L$-doublet sneutrino VEVs $\langle\tilde{\nu}\rangle \ll m_Z$ coming from the requirement that the corresponding neutrinos do not have large masses. It is somewhat difficult to understand why such a small VEV should occur, since the scalar potential which produces it must include soft sneutrino (mass)^2 terms of order $m_{\rm soft}^2$. One can get around this by instead introducing a new gauge-singlet chiral supermultiplet with L$=-1$. The scalar component can get a large VEV, which can induce L-violating terms (and in general B-violating terms also) in the low-energy effective superpotential of the MSSM.[127]

In any case, if R-parity is violated, then the LSP will decay, completely altering the signals for supersymmetry. The type of signal to look for depends on the form of R-parity violation. If there are L-violating terms of the type λ and/or λ' as in eq. (5.7), then the final states from $\widetilde{N}_1$ decay will always involve a charged lepton or a neutrino plus either a pair of additional charged leptons or a pair of jets. Two such decays are shown in Fig. 20a,b, but there are others. Signals for supersymmetry will therefore always include leptons or large missing energy, or both. On the other hand, if terms of the form λ'' in eq. (5.8) are present instead, then there are baryon-number

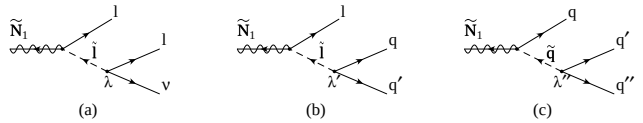

Figure 20: Decays of the $\widetilde{N}_1$ LSP in models with R-parity violation [see eqs. (5.7) and (5.8)].

violating decays $\widetilde{N}_1 \to qq'q''$ from graphs like the one shown in Fig. 20c. In that case, supersymmetric events will always have lots of hadronic activity, and will only have missing energy signatures when the other parts of the decay chains happen to include neutrinos. This could make the discovery and study of supersymmetry very difficult. There are other possibilities, too, because if R-parity is violated, then the decaying LSP need not be $\widetilde{N}_1$, and sparticles which are not the LSP can in principle decay directly to Standard Models quarks and leptons. If λ' is non-zero, then squarks can be produced as resonances at the $e^{\pm}p$ collider at HERA. A complete survey of the possibilities would be far too complicated to present here.

10.2 The next-to-minimal supersymmetric standard model

The simplest possible extension of the particle content of the MSSM is to add a new gauge-singlet chiral supermultiplet. The resulting model is often called the next-to-minimal supersymmetric standard model (NMSSM).[128] The most general possible superpotential for this model is given by

$$W_{\rm NMSSM} = \frac{1}{6}kS^3 + \frac{1}{2}\mu_S S^2 + \lambda S H_u H_d + W_{\rm MSSM}, \qquad (10.2)$$

where S stands for both the new chiral supermultiplet and its scalar component. (There could also be a term linear in S in $W_{\rm NMSSM}$, but this can always be removed by redefining S by a constant shift.)

One of the virtues of the NMSSM is that it can provide a solution to the μ problem mentioned in sections 5.1 and 7.2. To understand this, suppose we set $\mu_S = \mu = 0$ so that there are no mass terms or dimensionful parameters in the superpotential at all. Then an effective μ-term for $H_u H_d$ will still arise from the third term in eq. (10.2) if S gets a VEV, with $\mu = \lambda\langle S\rangle$. The absence of dimensionful terms in $W_{\rm NMSSM}$ can be enforced by introducing a new symmetry (in various different ways). The soft terms in the lagrangian give a contribution to the scalar potential which can be written as

$$V_{\rm soft}^{\rm NMSSM} = (\frac{1}{6}a_k S^3 + a_\lambda S H_u H_d + {\rm c.c.}) + m_S^2|S|^2 + V_{\rm soft}^{\rm MSSM}, \qquad (10.3)$$

where a_k and a_λ have dimensions of mass. One may now set $b = 0$ in $V_{\rm soft}^{\rm MSSM}$, because an effective value for b will be generated, equal to $a_\lambda\langle S\rangle$. If the new parameters k, λ, a_k and a_λ are chosen correctly, then phenomenologically acceptable VEVs will be induced for S, H_u^0, and H_d^0. A correct treatment of this requires the inclusion of one-loop radiative corrections. But the important point is that the scale of the VEV

$\langle S \rangle$, and therefore the effective value of μ, is then determined by the soft terms of order $m_{\rm soft}$, instead of being a free parameter which is conceptually independent of supersymmetry breaking.

