AMPLIFICATION OF SUB-MICROSCOPIC EVENTS

Having completed our survey of the basic physics required for an understanding of particle detection techniques, we will now turn our attention to the specifics of particle detectors. We will see in the next several lectures that there are a variety of types of detectors, each one being characterized by a number of advantages and disadvantages. However, one thing all these detectors have in common is that the first process in the chain of events which generates a macroscopic signal indicating the detection of a particle or photon is the *jiggling* of one or more electrons in the detecting medium. In scintillators or Cerenkov radiators the jiggled electrons generate photons which are subsequently detected. In gas ionization, proportional and Geiger counters the jiggled electrons actually escape their parent atoms and are subsequently captured on electrodes. A similar process occurs in semi-conductor diode detectors whereby the jiggled electrons are elevated to the conduction band which allows them to be collected at electrodes. Jiggled electrons cause condensation centers in cloud chambers and vaporization centers in bubble chambers. Even nuclear emulsions and dielectric track detectors can trace their detection mechanisms to a disturbed electron which induces a chemical transformation which can later be visualized.

The value of a particle detector is usually severely limited unless there is some means by which the primary jiggled electron signal can be amplified. This is not to say that such amplification is absolutely essential however. In the early part of this century, the electroscope and zinc sulfide scintillation screens were two popular detectors which did not require amplification. However these detectors were not very sensitive. Two early detectors which utilized forms of amplification were the Wilson cloud chamber and photographic film. For the cloud chamber the amplification was brought about by the macroscopic condensation of droplets of liquid about condensation centers consisting of ions. This amplification rendered the cloud chambers sufficiently sensitive for the observation of rapidly moving, singly charged particles. Photographic film also amplifies electronic signals by means of chemical processing of latent, invisible radiation damage. Early films were inadequate to the task of seeing minimum ionizing particles due to technical difficulties involving emulsion grain size and density but these problems were overcome in the 1940's with the subsequent emergence of the emulsion as the premier tool of particle physics for a number of years. At about the same time that emulsion was making a name for itself, two other developments were taking place which also involved signal amplification and which would expand the scope and power of particle detection techniques even more so than nuclear emulsions had. One of these developments was the invention of the transistor. In time this would lead to electronic amplifiers and timing circuits vastly superior to the earlier vacuum tube models which had done much to enhance the application of various gas ionization detectors. The other development was that of high sensitivity photomultiplier tubes. These devices ushered in the era of scintillation counters and Cerenkov radiators, two detectors which to this date are unsurpassed in a number of applications.

We will begin our formal discussion of amplification of detector signals (in particular, of electronic amplification; we will reserve for a later date discussion of the various chemical amplification techniques) with a comprehensive summary of the properties of photomultiplier tubes (PMT's). PMT's occupy a crucial role in modern experiments in that they are the point of contact between the physicist and the electronics engineer. As is often true with such points of contact, there can exist a certain degree of nebulosity with regard to the responsibility for PMT performance. We feel that it is important for the physicist to assume complete responsibility for this task. Of course the scientist must, in the final analysis, accept responsibility for all aspects of an experiment but in practice it is not always possible to oversee everything. Nevertheless, PMT performance is so crucial, and is so subject to environmental conditions, that it behooves the scientist to think very carefully about PMT selection and implementation.

Photomultiplier Tubes

In very succinct terms, a PMT converts a light signal into an electric signal. There are a number of PMT designs and configurations but all have in common the following features:

1. A *window* to allow the passage of light from an external environment into the PMT
2. A *photocathode* material to convert the incident photon into a free electron with a large efficiency
3. An evacuated PMT interior to allow the motion of the liberated electron with a minimal amount of energy degradation
4. A sequence of *dynodes*, each one at a more positive electrical potential than its predecessor
5. An *anode*, which collects the electrons ejected from the last dynode

Very simply, the incident photon passes through the window, strikes the photocathode and is converted into a free electron which is accelerated to the first dynode where it strikes with sufficient energy to liberate more than one electron. Each of these liberated electrons subsequently passes down the remaining part of the dynode chain, undergoing similar amplification. At high enough PMT voltage, the net current gain can be as much as ten million. We will now discuss in greater detail the various components outlined above.

Windows

PMT windows are made of glass and usually limit the maximum detectable photon energy due to the UV absorption by the glass. For normal, inexpensive glass, the UV cutoff is in the neighborhood of 350 nm. Certain applications require greater UV transmittance than this, however, and this can be achieved with quartz windows which extend the cutoff to about 160 nm. The quartz windows are much more expensive than ordinary glass.

Photocathode Materials

We earlier discussed the photoelectric effect as it related to K-shell ionization. Since PMT's must observe visible photons ($\hbar\omega<3.5$eV) it is apparent that this simple discussion will not suffice. In addition to a failure to take into account the proper final state of the electron, the simplified discussion presented earlier was incapable of describing the motion of the electron through the photo-emissive material or the escape of the electron over the potential barrier at the vacuum-photocathode interface. We will not go into the details here, but suffice it to say that semi-conductors turn out to be vastly superior to metals as photo-emissive substances in the optical region. Semi-conductors absorb a much higher fraction of incident light, have lower energy thresholds, and allow for photoelectron escape from greater depths than do metals. The design and fabrication of improved photocathode materials is a complex, ongoing endeavor. To give an example for this we note that certain photocathodes consist of mixtures of several elements which take on optimal semi-conductor properties at a fixed ratio of ingredients. Changing the ratio slightly can destroy the semi-conducting character of the mixture.

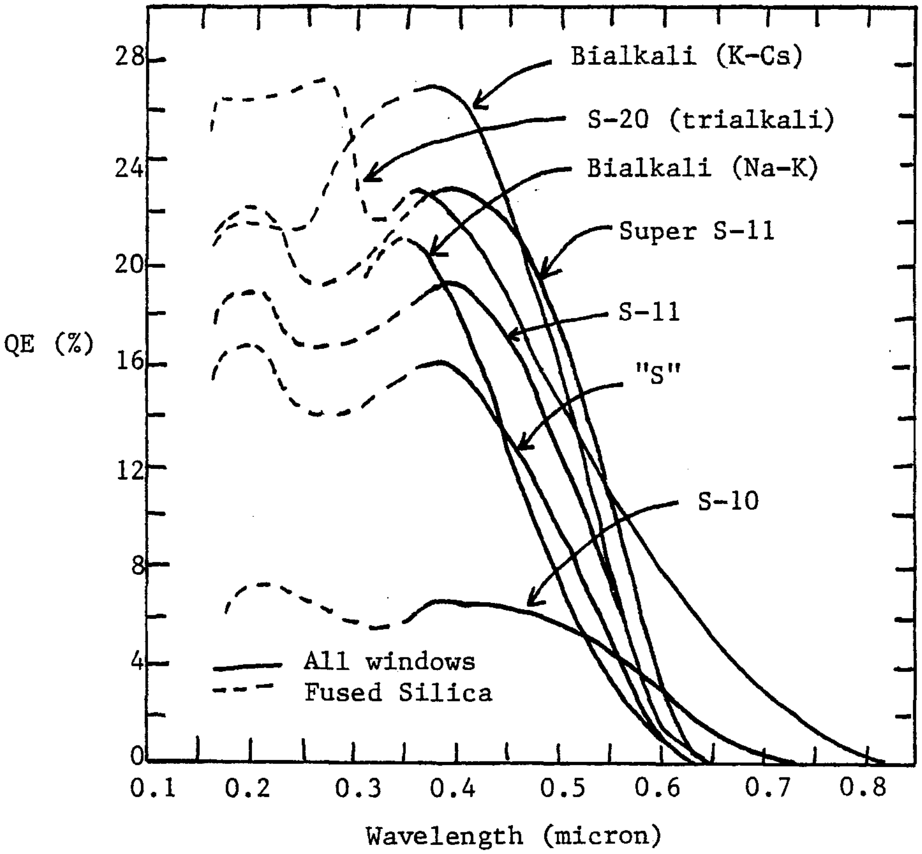
Photocathodes exist in a variety of configurations. They can be used as opaque sheets from which secondary electrons are withdrawn from the same side as the incident photon impinged or they can be affixed to the interior of the PMT window so that the photoelectrons are ejected on the opposite side. It is obvious that photocathode thickness is critically important for these latter, transparent, photocathodes. If the cathode is too thick, too much light is absorbed at a depth from which electron escape is impossible. If the cathode is too thin, too much light passes through the cathode without interacting. It turns out that transparent photocathodes are much more convenient to use with scintillators and Cerenkov counters than are the opaque photocathodes.

