CBX 95 - 08
Harry Nelson
Tim Nelson
Mike Witherell
March 24, 1995

Partial Reconstruction of $B \rightarrow D^* X$

Abstract

We consider the reconstruction of the decay sequence $B \rightarrow D^* X$, followed by $D^* \rightarrow D\pi_{\rm slow}$, where only the X and the $\pi_{\rm slow}$ are reconstructed, and the D is unmeasured.

1 Introduction

We consider reconstruction of the decay sequence:

$$B \rightarrow D^* X \ , \ D^* \rightarrow D\pi_{\rm slow}. \tag{1}$$

where both X and $\pi_{\rm slow}$ are reconstructed, and the D is not reconstructed. Our goals are to obtain high statistics samples of $B \rightarrow D^* X$, and unique samples involving particular final states X.

2 Variables and Constraints

There are 5 particles in the sequence described by Eq. 1 (B,D^*,X,D,π). One needs 4 variables (a 4-momentum) to describe each particle.

A total of 20 = 5×4 variables are then needed to completely describe the sequence described in Eq. 1.

The constraints we exploit are:

1. From energy-momentum conservation in the 2 decays, $4 \times 2 = 8$ constraints.

2. The B mass of $5279 \pm 2\,\mathrm{MeV}$. (1 constraint)

3. The B energy, or the beam energy of CESR at the $\Upsilon(4S)$, $E_B = 5290 \pm 3\,\mathrm{MeV}$. (In practice, one would use the CESR beam energy for the event undergoing reconstruction.) (1 constraint)

4. The D^* mass of $2010.1 \pm 0.6\,\mathrm{MeV}$ for D^{*+} and $2007.1 \pm 1.4\,\mathrm{MeV}$ for the D^{*0}. (1 constraint)

5. The 4 reconstructed variables that describe the X. These may be taken as the mass of the X, which may itself be reconstructed from its decay products, and the 3 momentum components of X. (4 constraints)

6. The 4 reconstructed variables that describe the $\pi_{\rm slow}$, which may be taken as the pion mass, $139.57\,\mathrm{MeV}$ for the π^+ and $134.97\,\mathrm{MeV}$ for the π^0, and the 3 measured momentum components. (4 constraints)

7. The mass of the D, which is $1869.3 \pm 0.5\,\mathrm{MeV}$ for the D^+, and $1864.5 \pm 0.5\,\mathrm{MeV}$ for the D^0. (1 constraint)

There are $8+1+1+1+4+4+1 = 20$ constraints. So there are just enough to cover the variables and produce a complete solution for the kinematics; in other words, one can make a '0C' fit. However, the solution is quadratic, meaning that:

1. The solutions are, in general, complex valued. When a non-zero imaginary part is required, then there is no physically acceptable solution.

2. For most cases with a physically acceptable (real) solution, there are actually *two* equally acceptable solutions; in other words, a "quadratic ambiguity".

Because there are no extra constraints in this reconstruction, one cannot make the 'usual' plots that show evidence of a signal, like $\Delta E = E_{\rm rec} - E_B$ and $M_{\rm rec} - m_B$. There are a number of methods to establish a signal, however:

1. Use of characteristic shapes in some distributions of kinematic variables, such as $\cos\phi$ (ϕ defined in Fig. 3).

2. Use of Dalitz plots to search for resonances between the D^* and X when X is not single body

3. Further partial reconstruction of the D

4. Further partial reconstruction of the 'other' B

3 Physical Constraints and Reconstruction

Eliminating the cases in which there is no physically acceptable (real) solution for the kinematics is one of the most powerful tools in eliminating backgrounds. To do this most effectively we must first understand the essence of these cases.

Given:

1. A measured $\vec{p}_\pi$, hypothesized to be the slow pion.

2. A measured $\vec{p}_X$ and m_X, hypothesized to come from $B \to D^* X$.

3. The constraints to the hypothesized decay sequence $B \to D^* X$, $D^* \to D\pi_{\mathrm{slow}}$.

   (a) E_B
   (b) m_B
   (c) m_{D^*}
   (d) m_D
   (e) m_π

There are three basic ways in which an inconsistency with the hypothesized decay sequence can be deduced:

1. Given the measured $\vec{p}_X$ and m_X, 4-momentum *cannot* be conserved in the decay $B \to D^* X$.

2. Given the measured E_X and $\vec{p}_\pi$, 4-momentum *cannot* be conserved in the decay $D^* \to D\pi_{\mathrm{slow}}$.