The NMSSM contains, besides the particles of the MSSM, a real $P_R = +1$ scalar, a real $P_R = +1$ pseudoscalar, and a $P_R = -1$ Weyl fermion "singlino". These fields have no gauge couplings of their own, so they can only interact with Standard Model particles by mixing with the neutral MSSM fields with the same spin and charge. The real scalar mixes with the MSSM particles h^0 and H^0, and the pseudo-scalar mixes with A^0. One of the effects of replacing the μ term by the dynamical field S is to raise the upper bound on the lightest Higgs mass, for a given set of the other parameters in the theory. However, the bound in eq. (7.43) is still respected in the NMSSM (and any other perturbative extension of the MSSM), provided only that the sparticles that contribute in loops to the Higgs mass are lighter than 1 TeV or so. The odd R-parity singlino mixes with the four MSSM neutralinos, so there are really five neutralinos now. In many regions of parameter space, mixing effects involving the singlet fields are small, and they essentially just decouple. In that case, the phenomenology of the NMSSM is nearly indistinguishable from that of the MSSM. However, if any of the five NMSSM neutralinos (and especially the LSP) has a large mixing between the singlino and the usual gauginos and higgsinos, then the signatures for sparticles can be altered in important ways.[129]

10.3 Extra D-term contributions to scalar masses

Another way to generalize the MSSM is to include additional gauge interactions. The simplest gauge extension of the MSSM introduces just an additional abelian gauge symmetry, which we can call $U(1)_X$. As long as $U(1)_X$ is broken at a very high mass scale, then the corresponding vector gauge boson and gaugino fermion will be very heavy and will decouple from physics at the TeV scale and below. If so, one might suppose that all effects following from the existence of $U(1)_X$ will be completely negligible for collider experiments in the foreseeable future. However, this is not necessarily so, because as long as the MSSM fields carry $U(1)_X$ charges, the breaking of $U(1)_X$ at a very high energy scale can leave its imprint on the soft terms of the MSSM.[130]

To see how this works, let us consider the scalar potential for a model in which $U(1)_X$ is broken. Suppose that the MSSM scalar fields, denoted generically by ϕ_i, carry $U(1)_X$ charges x_i. In order to break $U(1)_X$, we also introduce a pair of chiral supermultiplets with $U(1)_X$ charges ± 1, denoted S_+ and S_-. These fields are singlets under the Standard Model gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$, so that when they get VEVs, they will just break $U(1)_X$. An obvious guess for the superpotential containing S_+ and S_- is $W = M S_+ S_-$, where M is a supersymmetric mass. However, unless M vanishes or is very small, it will yield positive-semidefinite quadratic terms in the scalar potential of the form $V = |M|^2(|S_+|^2 + |S_-|^2)$ which will force the minimum to be at $S_+ = S_- = 0$. Since we want S_+ and S_- to obtain VEVs, this is unacceptable. Therefore we assume that M is 0 (or very small) and that the leading

contribution to the superpotential comes instead from a nonrenormalizable term, say:

$$W = \frac{\lambda}{2M_{\rm P}} S_+^2 S_-^2 . \tag{10.4}$$

(Non-renormalizable terms in the superpotential obey the same rules as we found before; in particular they must be analytic functions of the chiral superfields. See the Appendix for more details on non-renormalizable lagrangians in supersymmetric theories.) The equations of motion for the auxiliary fields are then $F^*_{S_+} = -\partial W/\partial S_+ = -(\lambda/M_{\rm P}) S_+ S_-^2$ and $F^*_{S_-} = -\partial W/\partial S_- = -(\lambda/M_{\rm P}) S_- S_+^2$, and the corresponding contribution to the scalar potential is

$$V_F \;=\; |F_{S_+}|^2 + |F_{S_-}|^2 + \ldots \;=\; \frac{|\lambda|^2}{M_{\rm P}^2}\Big(|S_+|^4 |S_-|^2 + |S_+|^2 |S_-|^4\Big) + \ldots . \tag{10.5}$$

