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How did we built our hyperbolic mirror omnidirectional camera - practical issues and basic geometry

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Like a lot of other researchers in the field of mobile robotics we decided to build an omnidirectional vision system by placing a hyperbolic mirror in front of a regular perspective camera. In this paper we give an account of the practical issues we encountered when assembling and calibrating our omnidirectional camera system. Hopefully our experiences can help others that want to build their own system.
Keywords: omnidirectional camera, camera calibration.

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1 Introduction

For a lot of applications it is beneficial to have a vision system that can observe a large part of the scenery. An inexpensive way to build a camera with a wide field of view is by using a standard camera with a standard lens having a rather narrow field of view and combining it with a mirror designed and placed in front of the camera in such a way to increase the field of view. Various convex mirror shapes can be chosen, but only a few will result in a camera system that has a so called single viewpoint (central camera), which entails that points in the image resulted from light rays intersecting in a unique world point [3, 6]. A common choice is the hyperbolic mirror combined with a standard camera with a standard lens, which we have used in our setup.

In this paper we will address some practical issues with assembling such a camera-lens-mirror system. First, in Section 2, a formal description is given of a general hyperbolic mirror, which is needed for calibrating the system and projecting image points to points in world coordinates. Section 3 covers the design choices we made when acquiring the mirror, the camera and the lens, and describes the basic camera calibration we performed. In Section 4 we discuss the problems and our experience with properly placing the mirror with respect to the camera in order to construct a system that behaves as a central camera. Finally, in Section 5 we give a short overview of the basic geometry of the camera-lens-mirror system.

2 Model of the mirror

The hyperbolic mirror has the form of a hyperboloid:

$$\frac{(z+e)^2}{a^2} - \frac{x^2+y^2}{b^2} = 1, \tag{1}$$

where a and b are mirror parameters denoting the semimajor and semiminor axis respectively [11] and $e = \sqrt{a^2+b^2}$ stands for mirror eccentricity. The coordinates of the 3D points are given by $X = \begin{pmatrix} x & y & z \end{pmatrix}^T$. The hyperboloid described by (1) has two surfaces and two corresponding "focal points" F_1 and F_2. The first focal point , F_1, is located at the origin and second one, F_2, at $\begin{pmatrix} 0 & 0 & -2e \end{pmatrix}^T$. In Figure 1 we present a side view of the hyperboloid function. The shape of the mirror is the shape of one side of the function while the other side is plotted using a dashed line. A hyperbolic mirror is typically placed in such a way that the optical center of the camera, known as the camera center, coincides with the second focal point F_2 as shown in Figure 1. Such placement is important since the whole camera-lens-mirror system becomes a wide central camera with its camera center located at the first mirror focal point F_1. The term "central camera" means that the world 3D points are projected to the image surface by central projection. The central projection is obtained by intersecting the imaging surface with the lines that connect the 3D points with a single point that is called the camera center. It is possible to deal with general mirror shapes and placements [4] but the geometry of the central cameras is usually simpler. Furthermore, there is a large amount of work considering the central projection model (the standard cameras) which can be directly applied (e.g. [6]).

3 Components

The first step in assembling the system is choosing an appropriate mirror and an appropriate lens for the camera. In this Section we discuss which characteristics are important and give an account of the choices we made. Also we describe the basic calibration of the camera-lens system.

3.1 Choosing a mirror

There is a number of producers of the hyperbolic mirrors in the market (e.g Panorama Eye [9] , Kaidan [1]). Except for the quality of the mirror probably the most important parameter is the maximal upward view angle indicated in Figure 1.

We already owned a mirror acquired from Accowle Company, Ltd. (see [9]). In [7], a report is given of the designing and manufacturing process that resulted in the production of the type of mirror we have.

Our specific mirror has a maximal upward view angle of 30°. Furthermore, the parameters of our mirror given by the producer are $a = 42.0882$ and $b = 25.0915$. The diameter of the outer rim of the mirror was 61mm.

3.2 Choosing a lens

Apart from the normal lens properties that should be taken into account, such as achromaticity and low lens aberrations, special care must be taken when choosing a lens for omnidirectional vision sensor. Below, we will explain which characteristics are to be preferred and why.