3. Even when 4-momentum can be conserved in both (1) and (2), no D^* *direction* may exist that is consistent with *both* (1) and (2).

3.1 The decay $B \to D^* X$

Here, we are given:

1. m_B, E_B(assumed), thus $|\vec{p}_B|$ as well
2. m_{D^*}(assumed)
3. $\vec{p}_X$and m_X(reconstructed)

So by *requiring* energy conservation we can calculate the energy of the D^* given by $E_{D^*} = E_B - E_X = E_B - \sqrt{{m_X}^2 + |\vec{p}_X|^2}$. The most basic constraint is that E_{D^*} should exceed m_{D^*}:

$$E_{D^*} > m_{D^*} \tag{2}$$

This is a weak constraint. If true, one can compute the magnitude of the momentum of the D^* in the lab frame,

$$|\vec{p}_{D^*}| = \sqrt{{E_{D^*}}^2 - {m_{D^*}}^2}. \tag{3}$$

Once $|\vec{p}_{D^*}|$ has been computed using conservation of energy we can examine whether the hypothesized decay *can* conserve the spacelike components of momentum as well: $\vec{p}_{D^*} = \vec{p}_B - \vec{p}_X$. The three momentum vectors must satisfy the triangle inequality: None of the momenta can be greater than the sum of the other two. The power of this requirement comes from the relatively small size of $|\vec{p}_B|$ compared to the other two.

A simple way of parameterizing this is to calculate the cosine of one of the interior angles of the triangle formed by $|\vec{p}_{D^*}|$, $|\vec{p}_B|$ and $-\vec{p}_X$. We shall choose the angle between $-\vec{p}_X$ and the D^* directions as shown in Fig. 1.

Notice that since we know the direction of the X system, we have the D^* direction constrained to a cone about the $-\vec{p}_X$ direction.

The cosine of the opening angle of this cone, in terms of lab quantities is given by:

$$\cos\theta_{-XD^*} = \frac{{m_B}^2 - {m_{D^*}}^2 - {m_X}^2}{2|\vec{p}_{D^*}||\vec{p}_X|} - \frac{1}{\beta_{D^*}\beta_X} \tag{4}$$

For cases where the triangle does not close this gives $|\cos\theta_{-XD^*}| > 1$. The requirement for a physical solution, or "physicality", is then that $|\cos\theta_{-XD^*}| \leq 1$.

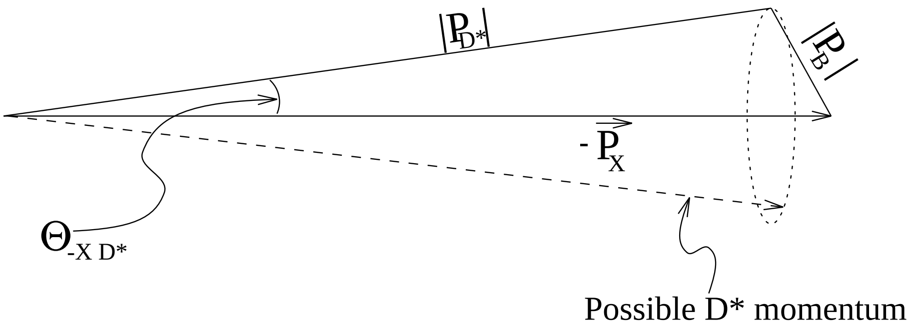

Figure 1: The $-X$–D^* cone angle as described in the text. Notice that because $|\vec{p}_B| \ll |\vec{p}_{D^*}|$ or $|\vec{p}_X|$ the D^* must be nearly back to back with X for signal.