Here the ellipses represent other terms that are higher order in $1/M_{\rm P}$ (see Appendix), which we can safely ignore. In addition, there are soft terms which must be taken into account:

$$V_{\rm soft} = m_+^2 |S_+|^2 + m_-^2 |S_-|^2 - \left(\frac{a}{2M_{\rm P}} S_+^2 S_-^2 + \text{c.c.}\right) . \tag{10.6}$$

The terms with m_+^2 and m_-^2 are soft masses for S_+ and S_-. We can assume that they come from a minimal supergravity framework at the Planck scale, but in general they will be renormalized differently, due to different interactions for S_+ and S_- which we have not bothered to write down in eq. (10.4) because they involve fields that will not get VEVs. The last term is a "soft" term exactly analogous to the ones appearing in the second line of eq. (4.1), with a of order $m_{\rm soft}$. The coupling $a/2M_{\rm P}$ is actually dimensionless, but should be treated as soft because of its origin and its tiny magnitude. Such terms arise from the supergravity lagrangian in an exactly analogous way to the usual soft terms. Usually one can just ignore them, but this one plays a crucial role in the gauge symmetry breaking mechanism. The scalar potential for terms containing S_+ and S_- is now:

$$V = \frac{1}{2} g_X^2 \Big(|S_+|^2 - |S_-|^2 + \sum_i x_i |\phi_i|^2\Big)^2 + V_F + V_{\rm soft} . \tag{10.7}$$

The first term is the square of the $U(1)_X$ D-term [see eqs. (3.74) and (3.75)], and g_X is the $U(1)_X$ gauge coupling. The scalar potential eq. (10.7) has a nearly D-flat direction, because the D-term part vanishes for $\phi_i = 0$ and any $|S_+| = |S_-|$. Therefore, if $m_+^2 + m_-^2 < 0$, the point $S_+ = S_- = 0$ will be destabilized and S_+ and S_- can obtain large VEVs. Without loss of generality, we can take a and λ to both be real and positive for purposes of minimizing the scalar potential. As long as $a^2 - 8\lambda^2(m_+^2 + m_-^2) > 0$, the global minimum of the potential occurs for

$$\langle S_+ \rangle^2 \approx \langle S_- \rangle^2 \approx \frac{a M_{\rm P}}{6\lambda^2}\Big[1 + \sqrt{1 - 6\lambda^2(m_+^2 + m_-^2)/a^2}\Big] \tag{10.8}$$

(with $\langle\phi_i\rangle = 0$), so $\langle S_+\rangle \approx \langle S_-\rangle \sim \mathcal{O}(\sqrt{m_{\rm soft} M_{\rm P}})$. The V_F contribution is what stabilizes the scalar potential at very large field strengths. The VEVs of S_+ and S_- will be much larger than 1 TeV as long as a is not too small. Therefore the $U(1)_X$ gauge boson and gaugino can be very heavy, with masses of order $g_X\langle S_\pm\rangle$, and play no role in collider physics.

However, there is also a small deviation from $\langle S_+\rangle = \langle S_-\rangle$, as long as $m_+^2 \neq m_-^2$. At the minimum of the potential with $\partial V/\partial S_+ = \partial V/\partial S_- = 0$, the leading order difference in the VEVs is given by

$$\langle S_+\rangle^2 - \langle S_-\rangle^2 = -\frac{1}{g_X}\langle D_X\rangle \approx \frac{1}{2g_X^2}(m_-^2 - m_+^2) \tag{10.9}$$

assuming that $\langle S_+\rangle$ and $\langle S_-\rangle$ are much larger than their difference. After integrating out the S_+ and S_- by replacing them with their equations of motion expanded around the minimum of the potential, one finds that the MSSM scalars ϕ_i each receive a correction to their (mass)^2 given by