Field of view The camera-lens field of view, depending on the focal length of lens, and the size of the mirror will determine which part of the mirror will be visible in the images. Typically a field of view is chosen such that the whole mirror is visible. There are also lenses with variable field of view, which cost a little bit more. With such a lens it is possible to tune the field of view so the mirror will perfectly fit in the image. In our case we choose a lens with a fixed field of view, that was somewhat smaller than the perfect size. This results in an image which does not capture the whole mirror, but has the advantage the part of the mirror that is visible has a higher resolution (also see Figure 2).

Minimal working distance The mirror is often designed in such a way that second mirror focal point is located close to the mirror itself. Because the optical center of the lens (i.e. camera center) is positioned on this second focal point, the camera is also close to the mirror. For our mirror the optical center was about 11 cm from the mirror. The goal is to get a sharp image of the scenery reflected in the mirror. To get a sharp image of a point in the scenery the lens has to be focused on a plane just behind the point where the ray of the scenery point intersects with the mirror [2]. Thus in order to get a sharp image of the scenery the lens has to be able to focus on the mirror.

Depth of field The mirror has a certain depth. It should be possible to obtain sharp image of the whole mirror. Unfortunately most lenses have small depth of field at short distances and often only a part of the mirror can be seen sharply. The depth of field can be somewhat increased if the amount of light that pass through the lens is reduced by closing the iris but this can lower the quality of the images.

3.3 Lens-camera model and calibration

A world point X (now in homogeneous coordinates) is projected to the image point $\mathbf{x}_{im} = \begin{pmatrix} x_{im} & y_{im} & 1 \end{pmatrix}^T$ using the standard perspective camera equations:

$$\mathbf{x}_{im} = PX = KR\,[\,I\,|\,C\,]\,X \tag{2}$$

where P is the 3×4 projection matrix that can be decomposed as described above. Here C is a 3 dimensional vector that represents the position of the camera center. The I above denotes

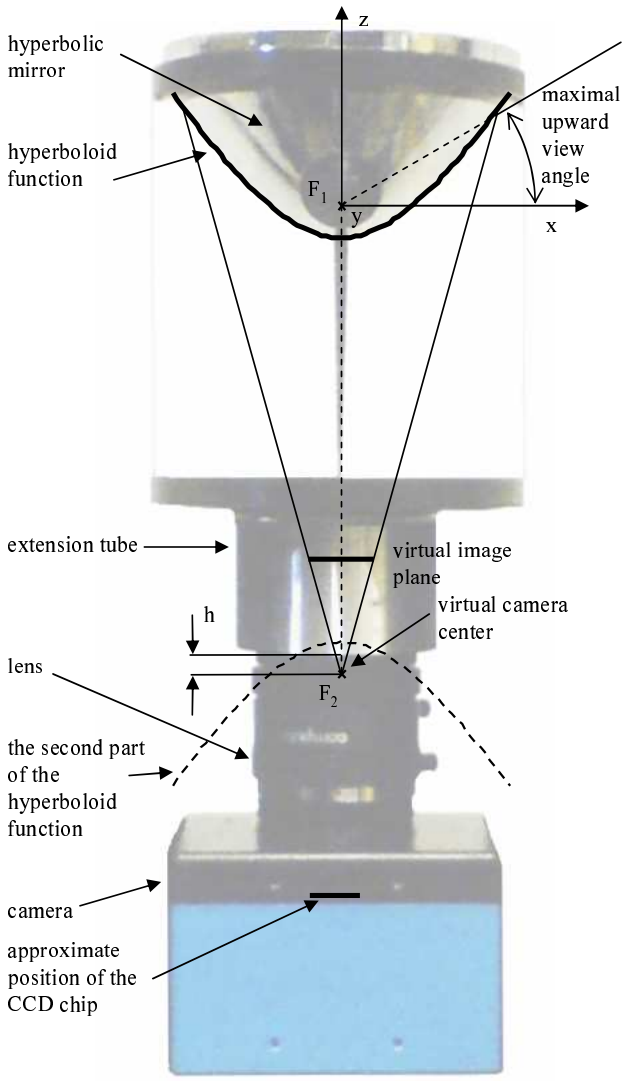


Figure 1: A side view of our camera-mirror system. At the bottom the regular firewire camera is visible with the lens attached to it. On the lens a hollow extension tube is mounted which positions the mirror on the right distance. On top of the image of the camera system the hyperboloid function is plotted with the first focal point, F_1, at the upper part, located inside the mirror, and the second focal point, F_2, coinciding with the optical center inside the lens.

the 3×3 identity matrix. The matrix R is a 3×3 rotation matrix that describes the rotation of the camera with respect to the world frame. The matrix K is the camera calibration matrix:

$$K = \begin{pmatrix} f_x & s & x_0 \\ 0 & f_y & y_0 \\ 0 & 0 & 1 \end{pmatrix} \tag{3}$$

where f_x and f_y are the scale factors in x and y directions, s is skew and (x_0, y_0) are the image coordinates of the projection of the camera optical center. There is a large set of camera calibration techniques that can be used to estimate the matrix K [6, 12, 5]. Skew s is often close to 0 since the pixels are usually rectangular and $f_x = f_y$ for square pixels.