For signal, typically, $\theta_{-XD^*} \approx 0.1$. The lower limit on $\cos\theta_{-XD^*}$ depends on m_X and $|\vec{p}_X|$. To account for this easily, it is convenient to work in the B rest frame. With $\gamma_B = E_B/m_B$, $\beta_B = \sqrt{1 - 1/\gamma_B{}^2}$, and using the quantities that describe the D^* and the X in the B rest frame:

$$\begin{aligned}
E^*_{D^*} &= ({m_B}^2 + {m_{D^*}}^2 - {m_X}^2)/2m_B \\
E^*_X &= ({m_B}^2 + {m_X}^2 - {m_{D^*}}^2)/2m_B \\
|\vec{q}_X| = |\vec{q}_{D^*}| &= \lambda({m_B}^2, {m_X}^2, {m_{D^*}}^2)/4{m_B}^2 \\
\lambda(a,b,c) &= a^2 + b^2 + c^2 - 2ab - 2ac - 2bc,
\end{aligned}$$

one can reconstruct the decay zenith of the X, θ^*_X:

$$\cos\theta^*_X = -\frac{\beta_B(E^*_X - E^*_{D^*})}{2|\vec{q}_X|} + \frac{|\vec{p}_X|^2 - |\vec{p}_{D^*}|^2}{2{\gamma_B}^2\beta_B m_B|\vec{q}_X|} \tag{5}$$

The physical constraint is then, for all m_X and $|\vec{p}_X|$:

$$-1 \le \cos\theta^*_X \le 1 \tag{6}$$

This is effectively a $m_X$dependent requirement on $|\vec{p}_X|$. As noted above, one benefits from the small value of β_B, which yields a small range of allowed $|\vec{p}_X|$.

3.2 The decay $D^* \to D\pi_{\rm slow}$

Here, we are given:

1. m_{D^*}, E_{D^*}(calculated in the last section).
2. m_{D^0}(assumed).
3. $\vec{p}_\pi$(reconstructed) and m_π(assumed).

So that here we have exactly the same information as above in the decay $B \to D^* X$. Similarly then, we have; with $E_\pi = \sqrt{{m_\pi}^2 + |\vec{p}_\pi|^2}$:

$$\begin{aligned} E_{D^0} &= E_B - E_X - E_\pi \\ E_{D^0} &> m_{D^0} \\ |\vec{p}_{D^0}| &= \sqrt{{E_{D^0}}^2 - {m_{D^0}}^2} \end{aligned}$$

Having required energy conservation to derive $|\vec{p}_{D^0}|$, again we turn around and check that 3-momentum can be conserved in the decay by asking that a triangle formed by the three momenta can close. Here, again, the power of this physicality requirement comes from the relatively small value of $\vec{p}_\pi$ compared to the other two.

We choose the cosine of the angle between the D^* and the $\pi_{\rm slow}$ directions to parametize the closure of this triangle as in Fig. 2.

The angle of this cone, in terms of lab quantities is given by:

$$\cos\theta_{\pi D^*} = -\frac{{m_{D^*}}^2 + {m_\pi}^2 - {m_{D^0}}^2}{2|\vec{p}_{D^*}||\vec{p}_\pi|} + \frac{1}{\beta_{D^*}\beta_\pi} \tag{7}$$

This angle is typically somewhat larger than the angle θ_{-XD^*}. Note again that in knowing the direction of the $\pi_{\rm slow}$ we have the D^* direction constrained to another cone of angle $\theta_{\pi D^*}$ about the $\pi_{\rm slow}$.

As in the case of $B \to D^* X$, $\cos\theta_{\pi D^*}$ has a lower limit that varies event by event. We again consider the decay zenith of the $\pi_{\rm slow}$ with respect to the D^* lab direction in the D^* rest frame.