The more commonly used photocathodes are silver-oxygen-cesium (Ag-O-Cs), cesium-antimony (Cs_3Sb), bialkali (K_2CsSb) and multialkali or trialkali ($(Cs)Na_2KSb$). Recently developed negative electron affinity materials (e.g. $GaAs_xP_{1-x}(Cs)$) which have improved sensitivities and extended spectral ranges are in use in opaque photocathodes.

The most important property of photocathodes for the experimental physicist is the *quantum efficiency*. This is defined as the ratio of secondary electrons ejected from the photocathode to the number of photons of a given wavelength incident on the external side of the PMT window:

$$\text{quantum efficiency} = \text{QE} = \frac{\text{ejected electrons}}{\text{incident photons}}$$

The quantum efficiencies for various types of transparent photocathodes are shown on the following page. The long wavelength drop in QE is due to the photocathode while the short wavelength drop is due to the window.



Another property of photocathodes which is of some importance and which should be taken into consideration during a PMT selection process is the cathode resistivity. If the cathode resistance is large enough, the IR drop across the cathode can cause sufficient potential fluctuations to significantly affect the first dynode collection efficiency and subsequently introduce PMT non-linearity for large cathode currents. This is a problem only for tubes with semitransparent photocathodes, particularly of the CsSb or bialkali type. Maximum cathode currents for several configurations are given below:

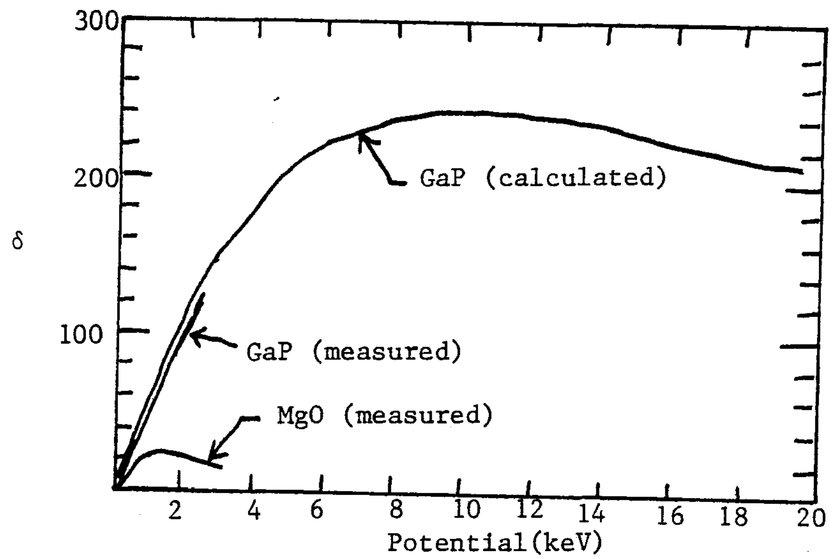
Cathode Material	Temperature (° C)	Maximum Cathode Current (amp)
Semitransparent bialkali up to 1 inch diameter	22	10^{-8}
Semitransparent bialkali up to 1 inch diameter	-100	10^{-10}
Bialkali larger than 1 in	22	10^{-9}
Bialkali larger than 1 in	-100	10^{-11}
Opaque CsSb		$10^{-4}/cm^2$
Semitransparent CsSb	22	$10^{-6}/cm^2$
Multialkali, Ag-O-Cs	22	$10^{-4}/cm^2$

Dynode Chains and Secondary Emission

The photoelectrons ejected from the photocathode will be accelerated to the first dynode by an applied potential. If these electrons strike the dynode with sufficient energy they will cause the secondary emission of additional electrons. The *secondary emission yield* can be defined for any dynode as:

$$\text{secondary emission yield} = \delta = N_s/N_p$$

where N_s is the average number of secondary electrons emitted for N_p primary electrons incident upon the dynode. The physics of secondary emission is similar to that of photoemission and we will not go into that here. It is adequate to note that δ is proportional to the inter-dynode potential for a large range of potentials. We plot below values of δ for various potentials for a common secondary emission material MgO as well as for GaP(Cs), a recently developed negative electron affinity material.



Most phototubes operate with interdynode potentials well below a keV so that δ is in the linear region even for MgO.

Before we go on to the various types of divider chains, we should mention something about the time lag in photoemission and in secondary emission. Thus far, only upper limits can be provided by experiments and these are of the order of 100 picoseconds or less for both photo- and secondary emission.

There are a number of geometrical configurations for dynode structures. Among these we might mention the following:

1. *circular cage dynodes* - this is a compact structure which has fast time response due to its small size and high field strengths

2. *venetian blind dynodes* - large area dynode system for use with planar photocathodes; rugged, but relatively slow response
3. *box and grid dynodes* - large area dynodes; time response similar to that of venetian blind tube
4. *linear multiplier* - better time response than either box and grid or venetian blind system

The above dynode types are generally all used with a semi-transparent photocathode. If PMT speed is unimportant, one is generally better off using venetian blind types as these are relatively inexpensive and are less subject to transient dynode heating effects than the faster tube types (linear multiplier or circular cage). The dynode heating effects contribute to rate dependence of the PMT gain.

We will now turn our attention to various aspects of PMT performance.

Pulse Timing

We have already mentioned one facet of pulse timing, namely the lag between excitation and either photoemission or secondary emission, and have seen that this is faster than can be presently measured. Another facet is the variation of time response for photons striking different positions on the photocathode. For planar photocathodes this time difference can amount to as much as 10 nanoseconds. Curved photocathodes are much better in this regard and for this reason fast phototubes generally have curved windows. Pulse rise time and electron transit time are two other parameters related to the speed of a PMT. The pulse rise time is defined to be the time for the anode pulse to rise from 10 to 90% of its maximum value for a delta function illumination of the cathode. For fast tubes (circular or linear) this is typically less than 2 nanoseconds while for venetian blind types it is of the order of 20 nanoseconds. The electron transit time is just the time delay between illumunation of the cathode and the arrival of the electron burst at the anode. For fast tubes this time is of the order of 25 nanoseconds while it can be as large as 100 nanoseconds for large venetian blind tubes. For timing applications, one can do better than the pulse rise time if the tube behavior is consistent. Thus, if one takes into account such effects as variations in pulse height, it is possible to achieve timing FWHM's of the order of 200 picoseconds.

Output Linearity

If we assume that the PMT is not being overdriven at the cathode and that the divider circuit and external power supply are adequate, the limit to PMT current linearity is provided by space charge effects in the space between the last two dynodes. Typically this occurs for anode currents of the order of 50 milliamps although large variations between tubes is possible. Since the maximum output current which is allowed by space charge limitations goes as the 3/2 power of the voltage, one can increase the range of linear output by using an unbalanced divider chain which applies more voltage to the last few dynodes than to earlier dynodes. For currents below space charge limited values tubes typically deviate from integral linearity by no more than a couple of per cent although with some care linearities to a fraction of a per cent can be achieved over many orders of magnitude in both cathode and anode current.

