OBSERVATIONS OF NEUTRAL V-PARTICLE DECAYS
WITH THE 48" MAGNET CLOUD CHAMBERS

Thesis by
Victor A. J. van Lint

In Partial Fulfillment of the Requirements
for the Degree of
Doctor of Philosophy

California Institute of Technology
Pasadena, California
1954
ACKNOWLEDGEMENTS

Most of the experimental data included in this thesis were obtained with the new 48" magnet cloud chambers at Caltech. The design and construction of the equipment required the efforts of a large number of people, only a few of whom can be named here.

Professors C. D. Anderson, E. W. Cowan, and R. B. Leighton were the guiding hands of the entire project. Their experience with many preceding cloud chambers, together with a sound knowledge of fundamental principles, enabled them to design this magnet cloud chamber equipment for efficient operation. The fact that the cloud chambers yielded the first useful photographs one year after the laboratory was first occupied is a compliment to their design. All major modifications and additions were discussed with them and they invariably contributed suggestions of great practical value. They also contributed greatly to the results of the V-particle analysis by discussing the data and their interpretation.

Many of the measurement techniques discussed in Section II have been adapted from those used by Dr. Leighton with the 18" magnet cloud chambers. The larger dimensions of the new equipment, as well as the desirability of achieving the maximum precision in reducing the available data, necessitated the introduction of some small modifications. The net result remains one of the fastest available tech-
niques for analyzing V-particles which nevertheless retains the precision inherent in the photographs.

Foremost among the many graduate students who worked with the author was Mr. George H. Trilling. He joined the project at its beginning and has contributed greatly to the construction and operation of the equipment and the analysis of the data. He has also participated in many fruitful discussions concerning improvements to the equipment, interpretation of the data, and many other problems in physics.

Other graduate students who contributed greatly to the construction work and the operation of the equipment include Mr. Carl A. Rouse and Mr. Arnold A. Strassenburg. Their assistance with the operation of the equipment enabled the author to spend much of his time analyzing the data.

Dr. R. V. Adams assisted the project during the summer of 1953 by analyzing many V-particle photographs. His contribution to the results through discussions of improvements in the measurement technique and interpretation of the data was also extremely valuable.

No large construction job can be undertaken efficiently without the assistance of capable draftsmen and machinists. Mr. R. B. Pollock made most of the design drawings and supervised many of the machining operations. The author is particularly indebted to Mr. P. F. Fuselier, whose advice on machine shop practice and contribution to
some of the precision machining work was invaluable. It is, of course, not possible to name the many others who participated in the construction work and thus helped to make the project a success, but their contributions are greatly appreciated.
ABSTRACT

This thesis is the first to result from operation of the 48" magnet cloud chambers at the California Institute of Technology. The apparatus is not described here in any detail. The techniques used to measure track curvatures and the positions of points inside the chambers are discussed in detail as is the analysis of the possible errors in these quantities. A formula is derived for computing the momentum of a particle from its measured curvature on the film. It includes corrections for the nonaxial components of the magnetic field and the effect of the conical projection involved in the photography. A method for correcting approximately for the variation of the magnetic field along the track is also presented.

A summary of the dynamics of the two-body decay of a neutral V-particle is presented. A discussion of the uses of plots of \( a \) and \( R_t \) for a two-body decay is included. The dynamics of some simple three-body decays are discussed. The distribution of the momenta of one of the decay products is derived, including only the effects of the density of states in phase space. An attempt is made to analyze the observable properties of a three-body decay when the two charged products are treated as products of a two-body decay.

The results from the analysis of the best cases of \( \Lambda^0 \) and \( \theta^0 \) decays observed in the 48" magnet cloud chambers are summarized. They include a value for the energy release in the \( \Lambda^0 \) decay of \( (34.8 \pm 1) \) Mev, and a preliminary discussion of the coplanarity and transverse momentum balance for those cases in which an origin for the \( V^0 \)-particle can be located. In addition, an excellent case of the decay of a neutral V-particle into two light secondaries with an energy release much lower than that for the usual \( \theta^0 \) decay scheme forms the basis for the discussion of all such "anomalous \( \theta^0 \)" decays observed in the three Caltech magnet cloud chambers.
# CONTENTS