This yields:

$$\cos\theta^*_\pi = -\frac{\beta_{D^*}(E^*_\pi - E^*_{D^0})}{2|\vec{q}_\pi|} + \frac{|\vec{p}_\pi|^2 - |\vec{p}_{D^0}|^2}{2{\gamma_{D^*}}^2\beta_{D^*}m_{D^*}|\vec{q}_\pi|} \tag{8}$$

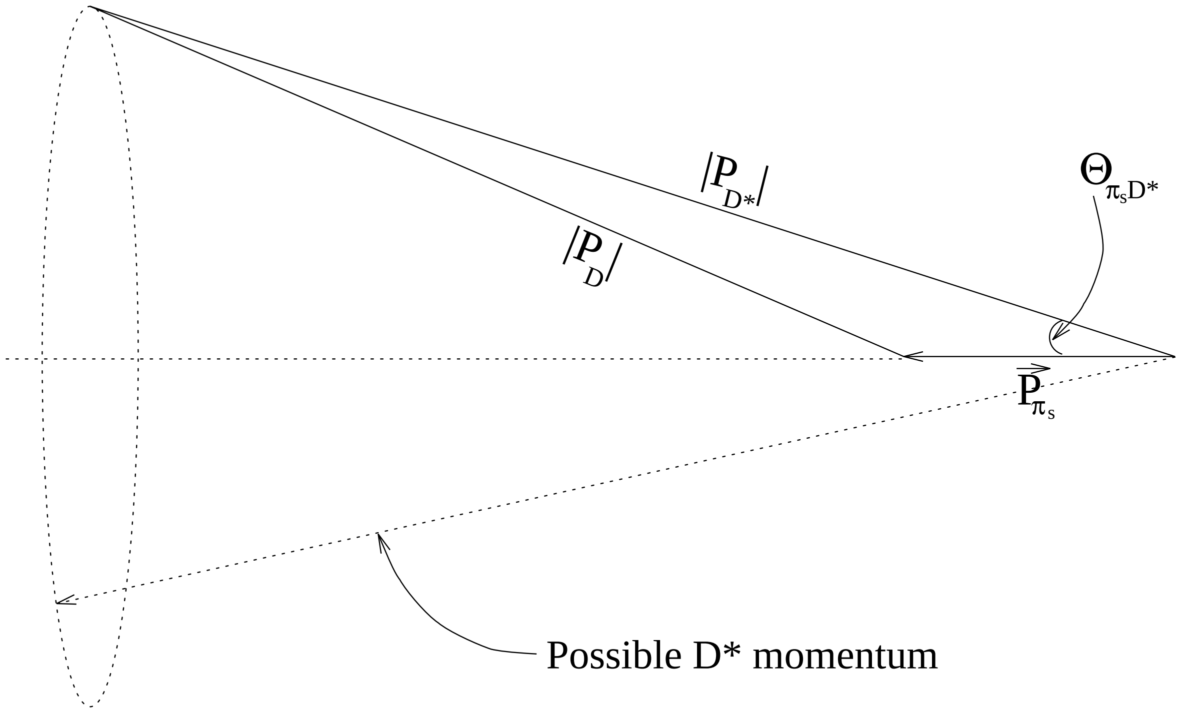

Figure 2: The $\pi_{\rm slow}$–D^* cone angle as described in the text. Notice that the D^* will be moving in a similar direction to the $\pi_{\rm slow}$ for signal.

and we require,

$$-1 \leq \cos\theta_\pi^* \leq 1 \tag{9}$$

Again, this is effectively an $E_X$dependent requirement on $|\vec{p}_\pi|$. One benefits from the small value of β_π^*, which produces $|\vec{p}_\pi|$ which is both small and small in range.

3.3 Solutions for D^* direction

The possible D^* momenta must have magnitude defined by Eq. 3, and must lie simultaneously on the cone defined in Fig. 1, and the distinct cone defined in Fig. 2. (The vertices of the cones are both at the origin on momentum space).

There are two possibilities:

1. These cones may not cross (no real solution for $\vec{p}_{D^*}$).
2. If they do cross, there will generally be two different but equally viable solutions (a quadratic ambiguity).

In order to parameterize the crossing of the two cones, both to reject cases where they do not cross as well as for the reconstruction of the D^*direction in the lab, we shall define an angle ϕ, as shown in Fig. 3. We define ϕ as the azimuthal angle about the $-X$–D^* cone where the two cones cross. In order that $\pm\phi$ describe both solutions. $\phi = 0$ is defined so that it lies in the $-X$–π_{slow} plane. It will be helpful if we have a method for calculating ϕ which continues smoothly when the cones fail to intersect.