$$\Delta m_i^2 = -x_i g_X \langle D_X\rangle\,, \tag{10.10}$$

in addition to the usual soft terms derived from the minimal supergravity boundary conditions and RG equations. The D-term corrections eq. (10.10) can be roughly of the order of $m_{\rm soft}^2$ at most, since they are all proportional to $m_-^2 - m_+^2$. Note that the result eq. (10.10) does not actually depend on our choice of the nonrenormalizable superpotential, as long as it produces the required symmetry breaking with large VEVs; this is a general feature. In a sense, the soft supersymmetry-breaking terms m_+^2 and m_-^2 have been recycled into a non-zero D-term for $U(1)_X$, which then leaves its "fingerprint" on the spectrum of MSSM scalar masses. The most important feature of the correction eq. (10.10) is that each MSSM scalar (mass)^2 obtains a correction just proportional to its charge x_i under the spontaneously broken gauge group, with a universal factor $g_X\langle D_X\rangle$. From the point of view of TeV scale physics, the quantity $g_X\langle D_X\rangle$ can simply be taken to parameterize our ignorance of how $U(1)_X$ got broken. Typically, the charges x_i are rational numbers and do not all have the same sign, so that a particular candidate $U(1)_X$ can leave a quite distinctive pattern of mass splittings on the squark and slepton spectrum.

The additional gauge symmetry $U(1)_X$ in the above discussion can stand alone, or may perhaps be embedded in a larger non-abelian gauge group. If the gauge group for the underlying theory at the Planck scale contains more than one new $U(1)$ factor, then each such factor can make a contribution exactly analogous to eq. (10.10). Additional $U(1)$ gauge groups are quite common in superstring models, so from that point of view one may be optimistic about the existence of the corresponding D-term corrections. Once one merely assumes the existence of additional $U(1)$ gauge groups at very high energies, it is quite unnatural to assume that such D-term contributions to the MSSM scalar masses should vanish, unless there is an *exact* symmetry which will enforce $m_+^2 = m_-^2$. The only question is whether or not the magnitude of the D-term contributions is significant compared to the usual minimal supergravity and RG contributions; it may very well not be. Note also that as long as the charges x_i

are family-independent, then from eq. (10.10) the squarks and sleptons with the same electroweak quantum numbers remain degenerate, maintaining the natural suppression of FCNC effects. So it is quite possible that efforts to understand the sparticle spectrum of the MSSM will need to take into account the possibility of D-terms from additional gauge groups.

11 Concluding remarks

In this primer, I have attempted to convey some of the more essential features of supersymmetry as it is known so far. One of the most amazing qualities of supersymmetry is that so much is known about it already, despite the present lack of direct experimental data. Even the terms and stakes of many of the important outstanding questions, especially the paramount issue "How is supersymmetry broken?", are already rather clear. That this can be so is a testament to the unreasonably predictive quality of the symmetry itself.

We have seen that sensible and economical models for supersymmetry at the TeV scale can be used as convenient templates for experimental searches. As summarized in section 7.6, two of the simplest possibilities are the "minimal supergravity" scenario with new parameters m_0^2, $m_{1/2}$, A_0, $\tan\beta$ and Arg(μ), and the "gauge-mediated" scenario with new parameters Λ, $M_{\rm mess}$, N_5, $\langle F\rangle$, $\tan\beta$, and Arg(μ). However, one should not lose sight of the fact that the only indispensable idea of supersymmetry is simply that of a symmetry between fermions and bosons. Nature may or may not be kind enough to realize this beautiful idea within one of the specific frameworks that have already been explored well by theorists.

The experimental verification of supersymmetry will not be an end, but rather a revolution in high energy physics. It seems likely to present us with questions and challenges which we can only guess at presently. The measurement of sparticle masses, production cross-sections, and decays modes will rule out some models for supersymmetry breaking and lend credence to others. We will be able to test the principle of R-parity conservation, the idea that supersymmetry has something to do with the dark matter, and possibly make connections to other aspects of cosmology including baryogenesis and inflation. Other fundamental questions, like the origin of the μ parameter and the rather peculiar hierarchical structure of the Yukawa couplings may be brought into sharper focus with the discovery of the MSSM spectrum. Understanding the precise connection of supersymmetry to the electroweak scale will surely open the window to even deeper levels of fundamental physics.