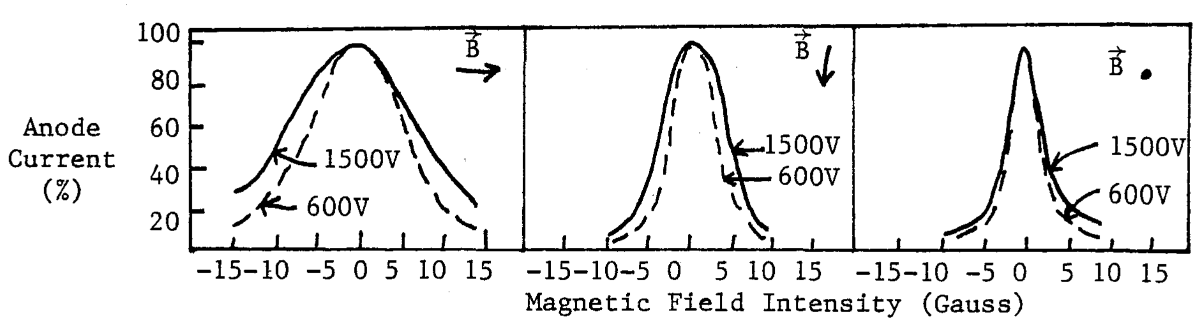
Environmental Effects and Handling of PMT's

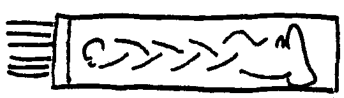
As might easily be imagined, it is very bad practice to expose a PMT to ambient lighting with high voltage applied to the tube. The amount of damage to the tube will depend on a number of variables such as tube type, light level, divider circuit etc. In our experience we have observed tubes being destroyed by intense particle beam induced scintillation and high rate light emitting diode (LED) signals while we have also exposed tubes to room lighting with high voltage applied for three hours with no deleterious effect. As a rule, one should always work with very muted lighting unless he is absolutely sure that the PMT is adequately shielded from light leaks and even then it is good practice to assume that a light leak can pop open at any moment.

It is also considered good practice to avoid exposing PMT's to room lighting (especially to fluorescent lights) or sun light even with the high voltage turned off. This type of exposure can induce phosphorescence which will keep the tube noisy for days and in the case of multialkali photocathodes can result in permanent loss of sensitivity. However, in the case of fabrication procedures, it is better to work with a tube in a well lit environment than it is to work in the dark and risk an accident due to poor seeing conditions.

When you work with a phototube for the first time the first question you will come up with is what polarity to apply to it. As we will discuss later there are certain applications where positive high voltage is preferred and some where negative high voltage is preferred. Each configuration has its own divider circuit and you cannot simply reverse the polarity by switching the power supply polarity. Our advice to you is this: don't worry too much if you accidentally apply the wrong polarity since it is unlikely for you to do damage to the tube. However if you ever turn on a tube at its nominal voltage and observe no response (as you would if the polarity was incorrect) do not indiscriminately start to increase the level of the voltage in the hopes of coaxing out a signal without first checking out every other possibility.

It is easy to guess that PMT's would be subject to large effects if in the presence of a magnetic field. We show below the effect on anode current of magnetic fields in several orientations.





Tube Orientation Relative to Arrows Above

The largest effect is seen to occur for those cases where the magnetic field is perpendicular to the axis of the PMT. One generally shields against ambient magnetic fields by placing the PMT in a mu-metal housing which can also serve as a shield for electric fields if electrically connected to the PMT cathode.

One of the most common failure modes of a PMT is for one of the electrical feed throughs to develop a leak, thereby destroying the PMT vacuum. A very convenient way to inspect for this is to check the color of the faceplate of the PMT. The color of a healthy photocathode is orange. If the tube has developed a leak, the atmosphere will have reacted with the cathode material which will subsequently lose its color.

It goes without saying that PMT stability is usually of the utmost importance. When operated under stable conditions with anode and cathode currents well below their maximum specifications PMT's will usually experience gain fluctuations of less than 1% over 24 hour time periods. However, if the PMT current is excessive, or if the PMT suffers some mechanical shock, as it might in transit, permanent loss of some fraction of the PMT gain is possible. Only rarely does such a gain reduction exceed 10% however. For long term operation, or for operation in an environment of unstable temperature or magnetic fields it is recommended that some kind of PMT calibration scheme be implemented. We will describe some such schemes later in the course when we discuss detector tests and calibrations. A final comment on PMT stability involves warmup time. All tubes require a certain amount of time after having the high voltage applied to achieve a stabilized gain. This is often less than 1 minute, although some tubes require more than 15 minutes. It is good practice to establish the warmup time before embarking on an extensive series of tests on a large number of PMT's. If this is not done, a lot of time could be wasted.

<u>PMT Amplification and Statistics</u>

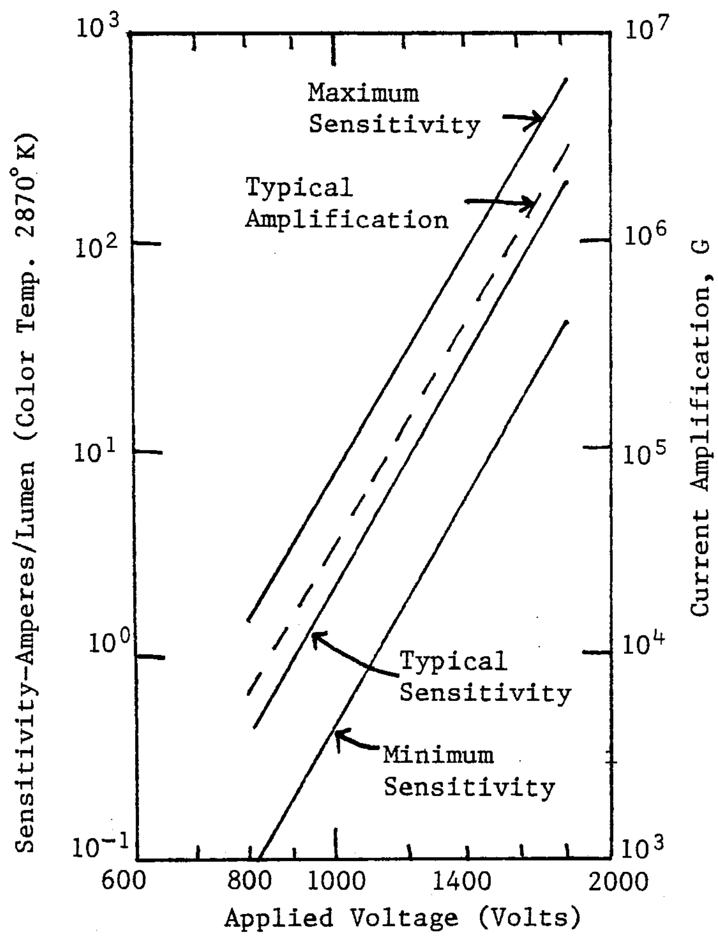
Although there will be some occasions for you to use PMT's in the DC mode, where an output anode current reflects a constant cathode illumination, most applications of PMT's in high energy physics, astrophysics, and nuclear physics are in the pulse mode. In this configuration, a pulse of light from a scintillator or Cerenkov radiator illuminates the PMT cathode for a short time (anywhere from a nanosecond to a microsecond, depending on the scintillator type), resulting in an injection of an average number, $\bar{N}$, of photoelectrons. For n dynodes, with the ith dynode's multiplication factor denoted by δ_i, we would expect there to be:

$$\bar{S} = \bar{N} \prod_{i=1}^{n} \delta_i$$

electrons at the anode. The PMT gain, G, is denoted by:

$$G = \prod_{i=1}^{n} \delta_i \, .$$

We have previously seen that $\delta_i \propto V_i$ where V_i is the potential between the ith dynode and its predecessor. The total PMT voltage will be given by the sum of all the V_i and in general, each V_i will be of the same order of magnitude. It is apparent then, that the voltage dependence of G will be very strong, very nearly given by $G \propto V^n$. This should be regarded only as an approximation. However the power law behavior is virtually universal. As an example, we show the gain dependence below as a function of applied voltage for an RCA 4517 PMT.



For the 4517 tube, the power law is 7.6, although the number of dynodes is 10. This difference is due to such factors as non-linear dynode gains and to differences in interdynode potentials along the dynode chain (the above graph is for a divider circuit recommended for the 4517 PMT which is indeed unbalanced). The important thing to keep in mind is that the strong dependence of gain on voltage implies that for a gain stability of $\sim$ 1%, one needs a power supply stability of $\sim$0.1%.

The sequence of processes which culminates in a signal at a PMT anode is clearly of a stochastic nature. For example, for a given number of photons liberated by a charged particle in a scintillator, there is an efficiency of much less than 100% for an individual photon to reach the PMT window. Once it reaches the window, the photon must pass through the window, interact in the cathode, and liberate a photoelectron which can then be accelerated to the first dynode. We have seen before that the probability for this to happen, which is just the quantum efficiency, is less than 30% for even the best phototubes. Each photoelectron is then accelerated to the first dynode where it may liberate from 0 to many secondary electrons. The process continues down the dynode chain.