We can compute phi as, (see Appendix for details.)

$$\cos\phi = \frac{\cos\theta_{\pi D^*} - \cos\theta_{-XD^*}\cos\theta_{-X\pi}}{\sin\theta_{-X\pi}\sin\theta_{-XD^*}} \tag{10}$$

When the two cones do not intersect, $|cos\phi| > 1$. When they do, we have both $\pm\phi$ as solutions.

The directions of the two D^* solutions in the lab can be reconstructed with the help of three special unit vectors:

1. The first in the direction of $-\vec{p}_X$ ($\hat{z}$ in Appendix):

$$\hat{n}_1 \equiv -\frac{\vec{p}_X}{|\vec{p}_X|}$$

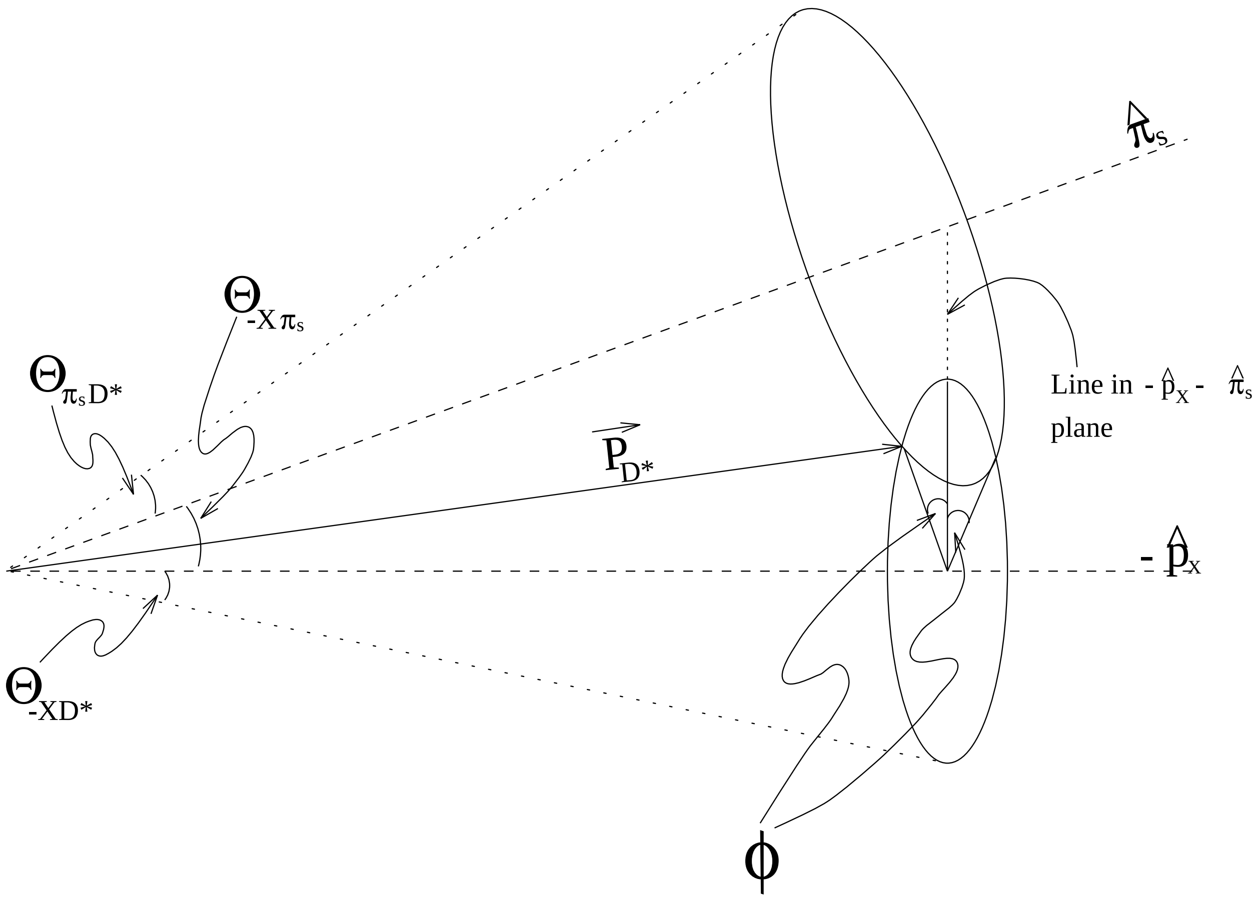

Figure 3: The angle ϕ which defines the possible solutions for the D^* direction as described in the text.