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Appendix: Nonrenormalizable supersymmetric lagrangians

In section 3, we discussed only renormalizable supersymmetric lagrangians. However, like all known theories that include general relativity, supergravity is nonrenormalizable as a quantum field theory. It is therefore clear that nonrenormalizable interactions must be present in any low-energy effective description of the MSSM. Fortunately, these can be neglected for most phenomenological purposes, because nonrenormalizable interactions have couplings of negative mass dimension, proportional to powers of $1/M_{\rm P}$ (or perhaps $1/\Lambda_{\rm UV}$, where $\Lambda_{\rm UV}$ is some other cutoff scale associated with new physics). This means that their effects at ordinary energy scales E accessible to experiment are typically suppressed by powers of $E/M_{\rm P}$ (or by powers of $E/\Lambda_{\rm UV}$). For energies $E \lesssim 1$ TeV, the effects of nonrenormalizable interactions are therefore usually too small to be interesting.

Still, there are several reasons why one might be interested in nonrenormalizable contributions to supersymmetric lagrangians. First, some very rare processes (like proton decay) can only be described using an effective MSSM lagrangian which includes nonrenormalizable terms. Second, one may be interested in understanding physics at very high energy scales where the suppression associated with nonrenormalizable terms is not enough to stop them from being important. For example, this could be the case in the study of the very early universe, or in understanding how additional gauge symmetries get broken. Third, the nonrenormalizable interactions may play a crucial role in understanding how supersymmetry breaking is transmitted to the MSSM. Finally, it is sometimes useful to treat strongly-coupled supersymmetric gauge theories using nonrenormalizable effective lagrangians, in the same way that chiral effective lagrangians are used to study hadron physics in QCD. Unfortunately, we will not be able to treat these rather complicated subjects in any sort of systematic way. Instead, we will merely sketch for the reader a few of the key elements that go into defining a nonrenormalizable supersymmetric lagrangian, so that they may hopefully seem slightly less mysterious when encountered in other works. More detailed treatments may be found in Refs.[17,20]

Let us consider a supersymmetric theory containing gauge and chiral supermultiplets whose lagrangian may contain terms that are nonrenormalizable. It turns out that the part of the lagrangian containing terms up to two spacetime derivatives is completely determined by specifying three independent functions of the scalar fields (or equivalently,[†] of the chiral superfields). They are:

- The superpotential $W(\phi_i)$, which we have already encountered in the case of renormalizable supersymmetric lagrangians. It must be an analytic function of the superfields treated as complex variables; in other words it depends only on the ϕ_i and not on the ϕ^{*i}. It has dimensions of $(\text{mass})^3$.

[†] The reader will lose nothing here by considering them as functions of the scalar fields; however, in a more sophisticated treatment some value would be lost.

- The *Kähler potential* $K(\phi_i, \phi^{*i})$. Unlike the superpotential, the Kähler potential is a function of both ϕ_i and ϕ^{*i}. It is real, and has dimensions of $(\text{mass})^2$. In the special case of renormalizable theories, we did not have to discuss the Kähler potential explicitly, because at tree-level there is only one possibility for it: $K = \phi^{i*}\phi_i$ (with the index i summed over as usual).

- The *gauge kinetic function* $f_{ab}(\phi_i)$. Like the superpotential, this is an analytic function of the ϕ_i treated as complex variables. It is dimensionless and symmetric under interchange of its two indices a, b, which run over the adjoint representations of the gauge groups of the model. In the special case of renormalizable supersymmetric lagrangians, it is just a constant (independent of the ϕ_i), and is equal to the identity matrix divided by the gauge coupling squared: $f_{ab} = \delta_{ab}/g_a^2$. More generally, it also determines the nonrenormalizable couplings of the gauge supermultiplets.

The whole lagrangian with up to two derivatives can now be written down in terms of these functions. This is a non-trivial consequence of supersymmetry, because many different individual couplings in the lagrangian are determined by the same three functions. This applies not only to theories with an ultraviolet cutoff like supergravity, but also to effective theories where one has integrated out ultraviolet degrees of freedom.