Our goal is to calculate the probability function $P(S|L)$, where L is a fixed number of primary photons, each of which is in principle capable of generating a photoelectron, and S is the number of electrons collected at the anode. In what follows then, we regard L as fixed, but intermediate numbers, such as the number of photoelectrons, must be considered to be random variables.

It should be apparent that the production of N photoelectrons is a binomial process. For each of the L photons there is a probability p for generating a photoelectron. In practice, p is much less than 1 so that it is usually a good approximation to consider the production of photoelectrons as a Poisson process with the corresponding standard deviation of the number of photoelectrons produced of $\sigma = \sqrt{Lp} = (\overline{N})^{\frac{1}{2}}$. If $\overline{N} > 5$ it is not a bad approximation to assume that the probability function for the production of N photoelectrons is Gaussian.

Now let N_1 be the number of secondary electrons emitted from the first dynode, and let $P(N_1|N)$ be the conditional probability for the ejection of N_1 secondary electrons given N photoelectrons. The single photoelectron function, $P(N_1|1)$, varies among different types of dynodes currently in use. Its form ranges from that of a Poisson distribution to that of an exponential distribution. One might expect that Poisson statistics should apply given the following simple argument: as the accelerated photoelectron plows into the dynode there is a probability per unit length for the production of secondary electrons; but this is just the definition of a Poisson process. The deviations from Poisson distributions observed in some dynode chains have been explained by non-uniformities of response over individual dynode surfaces. One can take this variability of secondary response into account by utilizing the *Polya distribution* for single photoelectron response. This distribution can be shown to be a composite of a number of different Poisson processes each having a different mean value. Thus, it naturally accounts for the possibility of dynode non-uniformities and by variation of the *Polya parameter* b, one can generate any distribution from an exponential to a Poisson. The Polya distribution has the following form:

$$P(n,b) = \frac{\mu^n}{n!}(1 + b\mu)^{-n-1/b} \prod_{j=1}^{n-1} (1 + jb)$$

As $b \to 0$, $P(n,b)$ approaches a Poisson distribution and as $b \to 1$, $P(n,b)$ approaches

$$\mu^n(1 + \mu)^{-(1+n)}\text{ , an exponential distribution.}$$

It can be shown that the mean for a Polya distribution is given by μ and that the variance is given by $(b\mu^2 + \mu)$.

It is apparent that in considering the statistics of PMT amplification we have a sequence of coupled devices which convert input numbers of particles into output numbers of particles. For simplicity, consider two such devices A and B (A may be the photocathode, and B the first dynode). Let $P_A(n)$ be the single particle input probability function for observing n output particles for A (if A is the cathode, this function has the value p for n=1 and is 0 for n>1). Similarly, let $P_B(n)$ denote the analogous function for device B. The *generating functions*, $Q_{A,B}$, are defined by:

$$Q_{A,B}(s) = \sum_{n=0}^{\infty} P_{A,B}(n)s^n \quad .$$

Now let $P_{AB}(n)$ be the probability function for the chained response of devices A and B, again with a single particle input into A. It can be shown that:

$$Q_{AB}(s) = \sum_{n=0}^{\infty} P_{AB}(n)s^n = Q_A(Q_B(s)) \quad .$$

It can also be shown that for an arbitrary probability function P, and corresponding generating function Q, the mean and variance can be expressed in terms of Q as:

$$\bar{n} = \frac{\partial Q}{\partial s}\Big|_{s=1}$$

$$\sigma^2 = \frac{\partial^2 Q}{\partial s^2}\Big|_{s=1} + \frac{\partial Q}{\partial s}\Big|_{s=1} - \left(\frac{\partial Q}{\partial s}\Big|_{s=1}\right)^2 \quad .$$

Therefore, $\bar{n}_{AB} = \frac{\partial Q_{AB}}{\partial s}\Big|_{s=1} = \frac{\partial Q_A}{\partial Q_B}\frac{\partial Q_B}{\partial s}\Big|_{s=1} = \frac{\partial Q_A}{\partial Q_B}\Big|_{s=1}\bar{n}_B \quad .$

For s=1, $Q_B=1$ so that $\bar{n}_{AB} = \bar{n}_A\bar{n}_B$. Thus, the average total gain is the product of the individual gains as might have been expected. Using the expression for variance, it can also be shown that:

$$\sigma_{AB}^2 = (\bar{n}_B)^2\sigma_A{}^2 + \bar{n}_A\sigma_B{}^2 \quad .$$

Now suppose there are L input particles instead of 1. The output number of particles from B is just the sum of L independent random variables so that:

$$\bar{n}_{AB}(L) = L\bar{n}_A\bar{n}_B \quad \text{and} \quad \sigma_{AB}^2(L) = L[(\bar{n}_B)^2\sigma_A^2 + \bar{n}_A\sigma_B{}^2]$$

with

$$\sigma_{AB}^2(L)/(\bar{n}_{AB}(L))^2 = \sigma_A{}^2/[L(\bar{n}_A)^2] + \sigma_B{}^2/[L\bar{n}_A(\bar{n}_B)^2] \quad .$$

For a Poisson photocathode process and a Polya secondary emission process:

$$\sigma_A{}^2/(\bar{n}_A)^2 = 1/p \quad \text{and} \quad \sigma_B{}^2/(\bar{n}_B)^2 = b + 1/\delta_1$$

where we have set $\bar{n}_B$ equal to the first dynode gain. Setting $Lp = \bar{N}$:

$$\sigma_{AB}^2(L)/(\bar{n}_{AB}(L))^2 = 1/\bar{N} + (b + 1/\delta_1)/\bar{N} \quad .$$

We see that the fractional variance for the electrons leaving the first dynode is the sum of the fractional variances associated with photoelectron production and with secondary electron production at the first dynode. This result is easily extended to a large number of dynodes:

$$\sigma_S^2(L)/[\bar{S}(L)]^2 = (1/\bar{N})[1 + (b + 1/\delta_1) + (b + 1/\delta_2)/\delta_1 + \cdots] \quad .$$

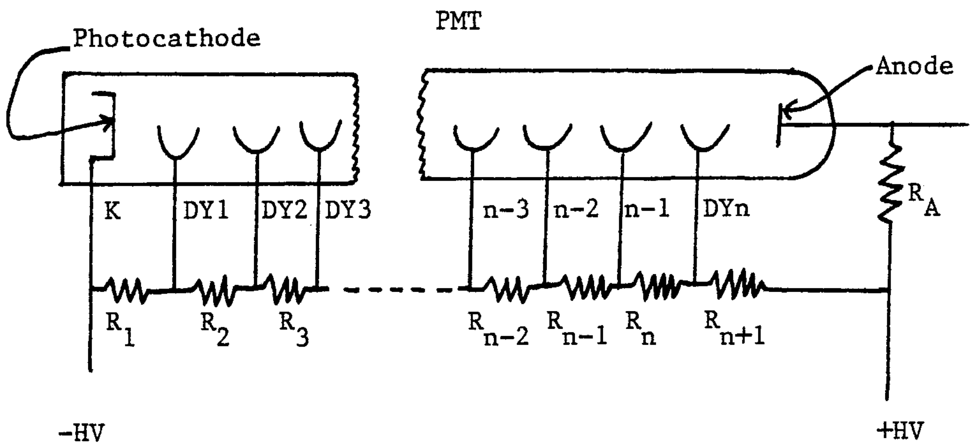
We have assumed that each dynode had the same Polya parameter. If we further assume that the Polya parameter is 0 (Poisson statistics) and that each dynode gain factor is the same, we easily see that:

$$\sigma_S^2(L)/[\overline{S}(L)]^2 = (1/\overline{N})\left(\frac{\delta}{\delta - 1}\right) \quad .$$

Thus, only if the dynode gain is much larger than 1 can we neglect the dynode contribution to the fluctuations of PMT signal. By examining the expression for the fractional variance of the anode signal on the previous page it is seen that by designing a tube with a Polya parameter of zero and setting a large voltage on the first dynode to increase the first dynode gain by as much as possible, one can minimize the dynode fluctuations. In such cases, one can actually resolve single photoelectron events from double photoelectron events and so on. In any case however, the fractional standard deviation of tha anode pulse height is proportional to $1/(\overline{N})^{\frac{1}{2}}$ for a given value of the high voltage. It is also true that for $\overline{N}$ larger than 5, the distribution in S is approximately Gaussian, as can be seen by application of the central limit theorem.