Most lenses are not ideal and will introduce additional distortions. A common type of distortion is the radial distortion that can be modeled by:

$$x_{im,d} = x_c + L(r)(x_{im} - x_c) \text{ and } y_{im,d} = y_c + L(r)(y_{im} - y_c) \tag{4}$$

where $r = \sqrt{(x_{im} - x_c)^2 + (y_{im} - y_c)^2}$ and $L(r) = 1 + k_1 r + k_2 r^2 + ...$ Here $x_{im,d}, y_{im,d}$ represent the actual image position after distortion. The distortion parameters $x_c, y_c, k_1, k_2, ...$ become additional parameters to be estimated during camera calibration. The uneven coefficients are usually omitted since they are expected to be zero for standard lenses. The radial distortion center is often assumed to coincide with the camera optical center $x_c = x_0, y_c = y_0$. Typically the lenses with wider field of view have larger distortions. A lens with a medium field of view is used for the camera-lens-mirror systems discussed here and the lens distortions can be well described by just a few coefficients. In our case using just $k2$ was enough.

4 Assembling and calibrating the camera-lens-mirror system

A common problem in assembling the system is placing the camera such that the camera (lens) center coincides with the second focal point of the hyperbolic mirror. Only then the whole camera-lens-mirror system can be described by the central camera model. The ideal procedure for finding the correct position of the camera with respect to the mirror would be: find the positioning for which the whole system can be "best" described as a central camera. The "best" can be defined as some reconstruction error of a known calibration object. However this is quite difficult to realize and the following two practical procedures are usually used.

4.1 Assembling by aligning with the rim of the mirror

A common procedure assumes that the outer rim of the mirror is at least partially visible in the camera images [3]. First, the perimeter of the outer rim is measured. The camera-lens is calibrated using some standard technique. The equations from Section 3.3 are used to predict how the outer rim of the mirror with known perimeter would look like if the camera and mirror are placed correctly. The camera and mirror are then aligned by aligning the predicted image of the rim and the current image, see Figure 2.

4.2 Assembling by calculating the camera optical center

The position of the second focal point, F_2, is often specified by the mirror producers or it can be calculated. In our case F_2 was at 116mm from the top rim of the mirror. Another way of assembling the system is to estimate the position of the camera (lens) center and then place the mirror such that the second focal point F_2 coincides with the camera center.

Most lenses have the optical axis in the middle. In our case it is possible to align the mirror axis with the lens axis relatively accurately by using just an extension tube, Figure 1. The

Figure 2: An example calibration image. The predicted image of the outer mirror rim, presented by the dashed circle, coincides with the actual outer rim that is partially visible in the image.

distance of the camera center from the top rim of the lens is denoted by h. The length of the tube plus the unknown h should be such that the camera center is placed at the mentioned 116mm from the top rim of the mirror.

A simple procedure to estimate h is the following. A calibration object is placed in front of the camera. Let the position of a point on the object be described in the camera coordinate system by $X_c = [x_c y_c z_c]^T$ and its image projection by x_{im}. We measure the perpendicular distance of the point from the top rim of the lens d and we have $z_c = d + h$. It is easy to show that from $\mathbf{x}_{im} = KX_c$ we get two linear equations where h,x_c and y_c are unknown. If we move the object approximately parallel to the camera axis such that x_c and y_c remain the same for each new distance d two more linear equations are obtained. As the result we get an over determined linear system that can be solved for the unknown h.