For example, in supergravity models the part of the scalar potential which does not depend on the gauge kinetic function can be found as follows. First, one may define the real, dimensionless "Kähler function":

$$G = \frac{K}{M_{\rm P}^2} + \ln\frac{W}{M_{\rm P}^3} + \ln\frac{W^*}{M_{\rm P}^3}. \tag{A.1}$$

(Just to maximize the confusion, G is also sometimes referred to as the Kähler potential. Also, many authors work in units with $M_{\rm P} = 1$, which simplifies the expressions but can slightly obscure the correspondence with the global superymmetry limit of large $M_{\rm P}$.) From G, one can construct its derivatives with respect to the scalar fields and their complex conjugates: $G^i = \delta G/\delta\phi_i$; $G_i = \delta G/\delta\phi^{*i}$; and $G_i^j = \delta^2 G/\delta\phi^{*i}\delta\phi_j$. Note that G_i^j really only depends on K. So using the same convention in which raised (lowered) indices i correspond to derivatives with respect to ϕ_i (ϕ^{*i}), we have $G_i^j = K_i^j/M_{\rm P}^2$, which is sometimes called the Kähler metric. The inverse of this matrix is denoted $(G^{-1})_i^j$, or equivalently $M_{\rm P}^2(K^{-1})_i^j$, so that $(G^{-1})_i^k G_k^j = (G^{-1})_k^j G_i^k = \delta_i^j$. In terms of these objects, the direct generalization of the F-term contribution to the scalar potential in ordinary renormalizable global supersymmetry turns out to be, after a complicated derivation: [70,71]

$$V = M_{\rm P}^4\, e^G \Big[G^i (G^{-1})_i^j G_j - 3\Big] \tag{A.2}$$

in supergravity. It can be rewritten in a slightly less compact form:

$$V = e^{K/M_{\rm P}^2}\left[(K^{-1})_j^i \Big(W^i + \frac{1}{M_{\rm P}^2} W K^i\Big)\Big(W_j^* + \frac{1}{M_{\rm P}^2} W^* K_j\Big) - \frac{3}{M_{\rm P}^2} W W^*\right] \tag{A.3}$$

where $K^i = \delta K/\delta\phi_i$ and $K_j = \delta K/\delta\phi^{*j}$. The order parameters for supersymmetry breaking (analogous to the auxiliary fields in the renormalizable, global supersymmetry case) turn out to be

$$F_i \;=\; -M_{\rm P}^2\, e^{G/2}\,(G^{-1})^j_i G_j \;=\; -e^{K/2M_{\rm P}^2}\,(K^{-1})^j_i\Big(W^*_j + \frac{1}{M_{\rm P}^2}W^* K_j\Big) \qquad \text{(A.4)}$$

in supergravity. In other words, local supersymmetry will be broken if one or more of the F_i obtain a VEV. The gravitino then absorbs the would-be goldstino and obtains a mass given by

$$m_{3/2}^2 = \frac{1}{3M_{\rm P}^2}\langle K^i_j F_i F^{*j}\rangle. \qquad \text{(A.5)}$$

Now if one assumes a "minimal" Kähler potential $K = \phi^{*i}\phi_i$, then $K^j_i = (K^{-1})^j_i = \delta^j_i$, so that expanding eqs. (A.3) and (A.4) to lowest order in $1/M_{\rm P}$ just reproduces the results $F_i = -W^*_i$ and $V = W^i W^*_i$ which were found in section 3.2 for renormalizable global supersymmetric theories [see eqs. (3.45)-(3.47)]. Equation (A.5) also reproduces the expression for the gravitino mass that was quoted in eq. (6.21).

The scalar potential eq. (A.2) does not yet include contributions from gauge interactions. The D-term contributions to the scalar potential are given by

$$V = \frac{1}{2}{\rm Re}\, f^{-1}_{ab}\,\widehat{D}^a\widehat{D}^b; \qquad \widehat{D}^a = -K^i(T^a)^j_i\phi_j, \qquad \text{(A.6)}$$

where ${\rm Re}\, f^{-1}_{ab}$ is the inverse of the real part of the gauge kinetic function matrix. In the case that $f_{ab} = \delta_{ab}/g_a^2$ and $K^i = \phi^{*i}$, this just reproduces the result of section 3.4 for the renormalizable global supersymmetry scalar potential, with $\widehat{D}^a = D^a/g^a$ being the D-term order parameter for supersymmetry breaking. If supersymmetry breaking takes place through F-term breaking, then it is often not necessary to include the supergravity effects on the D-terms explicitly. There are also many contributions to the lagrangian other than the scalar potential which depend on the three functions W, K and f_{ab}, which can be found in Ref.[71]