Voltage Dividers

The voltage steps between the various stages of a photomultiplier tube are usually provided by a resistive voltage divider as shown below.



With this type of divider only a single high voltage power supply is required. When constructing a voltage divider, the PMT user has two options concerning the wiring of the high voltage connector polarity. If the cathode is connected at ground potential, the anode is at *positive high voltage* and a coupling capacitor must be used to feed signals out from the anode to a preamplifier at ground potential. The use of coupling capacitors produces a shift in the baseline voltage at high rates. An advantage of this configuration is that the PMT shield, which must be at ground potential to prevent shock hazard, is at the same potential as the photocathode. When the photocathode is at a different potential than the shield, leakage currents passing through the glass can induce scintillations in the glass, increasing the dark noise of the tube and can, in some cases, permanently damage the photocathode. When the signal cannot be passed through a coupling capacitor (such as when one wishes to measure average current or in ultra high rate applications where baseline shift would be too severe) the anode is operated at ground potential (through R_A) and the photocathode must be operated at *negative high voltage*. In this case the shield surrounding the tube must be kept at this high negative voltage. Shock hazard can be prevented in one of two ways: surround the inner shield by a well insulated outer shield at ground potential or connect the shield to negative high voltage through a very high value resistor.

The resistance values of the dynode resistors can vary from $\sim 2 \times 10^4$ to $\sim 5 \times 10^6 \Omega$ per stage. The exact values used depends upon the application. For applications where the average current is high (such as for large pulses and/or high rates) lower values of resistance should be used but this requires a larger power supply and heating of the PMT by power dissipation in the divider chain can result. In applications where low average current is required (such as in low rate situations or where power is at a premium as in satellite, balloon or rocket packages) the higher values of resistance can be used but in no case should exceed $5 \times 10^6 \Omega$ per stage. At this point, leakage between the pins on the phototube base can introduce variations in the interstage voltage. Higher value resistors can be used if the PMT's are ordered without a base and then are scrupulously cleaned and potted with the divider. Such precautions are usually only required for satellite experiments though. The resistance values need only be selected for the average current required and capacitors can be used to supply current during the peak of a current pulse. These capacitors are connected to the last few dynodes where the current demands are the greatest. If Q is the charge demand at a given dynode associated with one pulse and V is the voltage across the capacitor, then a capacitance of $C = 100Q/V$ is required to keep the voltage at that dynode stable to 1%. These capacitors can either be connected in series from dynode to dynode or in a parallel configuration from each dynode to ground. The series connection is more convenient in that the capacitors can be wired directly on the socket and do not require a high voltage rating. The parallel connection is used when low inductance connections to the ground plane are required, as in applications requiring preservation of high linearity and pulse shape. Good high frequency wiring practices are important when wiring a divider, especially in fast (nanosecond) pulse work (a few mm of wire can have enough self inductance to cause ringing and distortion). In such applications, impedance mismatch between the anode and the output cable is minimized by using bypass capacitors connected with minimum lead length between the last two dynodes and the shield of the output cable.

The anode resistor, R_A, should be chosen so that pulses do not pile up on one another. If C_L is the capacitance seen by the anode (this includes the anode capacitance itself as well as the capacitance of the cable and measuring electronics) then the voltage developed across C_L during a pulse of total charge Q will be Q/C_L if the pulse duration is much smaller than $R_A C_L$. If the time between pulses is comparable to $R_A C_L$ however, pile up will occur. Thus, R_A should be chosen so that $R_A C_L$ is much less than the time between pulses but larger than the duration of the pulse.

Damage to the dynodes can result from exposure of the PMT to large amounts of light. When this type of exposure is anticipated, current limiting resistors in series with the dynode leads can be employed as safeguards.

In general, it is not wise practice to use a PMT divider unless you understand all the criteria that have gone into its design. It is the responsibility of the physicist to guarantee that the particular PMT and base he is working with is compatible with his particular experiment. Very often, the PMT manufacturer will provide optimal basing designs for specific applications.

PMT Noise

We have previously considered one source of PMT fluctuations, namely those encountered in stochastic processes. These fluctuations are unavoidable in a sense although they can be minimized with properly selected cathode-first dynode configurations and with good detector-PMT optical coupling. Other sources of PMT noise include light leaks and dark current. The former problem can be easily handled by carefully shielding the photocathode from ambient lighting by means of black paper and tape. The latter problem is due to a number of physical processes among which we might mention thermionic emission of photoelectrons, leakage currents from the divider chain, phosphorescence of the PMT window from a prior exposure to ambient lighting and radioactive contaminants of the PMT construction materials. Dark current can be controlled to a certain extent. For example one can scrupulously avoid exposing the tube to room lighting even with the high voltage turned off. Thermionic emission can be controlled by keeping the tube cool. Typical values of anode dark currents range from 0.1 to 100 nanoamps.

PMT Data

We provide for your convenience an abridged table of some of the many commercially available phototubes on the following page. It should be kept in mind that the state of the art in photomultiplier technology continues to advance even at this time. PMT manufacturers, such as RCA and EMI, are the best sources of up to date information.

TABLE 9-1. Properties of Some Commercially Available Photomultiplier Tubes.

A	B	C	D	E	F	H	J	K	L	M	P	Q	R	S
4516	1800	19	I	RCA	12.7	bia.	60	71	0.8(1500)	19.2	100.1	< 2		0.2
6199	1250	38	C	RCA	31.5	S-11	45	32	1(1000)	39.6	116.1	< 2.5	~25	4.5
6342A	1500	50	C	RCA	42.7	S-11	80	56	0.39(1250)	52.3	147.6	< 3.0	~25	4
9856	2000	50	V	EMI	43.9	bia.	70	75	0.71(900)	51.5	147	6.0		0.2
4523	2500	50	V	RCA	42.7	bia.	60	71	0.45(1500)	52.3	147.6	< 10	~50	0.5
50B01	2250	50	V	SRC	44	bia.	60	71	0.45(1500)	52.5	148			0.5
9815B	2500	50	I	EMI	46	bia.	65	78	0.78(1640)	53	149	1.6		1
8575	3000	50	I	RCA	45.7	bia.	85	97	10(2000)	53.3	145	< 2.5	~30	1
4524	2500	75	V	RCA	65.8	bia.	60	71	0.45(1500)	77.7	160.3	< 15	~55	4
75B01	2250	75	V	SRC	66	bia.	60	71	0.45(1500)	77.5	160			1
9758	2100	75	V	EMI	65	bia.	70	75	0.71(1100)	78	159	12		0.2
8055	2000	130	V	RCA	111.3	S-11	110	77	0.4(1500)	134.9	195.3	< 20	~100	4
4525	2500	130	V	RCA	111.3	bia.	67	80	0.4(1500)	134.9	195c3	< 20	~100	1.5
125B01	2500	130	V	SRC	119.4	bia.	67	80	0.45(1500)	134.9	195.3			1.5
4522	3000	130	I	RCA	114.3	bia.	77	88	30(2000)	133.4	295.9	< 3	~55	60
58AVP	3000	130	I	Amperex	110	S-11	70	56	100(2400)	131	269	2	45	2
9815B	2500	50	I	EMI	46	bia.	65	78	0.78(1640)	53	149	1.6		1
R647	1250	14	B	Hamamatsu	9	S-11	60		1.3(1000)	14	89			5
8850	3000	50	I	RCA	45.6	bia.	85	97	7.3(2000)	53.3	145	< 2.5	~30	0.6

A = model number
B = maximum voltage overall
C = nominal diameter (mm)
D = dynode structure: C = circular, V = venetian blind, I = in-line or linear
E = manufacturer
F = photocathode size (mm) diameter
H = spectral class
J = cathode luminous sensitivity in microamps per lumen
K = cathode radiant sensitivity in milliamps per watt at 400 nanometers
L = gain at given voltage (in parentheses) $\times 10^6$
M = maximum diameter (mm)
P = maximum length (mm)
Q = rise time at maximum voltage (nanoseconds)
R = transit time at maximum voltage (nanoseconds)
S = dark current (a very approximate number due to large variation in method of measurement between different manufacturers) (nanoamps)

We will now briefly describe the electronic processing of signals from detector current sources such as PMT's, gas ionization detectors or semi-conductor diode detectors.