2. The second perpendicular to $-\vec{p}_X$, and in the plane of $-\vec{p}_X$ and $\vec{p}_\pi$, and at $\phi = 0$:

$$\hat{n}_2 \equiv \frac{\vec{p}_\pi - (\vec{p}_\pi \cdot \hat{n}_1)\hat{n}_1}{|\vec{p}_\pi - (\vec{p}_\pi \cdot \hat{n}_1)\hat{n}_1|}$$

3. The third perpendicular to the first two, easily formed by taking the cross product of $\hat{n}_1$ on $\hat{n}_2$:

$$\hat{n}_3 \equiv \hat{n}_1 \times \hat{n}_2$$

Then, we get for the momentum of the D^* in the lab frame:

$$\vec{p}_{D^*} = |\vec{p}_{D^*}|(\cos\theta_{-XD^*}\hat{n}_1 + \sin\theta_{-XD^*}[\cos\phi^*_{D^*}\hat{n}_2 \pm \sin\phi^*_{D^*}\hat{n}_3])$$

With the solutions for $\vec{p}_{D^*}$, it is possible to reconstruct $\vec{p}_B$(and hence ϕ_B and θ_B), and $\vec{p}_{D^0}$.

4 Conclusion

This method of reconstruction is currently being applied to various modes using the CLEO dataset. These include the modes;

1. $B \to D^*\,\pi$
2. $B \to D^*\,\pi\,\pi$
3. $B \to D^*\,D^*_s$

with various other modes under consideration. Future CBX notes will document the results of these efforts.

A Solution for $\cos\phi$

The crossing of the two cones can be equivalently thought of as the intersection of the two planes defined by the bases of the two cones, with the sphere of constant $|\vec{p}_{D^*}|$. It is convenient to simply normalize by the magnitude of the momentum, and solve the problem on a unit sphere. To find the points of intersection, as shown in Fig. 4:

1. Place a $\hat{z}$ (above, was $\hat{n}_1$) direction along $-\vec{p}_X$
2. Place $\vec{p}_\pi$ in the $\hat{x}-\hat{z}$ (above, was the $\hat{n}_2-\hat{n}_1$) plane, at an azimuth $\phi=0$ in the $\hat{x}-\hat{y}$ (above, was $\hat{n}_2-\hat{n}_3$) plane.

then we have for the two altitudes of the cones, as shown in Fig. 4:

$$\begin{aligned} P_1 &= \cos\theta_{-XD^*} && (11)\\ P_2 &= \cos\theta_{\pi D^*} && (12)\end{aligned}$$

We first want the line that is the intersection of the planes,

$$z = P_1 \tag{13}$$

and,

$$z = -\tan\theta_{-X\pi}x + \frac{P_2}{\cos\theta_{-X\pi}} \tag{14}$$

This line is given by,

$$\begin{aligned} X_* &= \tfrac{\cos\theta_{\pi D^*}-\cos\theta_{-XD^*}\cos\theta_{-X\pi}}{\sin\theta_{-X\pi}} && (15)\\ Z_* &= \cos\theta_{-XD^*} && (16)\end{aligned}$$

Next, look at this line in a side view relative to Fig. 4, as shown in Fig. 5.

The radius, P_3, of the circle at the intersection of the unit sphere and the cone about the $-X$ direction, is:

$$P_3 = \sin\theta_{-XD^*} \tag{17}$$

We can compute phi as,

$$\begin{aligned} \cos\phi &= \tfrac{X_*}{\sin\theta_{-XD^*}} && (18)\\ &= \tfrac{\cos\theta_{\pi D^*}-\cos\theta_{-XD^*}\cos\theta_{-X\pi}}{\sin\theta_{-X\pi}\sin\theta_{-XD^*}} && (19)\end{aligned}$$