It should be noted that unlike in the case of global supersymmetry, the scalar potential in supergravity is *not* necessarily non-negative, because of the -3 term in eq. (A.2). This means that in principle, one can have supersymmetry breaking with a positive, negative, or zero vacuum energy. The last option might seem to be preferred phenomenologically by the absence of a cosmological constant, although it is not clear why the terms in the scalar potential should conspire to have $\langle V\rangle = 0$ at the minimum. Furthermore, it is not at all clear that $\langle V\rangle = 0$ really corresponds to the requirement of a vanishing observable, quantum-corrected cosmological constant.[131] In any case, with $\langle V\rangle = 0$ imposed as a constraint,[‡] eqs. (A.3)-(A.5) tell us that $\langle K^i_j F_i F^{*j}\rangle = 3M_{\rm P}^4 e^{\langle G\rangle} = 3e^{\langle K\rangle/M_{\rm P}^2}|\langle W\rangle|^2/M_{\rm P}^2$, and an equivalent formula for the gravitino mass is therefore $m_{3/2} = e^{\langle G\rangle/2}M_{\rm P}$.

[‡]We do this only to follow a popular example; as just noted we cannot endorse this imposition.

An instructive special case arises if we assume a "minimal" Kähler potential and divide the fields ϕ_i into a visible sector including the MSSM fields φ_i and a hidden sector containing a field X which breaks supersymmetry for us (and other fields that we need not treat explicitly). In other words, suppose that the superpotential and the Kähler potential have the form

$$W \;=\; W_{\rm vis}(\varphi_i) + W_{\rm hid}(X); \qquad \text{(A.7)}$$
$$K \;=\; \varphi^{*i}\varphi_i + X^*X. \qquad \text{(A.8)}$$

Now let us further assume that the dynamics of the hidden sector fields gives rise to non-zero VEVs

$$\langle X\rangle = xM_{\rm P}; \qquad \langle W_{\rm hid}\rangle = wM_{\rm P}^2; \qquad \langle \delta W_{\rm hid}/\delta X\rangle = w'M_{\rm P}. \qquad \text{(A.9)}$$

which defines a dimensionless quantity x and w, w' with dimensions of (mass). Requiring $\langle V\rangle = 0$ yields $|w' + x^*w|^2 = 3|w|^2$, and

$$m_{3/2} = \frac{|\langle F_X\rangle|}{\sqrt{3}M_{\rm P}} = e^{|x|^2/2}|w|. \qquad \text{(A.10)}$$

Now we suppose that it is valid to expand the scalar potential in powers of the dimensionless quantities $w/M_{\rm P}$, $w'/M_{\rm P}$, $\varphi_i/M_{\rm P}$, etc., keeping only terms that depend on the visible sector fields φ_i. It is not a difficult exercise to show that in leading order the result is:

$$\begin{aligned} V \;&=\; (W^*_{\rm vis})_i(W_{\rm vis})^i + m_{3/2}^2\varphi^{*i}\varphi_i \\ &\quad + e^{|x|^2/2}\Big[w^*\varphi_i(W_{\rm vis})^i \,+\, (x^*w'^* + |x|^2w^* - 3w^*)W_{\rm vis} + {\rm c.c.}\Big]. \end{aligned} \qquad \text{(A.11)}$$

A tricky point here is that we have rescaled the visible sector superpotential $W_{\rm vis} \to e^{-|x|^2/2}W_{\rm vis}$ everywhere, in order that the first term in eq. (A.11) is the usual, properly normalized, F-term contribution in global supersymmetry. The next term is a universal soft scalar (mass)^2 of the form eq. (6.28) with

$$m_0^2 \;=\; \frac{|\langle F_X\rangle|^2}{3M_{\rm P}^2} \;=\; m_{3/2}^2. \qquad \text{(A.12)}$$