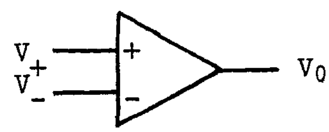
Operational Amplifiers

The operational amplifier (often referred to as an op-amp) is an extremely versatile analog circuit element that can be configured to provide a wide variety of functions using simple external components. Operational amplifiers can be made to add, subtract, multiply, divide, integrate, differentiate, and perform other complex transformations to signals provided at their inputs. Due to this versatility and their availability in low cost monolithic integrated circuit packages, operational amplifiers form the backbone of analog circuit design.

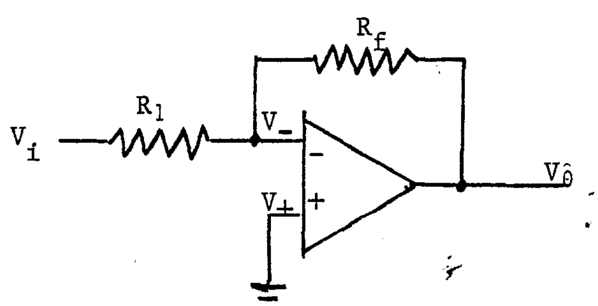
An operational amplifier has the following characteristics:

1. High voltage gain: $A = 10^3 - 10^9$ (in circuit analysis, A is allowed to become infinite)
2. High input impedance (a useful approximation is that no current is drawn at the inputs)
3. Low output impedance (the output voltage is virtually independent of the load)
4. High bandwidth

The following circuit model is used to represent an operational amplifier:



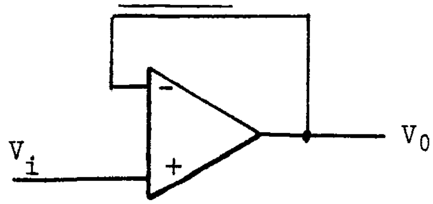
and its operation can be described mathematically by $V_0 = A(V_+ - V_-)$ where V_0 is the output voltage, $V_{+,-}$ are the voltages at the positive and negative inputs respectively, and A is the *open loop gain*. As a simple example of its use, we will turn this op-amp into an inverting amplifier through the use of *feedback*. We connect the output to the negative input through a resistor and ground the positive input as follows:



The circuit analysis proceeds as follows: $V_0 = A(V_+ - V_-) = -AV_-$. In order for this to be true for $A \gg 1$, V_- must be very small (this is sometimes expressed by saying that V_- is at *virtual ground*). If this is so, the requirement of no current being drawn at the input implies that $V_i/R_1 = -V_0/R_f$ or $V_0 = -V_iR_f/R_1$. Some other common examples of the use of operational amplifiers are shown below.

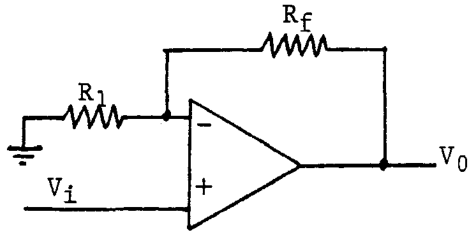
DESCRIPTION — CIRCUIT — FUNCTION

Voltage follower (converts high impedance input to low impedance output)



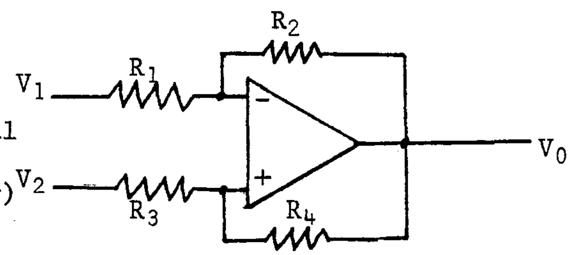
$V_0 = V_i$

Non-inverting amplifier



$V_0 = V_i(R_1 + R_f)/R_1$

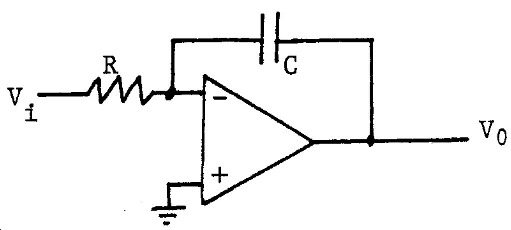
Differential amplifier (subtracter)



$V_0 = \frac{R_4}{R_1}\left(\frac{R_1 + R_2}{R_3 + R_4}\right)V_2 - \frac{R_2}{R_1}V_1$

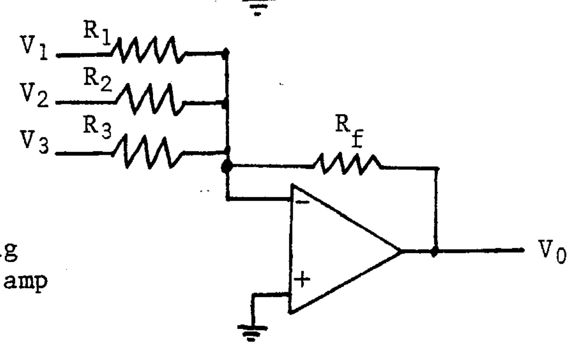
$= (V_2 - V_1)R_2/R_1$ if $R_1=R_3$ and $R_2=R_4$

Integrator



$V_0 = -\frac{1}{RC}\int_0^t V_i dt$

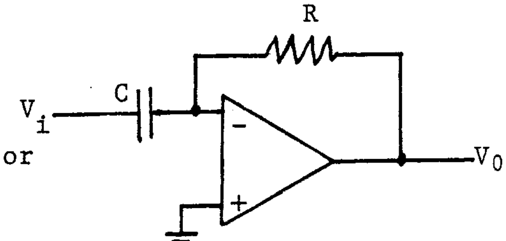
Inverting Summing amp



$V_0 = -R_f(V_1/R_1 + V_2/R_2 + V_3/R_3)$

$= -(V_1 + V_2 + V_3)$ if $R_f=R_1=R_2=R_3$

Differentiator



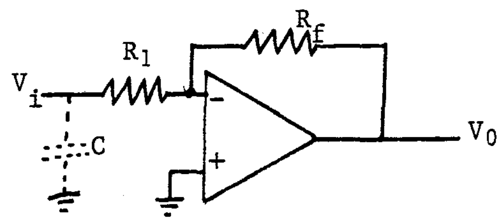
$V_0 = -RC \frac{dV_i}{dt}$

Many other applications of op-amps, including logarithmic amplifiers, multipliers, dividers, active filters, waveform generators, constant current sources etc. are possible. For a complete description see Tobey et al. Operational Amplifiers: Design and Applications, 1971.

Preamplifiers

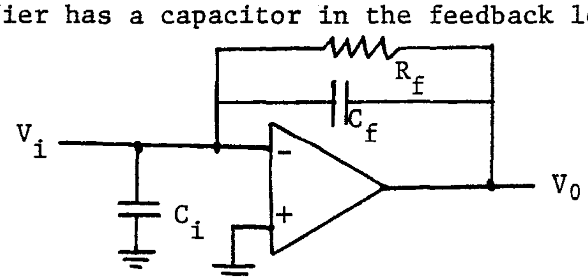
A preamplifier takes the signal from a detector and amplifies it in a way that preserves the maximum signal to noise ratio with a minimum of shaping. The optimal design of preamplifiers is dictated by the specific properties of the detector such as rise time, capacitance etc. The most common devices serving as input sources for preamplifiers are photomultiplier tubes, solid state detectors, and gas detectors. Three types of preamplifiers are commonly used with these detectors.

A *voltage sensitive preamplifier* is an ordinary linear amplifier which produces an output voltage proportional to the voltage presented on its input.



The input voltage will depend on the detector and cable capacitance C. For example, if R_1C is much larger than the duration of the current pulse then the maximum value of the input voltage, V_i, will be $\int idt/C$ where $Q = \int idt$ is the total charge delivered by the current pulse from the detector. Since $V_0 = -R_fV_i/R_1$, as we have seen before, the gain of such an amplifier depends on the capacitance of the detector and cabling. This is usually not desirable since the capacitance of many detectors fluctuates with changes in bias voltage and other physical parameters resulting in an unstable overall gain.