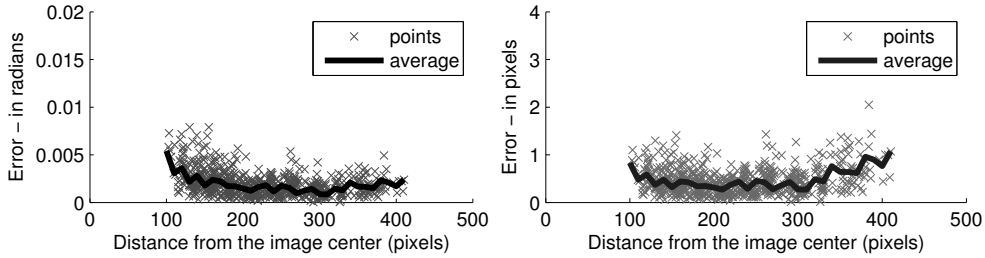
4.3 Calibration

Once the camera-lens is calibrated and the mirror properly placed there is in principle no need for further calibration. However, the presented simple assembling procedures introduce additional errors. Therefore it is often beneficial to reestimate some of the parameters. In [8] we find a simple procedure for parameter reestimation which is an extension of the standard camera calibration [5]. The parameter reestimation can be seen as the final fitting of the central camera model to the camera geometry. For the calibration results for our camera see Figure 3.

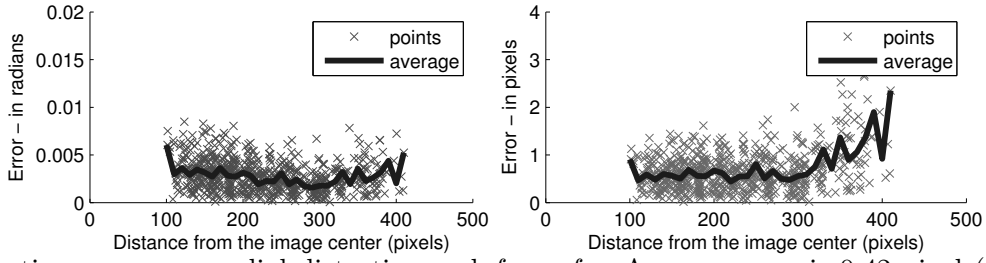
The parameter reestimation could sometimes lead to reasonable models even when there were large errors introduced in camera mirror positioning. An interesting reestimation procedure is given in [10] where also the shape of the mirror is reestimated using a polynomial approximation. More on dealing with non central cameras and approximate central models can be found in [4].

5 Single view geometry

We briefly present here the geometric model describing the whole camera-lens-mirror system. For a more elaborate discussion on this topic see [3].



a) Calibration errors using only the coefficient k_2 for the radial distortion. Average error is 0.28 pixel (standard deviation 0.23) and 0.0018 radians (standard deviation 0.0013).



b) Calibration errors, no radial distortion and $fx = fy$. Average error is 0.42 pixel (standard deviation 0.37) and 0.0028 radians (standard deviation 0.0016)

Figure 3: Calibration results. We used 10 images where the calibration object (see the top image) was placed at various orientations and positions around the camera.

5.1 World to image projection

A 3D world point X is first projected to the point X_m on the mirror surface. The projection is obtained by intersecting the mirror surface with the line connecting the origin F_1 and X. From (1) we get:

$$X_m = \begin{pmatrix} x & y & z & 1/\lambda(X) \end{pmatrix}^T, \tag{5}$$

where

$$\lambda(X) = \frac{b^2(-ez - a\sqrt{x^2+y^2+z^2})}{b^2z^2 - a^2(x^2+y^2))} \tag{6}$$

selects the correct intersection. The point X_m on the mirror is then projected to the image using the standard perspective camera equations $\mathbf{x}_{im} = KR\,[\,I\,|\,C\,]\,X_m$. The camera center should coincide with the second focal point of the mirror. Therefore we have $C = \begin{pmatrix} 0 & 0 & 2e \end{pmatrix}^T$. Usually the camera is aligned with the axis of the mirror and the R is the identity matrix.

5.2 Image to world projection

From an image point $\mathbf{x}_{im}$ we first construct a ray connecting the image point and the camera center at F_2. In the coordinate system centered at F_2 the image point corresponds to the point on the virtual imaging surface $X(F_2) = (KR)^{-1}\mathbf{x_{im}}$. The intersection of this ray with the mirror surface is given by (7) and (8) which are derived from (5) and (6).

$$X_m(F_2) = \begin{pmatrix} x & y & z & 1/\lambda(X) \end{pmatrix}^T, \tag{7}$$

where

$$\lambda(X) = \frac{b^2(ez + a\sqrt{x^2+y^2+z^2})}{b^2z^2 - a^2(x^2+y^2))}. \tag{8}$$

The mirror point X_m in the coordinate system centered at F_1 is

$$X_m(F_1) = X_m(F_2) - \begin{bmatrix} C \\ 0 \end{bmatrix}. \tag{9}$$

The ray connecting the origin F_1 and the mirror point $X_m(F_1)$ defines the 3D world points that project to the image point $\mathbf{x}_{im}$.

