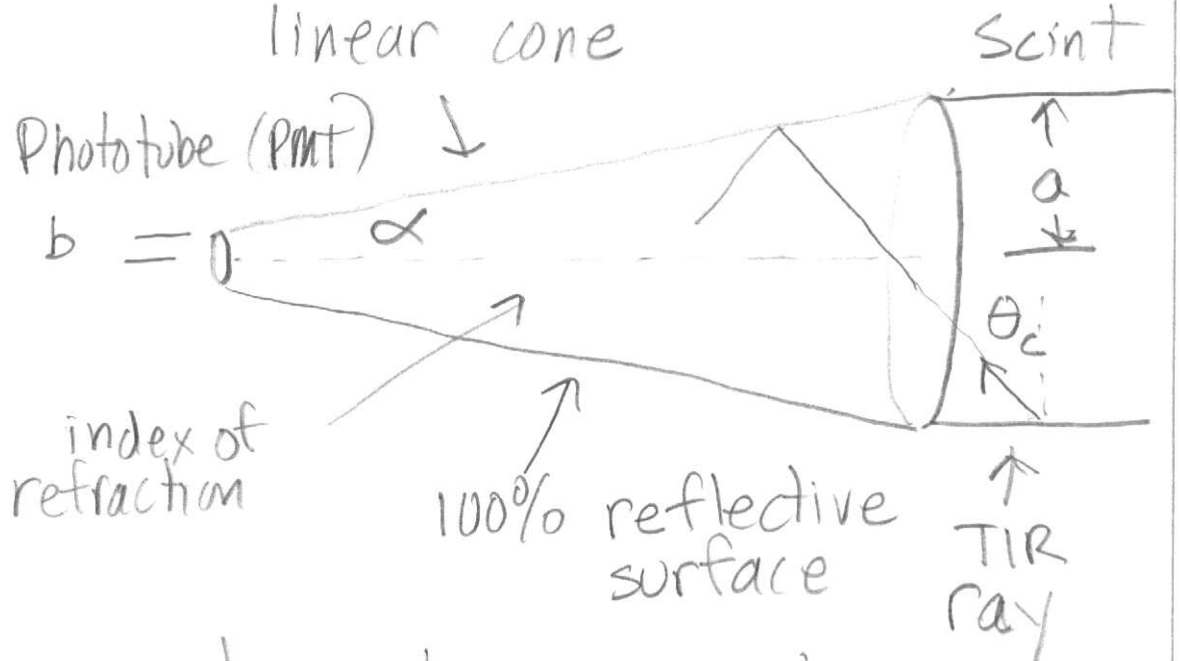
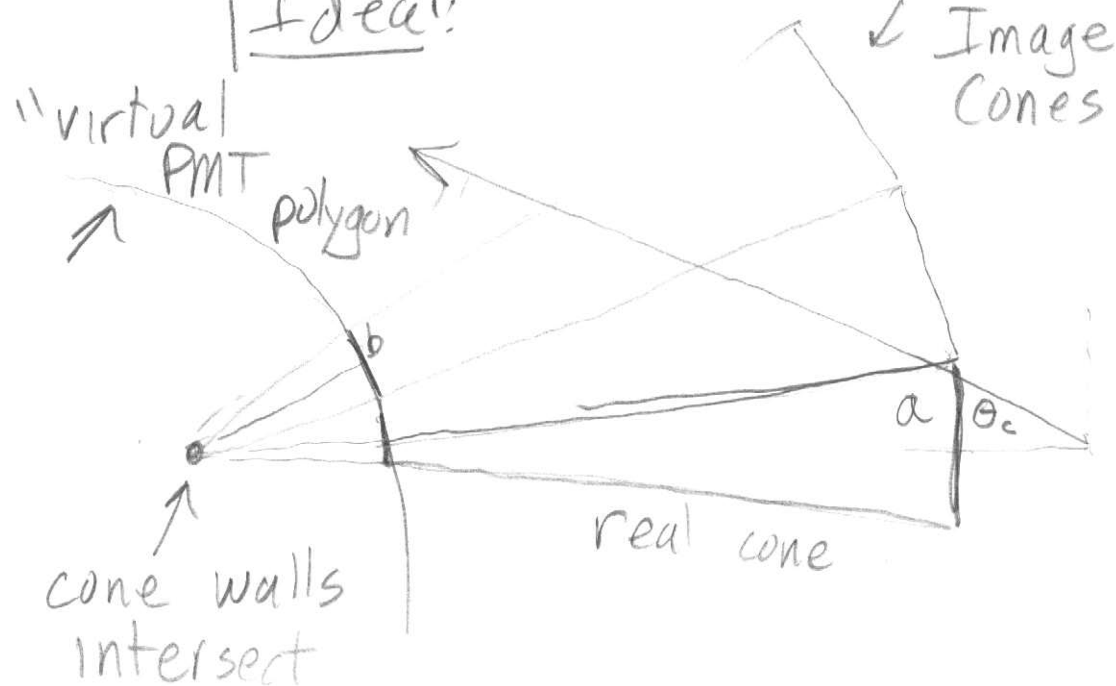
~100% Efficient V-trough Cone