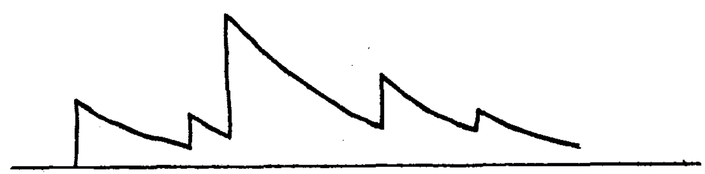
The *charge sensitive preamplifier* solves this problem and is used in nearly all applications where accurate pulse height analysis is important (we might mention however, that very often it is adequate to preceed the above voltage amplifier with a parallel RC circuit to ground for use with PMT signal analysis; this scheme is generally less expensive than a charge sensitive scheme would be). A charge sensitive amplifier has a capacitor in the feedback loop as shown below:



Here C_i is the capacitance of the detector and cabling etc. R_f is usually chosen to be large so that $R_fC_f \gg \tau$, the time duration of the current pulse. The output voltage V_0 is given by $V_0 = -AV_- = -AV_i$ and since the charge on C_f is just $Q_f = C_f(V_i - V_0)$ (we have ignored the discharge through R_f) and the charge on C_i is just $Q_i = C_iV_i$, where $Q_i + Q_f = Q$, the total charge delivered by the detector, we can write V_0 as:

$$V_0 = -AQ/(C_i + C_f(1 + A)) \quad .$$

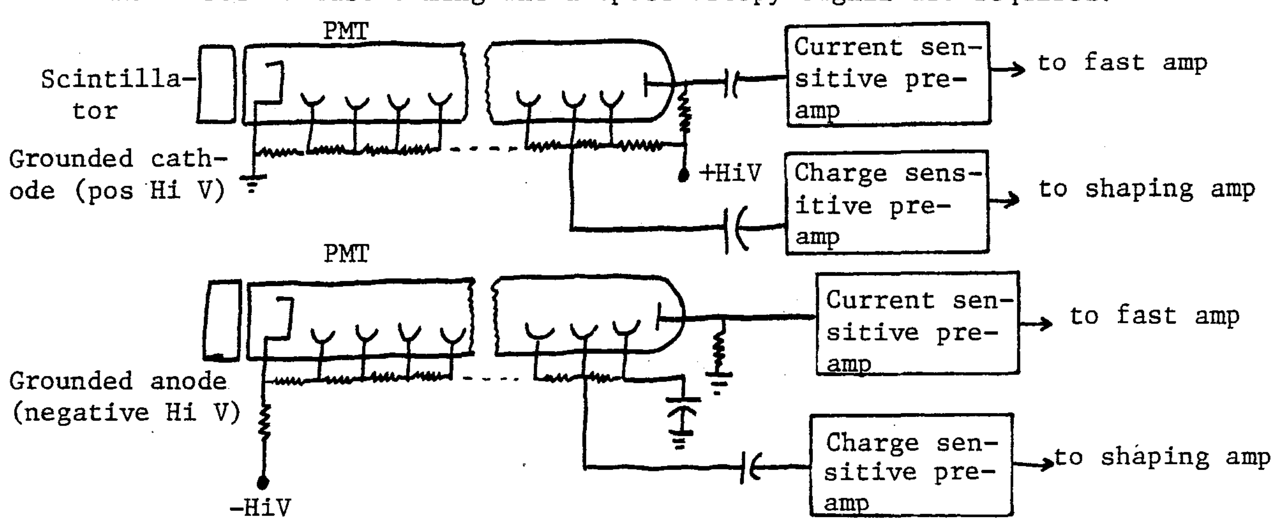
Notice that the charge collected on C_f is $Q_f = C_fV_i(1 + A)$ as if it had an effective capacitance of $C_f(1 + A)$. In the limit $A \to \infty$ all the charge Q gets dumped on C_f and the output voltage, $V_0 \simeq -Q/C_f$, is not sensitive to the detector and cable capacitance C_i. The output of a charge sensitive amplifier rises sharply with a time constant usually determined by the detector rise time and then falls slowly with a time constant given by R_fC_f. The larger R_fC_f is, the more linear will be the amplification, but at high rates these long *tail pulses* can *pile up* on one another as shown below.



Without suitable shaping to filter out the long tail, such pulses would be unsuitable for pulse height analysis or pulse height discrimination. Shaping amplifiers usually follow charge sensitive amplifiers for this reason.

When fast timing signals are required, but accurate pulse height analysis is not required, a *current sensitive preamplifier* is used. Current sensitive amplifiers are just fast voltage sensitive amplifiers with a resistor across the input (usually 50Ω). They produce a voltage pulse which closely follows the shape of the input current pulse and their output is capable of driving a cable with minimal distortion.

Shown below are typical configurations for charge sensitive and current sensitive preamplifiers used in conjunction with photomultiplier tubes where both a fast timing and a spectroscopy signal are required.

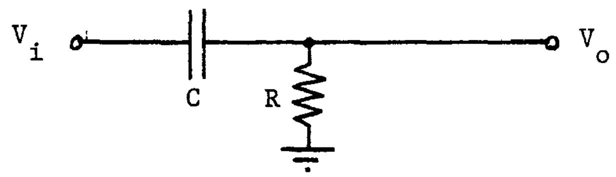


As can be seen, coupling capacitors must be used when signals are extracted at other than ground potential. These coupling capacitors can introduce a baseline shift at high rates and thus, DC coupling is preferred in discrimination of high rate pulses. The use of negative high voltage is inconvenient though, since a shield surrounding the photocathode must be held at this potential (to reduce spurious discharge). This presents a shock hazard unless complcated double shielding is employed. In the diagrams above, the negative timing signal is extracted from the anode, while a positive spectroscopy signal (positive because more electrons are leaving the dynode than striking it) is extracted from a dynode.

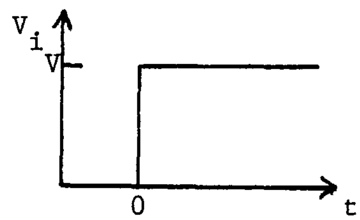
Shaping Amplifiers

The output of a charge sensitive preamplifier is not suitable for accurate pulse height analysis or discrimination. The fast rise and long flat tail make it difficult to latch on to the peak value and the pile up of additional pulses on the tails of others would result in a rate dependent pulse height. Since the amplification chain would have to respond to the high frequency components of the fast leading edge, high frequency noise would pass through, adding to the uncertainty with which the height of the peak could be measured. The role of a shaping amplifier is to filter out the very high and very low frequency components of the signal in a way that maximizes signal/noise and preserves the linearity of the maximum amplitude.

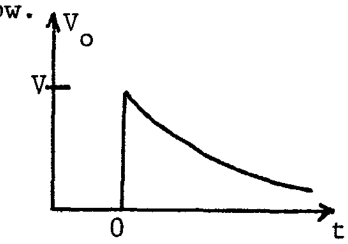
The most common technique for pulse shaping is through the use of resistor-capacitor networks (RC networks). The circuit below is called a *high pass CR filter* because it preferentially attenuates those components



of the input signal with frequencies smaller than $\sim(2\pi RC)^{-1}$. The attenuation factor (V_o/V_i) for the amplitude of a sinusoidal voltage of frequency f is $(1 + (2\pi RCf)^{-2})^{-\frac{1}{2}}$. The response for a step function input of height V at time t=0 for this circuit is shown below.