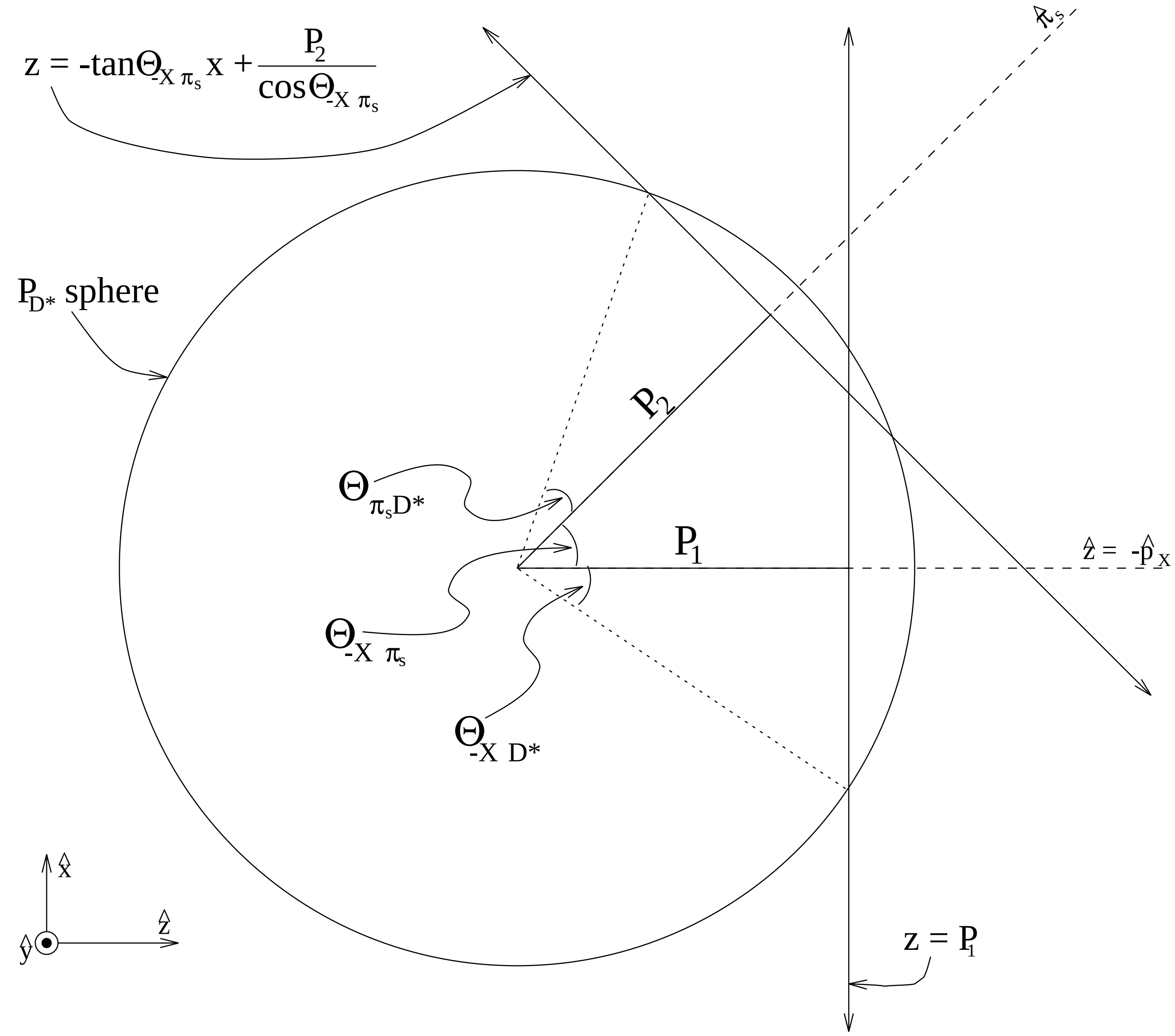

Figure 4: The intersection of two planes and a sphere used to solve for ϕ as described in the text. The first step is to find the x coordinate, X_*, of the intersection of the two planes shown.