The second line of eq. (A.11) just yields soft (scalar)^3 and (scalar)^2 analytic couplings of the form eqs. (6.29) and (6.30), with

$$A_0 \;=\; -\frac{\langle F_X\rangle}{M_{\rm P}}x^*; \qquad B_0 \;=\; \frac{\langle F_X\rangle}{M_{\rm P}}\Big(-x^* + \frac{1}{x + w'^*/w^*}\Big) \qquad \text{(A.13)}$$

since $\varphi_i(W_{\rm vis})^i$ is equal to $3W_{\rm vis}$ for the cubic part of $W_{\rm vis}$, and to $2W_{\rm vis}$ for the quadratic part. [If the complex phases of x, w, w' can be rotated away, then eq. (A.13) implies $B_0 = A_0 - m_{3/2}$, but there are many effects which can ruin this prediction.] The Polonyi model mentioned in section 6.3 is just the special case of this exercise in which $W_{\rm hid}$ is assumed to be linear in X.

However, there is no particular reason why W and K must have the simple form eq. (A.7) and eq. (A.8). Furthermore, we have not yet explained how gaugino masses arise from nonrenormalizable terms. This requires a non-minimal gauge kinetic function f_{ab}. If the gauge kinetic function can be expanded in powers of $1/M_{\rm P}$ as

$$f_{ab} = \delta_{ab}\Big[\frac{1}{g_a^2} + \frac{1}{M_{\rm P}} f_a^i \phi_i + \ldots\Big], \tag{A.14}$$

then it is possible to show that the gaugino mass induced by supersymmetry breaking is

$$m_{\lambda^a} = \frac{1}{2M_{\rm P}} {\rm Re}[f_a^i]\langle F_i\rangle. \tag{A.15}$$

The assumption of universal gaugino masses therefore follows if the dimensionless quantities f_a^i are the same for each of the three MSSM gauge groups; this can be automatic in certain GUT and superstring models. Similarly, the superpotential can be expanded with the schematic form

$$W = W_{\rm ren} + \frac{1}{M_{\rm P}}\phi^4 + \frac{1}{M_{\rm P}^2}\phi^5 + \ldots \tag{A.16}$$

where $W_{\rm ren}$ is the renormalizable superpotential with terms up to ϕ^3. It may also be possible to expand the Kähler potential like

$$K = \phi_i \phi^{*i} + \frac{1}{M_{\rm P}}(\phi^{*3} + \phi^{*2}\phi + \phi^*\phi^2 + \phi^3) + \ldots, \tag{A.17}$$

If one now plugs eqs. (A.16) and (A.17) with arbitrary hidden sector fields and VEVs into eq. (A.2), one obtains a general form like eq. (6.25) for the soft terms. It is only when special assumptions are made [like eqs. (A.7),(A.8)] that one gets the phenomenologically desirable results in eqs. (6.26)-(6.30). This is why it is often said that supergravity by itself does not guarantee universality of the soft terms. Furthermore, there is no guarantee that expansions in $1/M_{\rm P}$ of the form given above are valid or appropriate. In superstring models, the "dilaton" and "moduli" fields have Kähler potential terms proportional to $M_{\rm P}^2 {\rm ln}[(\phi + \phi^*)/M_{\rm P}]$. (The moduli are massless fields which do not appear in the tree-level perturbative superpotential. The dilaton is a special modulus field whose VEV determines the gauge couplings in the theory.)

Finally, let us mention how gaugino condensates can give rise to supersymmetry breaking in supergravity models. This requires that the gauge kinetic function has a non-trivial dependence on the scalar fields, as in eq. (A.14). Then eq. (A.4) is modified to

$$F_i \;=\; -M_{\rm P}^2\, e^{G/2}\,(G^{-1})_i^j G_j - \frac{1}{4}(K^{-1})_i^j \frac{\partial f_{ab}}{\partial \phi_j}\lambda^a\lambda^b + \ldots. \tag{A.18}$$

Now if there is a gaugino condensate $\langle\lambda^a\lambda^b\rangle = \delta^{ab}\Lambda^3$ and $\langle (K^{-1})_i^j \partial f_{ab}/\partial\phi_j\rangle \sim 1/M_{\rm P}$, then $\langle F_i\rangle \sim \Lambda^3/M_{\rm P}$. Then as above, the non-vanishing F-term gives rise to soft parameters of order $m_{\rm soft} \sim \langle F_i\rangle/M_{\rm P} \sim \Lambda^3/M_{\rm P}^2$, as in eq. (6.14).

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