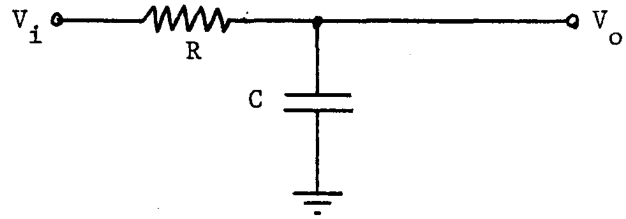
$V_i = 0,\ t < 0.\quad V_i = V,\ t \geq 0$



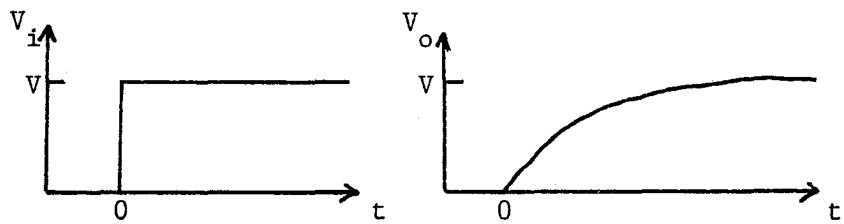
$V_o = 0,\ t < 0.\quad V_o = Ve^{-t/RC},\ t \geq 0$

The high pass CR filter is sometimes called a differentiator since for $RC \ll T$ (T is the smallest time over which the signal changes by an appreciable amount) the output voltage is given by $V_o = RC(dV_i/dt)$.

The circuit shown below, which results from swapping the positions of the resistor and capacitor, is called a *low pass RC filter* because it



preferentially attenuates those components of the input signal with frequencies larger than $\sim(2\pi RC)^{-1}$. The attenuation factor for the low pass filter is $(1 + (2\pi RCf)^2)^{-1/2}$, where f is the frequency of a sinusoidal input. The response of the low pass filter to a step function input is shown below.



$V_i = 0,\ t < 0.\quad V_i = V,\ t \geq 0 \qquad V_o = 0,\ t < 0.\ V_o = V(1 - e^{-t/(RC)}), t \geq 0.$

In the extreme where $RC \gg T$ (T is the time duration of the signal) the low pass filter performs as an integrator with $V_o = \int V_i dt/RC$.

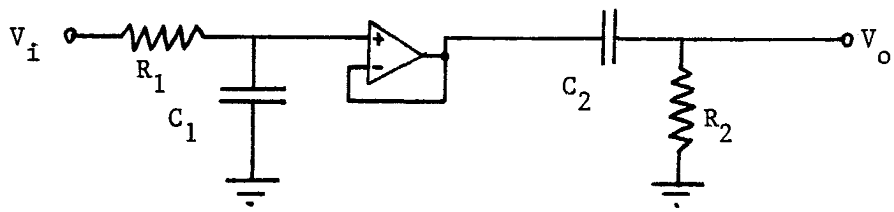
Neither of these two circuits acting alone is sufficient to produce a nicely shaped pulse for spectroscopy. Combinations of RC and CR filters can be used to provide suitable pulse shaping. Laplace transform techniques are the easiest way to analyse the response of these combinations. For the high pass filter we can write the equation

$$V_i = Q/C + V_o \quad \text{or} \quad dV_i/dt = V_o/(RC) + dV_o/dt.$$

Taking the Laplace transform of the equation on the right we have

$$s\mathcal{L}_i(s) = \mathcal{L}_o(s)/(RC) + s\mathcal{L}_o(s) = (s + 1/(RC))\mathcal{L}_o(s)$$

The transfer function is defined as $\mathcal{L}_o(s)/\mathcal{L}_i(s)$ and is given by $s/(s + 1/(RC))$ for the high pass filter. Similar analysis yields $1/(s + 1/(RC))$ for the transfer function of the low pass filter. The transfer function for a series of circuits separated by voltage followers (to prevent interaction) is just the product of the individual transfer functions. The output function V_o can be obtained by taking the inverse Laplace transform of the product of the transfer function with the Laplace transform of the input function. As an example, let us consider the CR-RC filter shown below.

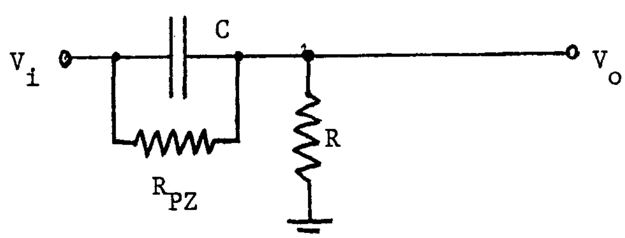


If we let $\tau_1 = R_1C_1$ for the low pass RC filter and $\tau_2 = R_2C_2$ for the high pass CR filter, the overalll transfer function is $s(s + 1/\tau_1)^{-1}(s + 1/\tau_2)^{-1}$. For a step function input at t = 0 of height V, $\mathcal{L}_i(s) = V/s$ and

$$V_o = V\mathcal{L}^{-1}\left[\frac{1}{(s + 1/\tau_1)(s + 1/\tau_2)}\right] = \begin{cases} V\dfrac{\tau_1\tau_2}{\tau_1 - \tau_2}\left(e^{-t/\tau_1} - e^{-t/\tau_2}\right) & \tau_1 \neq \tau_2 \\ Vte^{-t/\tau_1} & \tau_1 = \tau_2 \end{cases}$$

While the pulse shape from a CR-RC filter is better than that from a single stage filter, it is not optimal. Detailed analysis shows that the highest signal to noise ratio is achieved for a Gaussian shape. While it is impossible to achieve this in practice, an excellent approximation to a Gaussian can be obtained by following a CR filter with about 4 stages of RC filtering. A disadvantage of this CR-RC^n filter is that pile up is more of a problem at high rates because of the longer tail on the trailing edge of the output pulse.

Another problem that arises in shaping is that the charge sensitive preamplifier pulses are not perfect step functions but decay with a long time constant τ_i. In the analysis of the CR-RC filter we saw that the s in the numerator of the transfer function was cancelled by the 1/s arising from the Laplace transform of the input step function. The Laplace transform of the exponential function $\exp(-t/\tau_i)$ is $1/(s + 1/\tau_i)$ which only will cancel the s in the numerator when $\tau_i \to \infty$. The net effect of the s in the numerator is that the output of any CR-RC^n filter will have an undershoot following the main pulse which persists for a time comparable to the decay time τ_i of the charge amp. Thus, pulses that arrive within a time τ_i of the preceeding pulse will have a reduced amplitude. This problem can be remedied by modifying the high pass CR section of the filter in such a way that there will be no term involving s in the numerator of the transfer function. This can be accomplished by adding a resistor across the capacitor in the CR section as shown below.



The transfer function for this section becomes $(s + 1/(R_{PZ}C))/(s + 1/RC)$ instead of $s/(s + 1/RC)$. By setting R_{PZ} such that $R_{PZ}C$ is equal to τ_i the term $(s + 1/(R_{PZ}C))$ will just cancel the term $(s + 1/\tau_i)$ caused by the decay of the charge amp. This procedure is called *pole zero cancellation* since the pole at $s = -1/\tau_i$ is removed by the introduction of the resistor R_{PZ}. In a simplified sense, the introduction of the resistor allows a small fraction of the unmodified exponentiallly decaying input signal to feed through to the output, cancelling the undershoot. Shaping amplifiers usually have an adjustment slot for varying R_{PZ} to compensate for the specific decay constant of the user's charge amplifier.

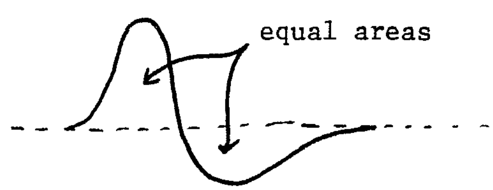
Input

Without pole zero cancellation

With pole zero cancellation

At low rates, and where accurate pulse height analysis is required, unipolar Gaussian shaping is ideal. Whenever AC coupling is employed, as in the case of the CR filter or if a coupling capacitor is required to transmit pulses out at ground level (as in the case of phototubes with positive high voltage described previosly) true unipolar pulses cannot be transmitted since no net charge can be transported across the capacitor. This results in a *baseline shift* which renders equal the areas above and below the zero voltage line. Without some sort of *baseline restoration* the peak voltage measured by a pulse height analyzer will be reduced by an amount that depends on the pulse rate. Various baseline restoration circuits are thus incorporated into amplifiers to correct this problem. At some very high rate the baseline restorer will no longer be able to keep the baseline at zero and in this case *bipolar* shaping is preferable even though the signal to noise ratio is not as good as for the unipolar pulses. Bipolar shaping amplifiers are designed so that the area under the pulse exactly cancels the area under the undershoot so that the baseline is automatically at zero a short time after the pulse has occurred.

Unipolar pulse



Bipolar pulse