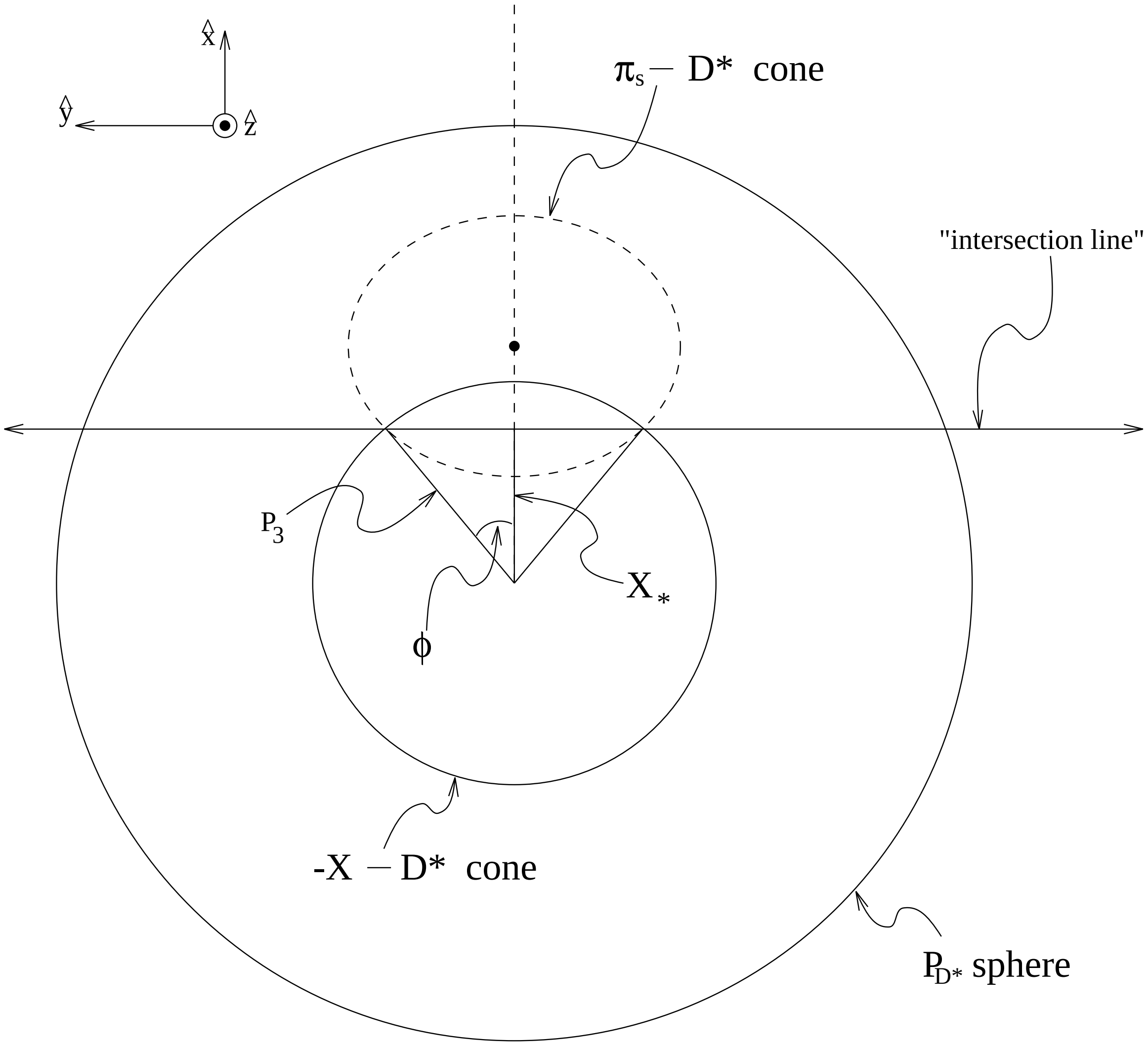

Figure 5: The solution for ϕ and thus the two possible D^* directions, at $\pm\phi$, as desribed in the text. Here we use the intersection of the two planes in Fig. 4, and the intersection of the $-X$–D^* cone with the unit sphere to define $\cos\phi$. Notice that a value of $\cos\phi$, with $|\cos\phi| > 1$, will result even when there is no intersection between the two cones.