6 Conclusions

We described a number of practical issues with building and calibrating an omnidirectional camera. Hopefully this report can be useful for anyone trying to build a wide view camera using a hyperboloid mirror and a regular camera. A MATLAB implementation of the basic geometric transformations is provided in the Appendix.

7 Appendix: Basic geometry - Matlab implementation.

Transforming a 3D world point to an image point:

```
[rXim]=W2Hyp(rXw,K,a,b)
%Input:
% rXw (3 x N) - set of N points in the world coordinates
% K   (3 x 3) - camera calibration matrix
% a,b         - hyperbolic mirror parameters
%Output:
% rXim (3 x N) - set of N points in the image coordinates - rXim(3,:)=1
e=sqrt(a^2+b^2);%eccentricity
Tc=[0 0 -2*e]';%translation from first focal point to the second one
Rc=eye(3);%this should be used if the camera was tilted
lambda=b^2*(-e*rXw(3,:)-a*sqrt(rXw(1,:).^2+rXw(2,:).^2+rXw(3,:).^2))./(b^2*rXw(3,:).^2-a^
rXm=repmat(lambda,3,1).*rXw;
rXim=K*Rc*(rXm-repmat(Tc,1,size(rXm,2)));
rXim=rXim./repmat(rXim(3,:),3,1);
```

Transforming an image point to a 3D ray (from the camera center):

```
function [rX]=Hyp2W(rXim,K,a,b)
%Input:
% rXim (3 x N)- set of N points in the image coordinates
% K   (3 x 3) - camera calibration matrix
% a,b         - hyperbolic mirror parameters
%Output:
% rX (3 x N) - set of N points in the mirror coordinates (on the mirror)
e=sqrt(a^2+b^2);%eccentricity
Tc=[0 0 -2*e]';%translation from first focal point to the second one
Rc=eye(3);%this should be used if the camera was tilted
rXc=inv(K*Rc)*rXim;
rXc=rXc./repmat(rXc(3,:),3,1);
lambda=b^2*(e*rXc(3,:)+a*sqrt(rXc(1,:).^2+rXc(2,:).^2+rXc(3,:).^2))./(b^2*rXc(3,:).^2-a^2
rX=repmat(lambda,3,1).*rXc+repmat(Tc,1,size(rXim,2));
```

References

[1] Kaidan 360 One VR. Website: http://www.kaidan.com.

[2] Simon Baker and Shree K. Nayar. A theory of single-viewpoint catadioptric image formation. *Int. J. Comput. Vision*, 35(2):175–196, 1999.

[3] R. Benosman and S.B. Kang. *Panoramic Vision*. Springer, 2001.

[4] B.Micusik and T.Pajdla. Autocalibration & 3d reconstruction with non-central catadioptric cameras. *In Proc. of IEEE Conference on Computer Vision Pattern Recognition*, 2004.

[5] Jean-Yves Bouguet. Camera calibration toolbox for Matlab, http://www.vision.caltech.edu/bouguetj/calib_doc/.

[6] R. Hartley and A. Zisserman. *Multiple View Geometry in Computer Vision*. Cambridge University Press, 2003.

[7] H. Ishiguro. Development of low-cost compact omnidirectional vision sensors and their applications. In *Proc. Int. Conf. Information Systems, Analysis and Synthesis*, pages 433–439, 1998.

[8] C. Mei. Omnidirectional calibration toolbox extension,http://www-sop.inria.fr/icare/personnel/christopher.mei/christophermeiphdstudenttoolbox.html.

[9] LTD. Panorama EYE, Seiwapro Co. Formerly named Accowle Company, Ltd. Website: http://www.accowle.com/english/index.html.

[10] D. Scaramuzza, A. Martinelli, and R. Siegwart. A flexible technique for accurate omnidirectional camera calibration and structure from motion. *In Proceedings of IEEE International Conference on Computer Vision Systems*, 2006.

[11] Juergen Wolf. Omnidirectional vision system for mobile robot localization in the robocup environment. Master's thesis, Graz, University of Technology, 2003.

[12] Z. Zhang. A flexible new technique for camera calibration. *IEEE Transactions on Pattern Analysis and Machine Intelligence*, 22(11):1330–1334, 2000.

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