100% Efficient V-trough Cone

Linear cone

Phototube (PMT) ↓

\[ b = 0 \]

Index of refraction

100% reflective surface

Can \( \alpha \) be chosen such that 100% of Totally Internally Reflecting (TIR) rays hit the phototube (PMT)?

Idea:

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"virtual PMT polygon

Virtual Image Cones

\[ cone \text{ walls intersect} \]

\[ real \text{ cone} \]
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\[ \theta_c \]

\[ TIR \text{ ray} \]
Key Diagram

\[ \delta = D \cos \theta_c \]
\[ = a \left( \frac{1}{\tan \alpha} + \tan \theta_c \right) \cos \theta_c \]
\[ \Delta = \frac{b}{\tan \alpha} \]

need: \( \delta < \Delta \)

\[ a \left( \frac{1}{\tan \alpha} + \tan \theta_c \right) \cos \theta_c < \frac{b}{\tan \alpha} \]

or \( a \cos \theta_c + a \sin \theta_c + \tan \alpha < b \)
Condition #1
\[ a \cos \theta_c < b \]
when condition #1 is satisfied,

Condition #2
\[ a \sin \theta_c \tan \alpha < b - a \cos \theta_c \]
\[ \tan \alpha < \frac{b}{a} \frac{1}{\sin \theta_c} - \frac{\cos \theta_c}{\sin \theta_c} \]

Example
\[ a = 4'' \]
\[ \sin \theta_c = \frac{1}{n} = \frac{1}{1.49} \]
\[ \theta_c = 42.1^\circ \]
\[ b > a \cos \theta_c = 0.74 \, a = 2.97'' \]
Make \( b = 3\frac{3}{4}'' = 3.25 \)
\[ \tan \alpha < \frac{3.25}{4} \frac{1}{\sin(42.1^\circ)} - \frac{\cos(42.1^\circ)}{\sin(42.1^\circ)} \]
\[ \tan \alpha < 0.106 \rightarrow \alpha < 6.05^\circ \]
The length \( L \) of this cone will be:

\[
\tan \theta = \frac{a-b}{L}
\]

\[
L = \frac{a-b}{\tan \theta} = \frac{0.75}{0.106}
\]

\[
L = 7.07\ "
\]

Should compare with a Winston cone.

(Generally, \( L \) for Winston cone is smaller).