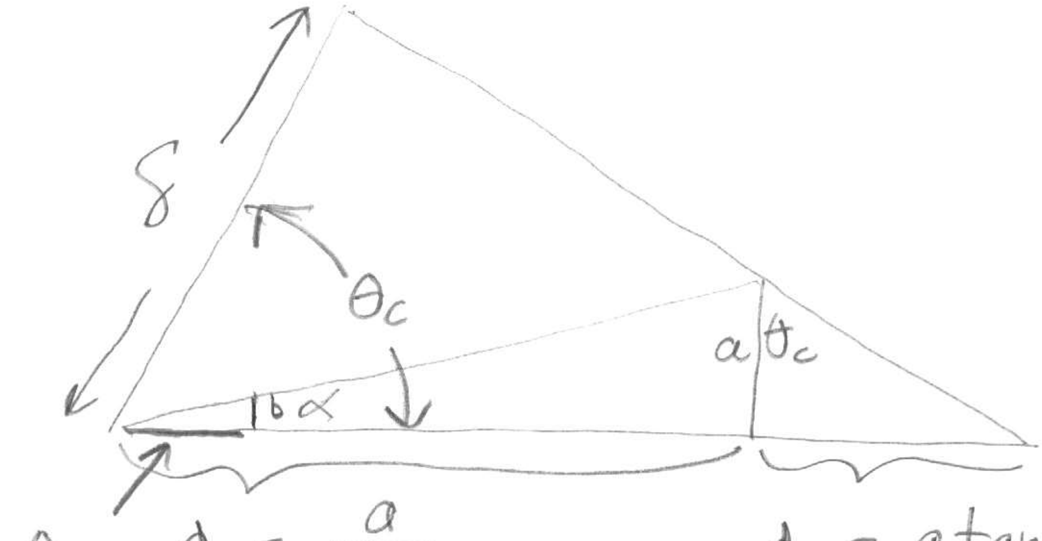


Can α be chosen such that 100% of Totally Internally Reflecting (TIR) rays hit the phototube (PMT)?

Idea!



Key Diagram



Δ $\quad d_1 = \dfrac{a}{\tan\alpha}$ $\qquad d_2 = a\tan\theta_c$

$$D = a\left(\frac{1}{\tan\alpha} + \tan\theta_c\right)$$

$$\delta = D\cos\theta_c$$
$$= a\left(\frac{1}{\tan\alpha} + \tan\theta_c\right)\cos\theta_c$$

$$\Delta = \frac{b}{\tan\alpha}$$

<u>need</u>: $\quad \delta < \Delta$

$$a\left(\frac{1}{\tan\alpha} + \tan\theta_c\right)\cos\theta_c < \frac{b}{\tan\alpha}$$

or $\quad a\cos\theta_c + a\sin\theta_c\tan\alpha < b$

Condition #1

$$\boxed{a\cos\theta_c < b}$$

(derivable from phase space args too)

when condition #1 is satisfied,

Condition #2

$$a\sin\theta_c \tan\alpha < b - a\cos\theta_c$$

$$\boxed{\tan\alpha < \frac{b}{a}\frac{1}{\sin\theta_c} - \frac{\cos\theta_c}{\sin\theta_c}}$$

Example

$a = 4''$

$\sin\theta_c = \frac{1}{n} = \frac{1}{1.49}$

$\theta_c \simeq 42.1°$

$b > a\cos\theta_c = 0.74a = 2.97''$

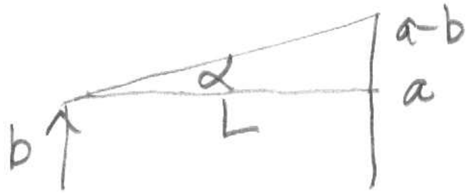
Make $b = 3\frac{1}{4}'' = 3.25$

$$\tan\alpha < \frac{3.25}{4}\frac{1}{\sin(42.1°)} - \frac{\cos(42.1°)}{\sin(42.1°)}$$

$\tan\alpha < 0.106 \rightarrow \alpha < 6.05°$

TOPS

The length L of this cone will be:



$$\tan\alpha = \frac{a-b}{L}$$

$$L = \frac{a-b}{\tan\alpha} = \frac{0.75}{0.106}$$

$$L = 7.07''$$

Should Compare With a Winston Cone..

(Generally, L for Winston Cone is smaller).