<table>
<thead>
<tr>
<th>SECTION</th>
<th>TITLE</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>Experimental Technique</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>Apparatus</td>
<td>1</td>
</tr>
<tr>
<td>B</td>
<td>Magnetic Field Measurement</td>
<td>4</td>
</tr>
<tr>
<td>C</td>
<td>Curvature Measurement</td>
<td>7</td>
</tr>
<tr>
<td>D</td>
<td>Co-ordinate Measurement</td>
<td>9</td>
</tr>
<tr>
<td>E</td>
<td>Estimation of Errors</td>
<td>14</td>
</tr>
<tr>
<td>II</td>
<td>Momentum Calculation</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>Formulation of the Problem</td>
<td>20</td>
</tr>
<tr>
<td>B</td>
<td>Exact Equation of the Track in Space in a Uniform Magnetic Field</td>
<td>21</td>
</tr>
<tr>
<td>C</td>
<td>Projection of the Track on the Rear Chamber Wall</td>
<td>23</td>
</tr>
<tr>
<td>D</td>
<td>Calculation of the Sagitta of the Projected Track</td>
<td>26</td>
</tr>
<tr>
<td>E</td>
<td>Measurement of the Direction of the Track</td>
<td>31</td>
</tr>
<tr>
<td>F</td>
<td>Effect of an Inhomogeneous Magnetic Field</td>
<td>32</td>
</tr>
<tr>
<td>G</td>
<td>Construction of the Tangent at One End of the Track</td>
<td>34</td>
</tr>
<tr>
<td>III</td>
<td>Decay Dynamics</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>Introduction</td>
<td>36</td>
</tr>
<tr>
<td>B</td>
<td>Notation</td>
<td>37</td>
</tr>
<tr>
<td>C</td>
<td>Two-Body Decay Dynamics</td>
<td>39</td>
</tr>
<tr>
<td>D</td>
<td>Relation Between Two-Body and Three-Body Decays</td>
<td>49</td>
</tr>
<tr>
<td>E</td>
<td>General Three-Body Decay</td>
<td>51</td>
</tr>
<tr>
<td>F</td>
<td>Uncoupled Three-Body Decay</td>
<td>53</td>
</tr>
<tr>
<td>G</td>
<td>Cascade Three-Body Decay</td>
<td>55</td>
</tr>
<tr>
<td>H</td>
<td>Three-Body Decay With One Heavy Secondary</td>
<td>58</td>
</tr>
<tr>
<td>IV</td>
<td>Preliminary Results On Neutral V-Particle Decays</td>
<td></td>
</tr>
<tr>
<td>A</td>
<td>Introduction</td>
<td>61</td>
</tr>
<tr>
<td>B</td>
<td>$A$ Decays</td>
<td>64</td>
</tr>
<tr>
<td>C</td>
<td>$\phi^0$ Decays</td>
<td>71</td>
</tr>
<tr>
<td>D</td>
<td>&quot;Anomalous $\phi^0$&quot; Decays</td>
<td>77</td>
</tr>
<tr>
<td>FIGURES</td>
<td></td>
<td></td>
</tr>
<tr>
<td>---------</td>
<td></td>
<td></td>
</tr>
<tr>
<td>NUMBER</td>
<td>TITLE</td>
<td>PAGE</td>
</tr>
<tr>
<td>1.</td>
<td>Cosmic Ray Laboratory</td>
<td>2</td>
</tr>
<tr>
<td>2.</td>
<td>Chamber Geometry</td>
<td>3</td>
</tr>
<tr>
<td>3.</td>
<td>Graphical Constructions</td>
<td>11</td>
</tr>
<tr>
<td>4.</td>
<td>Q-Plot</td>
<td>44</td>
</tr>
<tr>
<td>5.</td>
<td>$V^0$ Decay Yields</td>
<td>62</td>
</tr>
<tr>
<td>6.</td>
<td>Q-Values of $\Lambda^0$ Decays</td>
<td>66</td>
</tr>
<tr>
<td>7.</td>
<td>Photograph 14979 ($\Lambda^0$ Decay)</td>
<td>69</td>
</tr>
<tr>
<td>8.</td>
<td>Photograph 23023 ($\Lambda^0$ Decay)</td>
<td>70</td>
</tr>
<tr>
<td>9.</td>
<td>$\Delta P_T$ vs. $P_0\delta$ for $\Lambda^0$'s and $\phi$'s</td>
<td>72</td>
</tr>
<tr>
<td>10.</td>
<td>Photograph 9630 ($\phi$ Decay)</td>
<td>73</td>
</tr>
<tr>
<td>11.</td>
<td>Photograph 22332 ($\phi$ Decay)</td>
<td>74</td>
</tr>
<tr>
<td>12.</td>
<td>Q-Values of $\phi$ Decays</td>
<td>75</td>
</tr>
<tr>
<td>13.</td>
<td>Photograph 8796 (&quot;Anomalous $\phi$&quot; Decay)</td>
<td>78</td>
</tr>
<tr>
<td>14.</td>
<td>Photograph 12590 (&quot;Anom. $\phi$&quot; Decay)</td>
<td>80</td>
</tr>
<tr>
<td>15.</td>
<td>Photograph 19143 (&quot;Anom. $\phi$&quot; and $\phi$ Decays)</td>
<td>83</td>
</tr>
<tr>
<td>16.</td>
<td>Data on &quot;Anomalous $\phi$&quot; Decays</td>
<td>86</td>
</tr>
</tbody>
</table>
I. EXPERIMENTAL TECHNIQUE

A. Apparatus

The 48" magnet has an air gap which measures 46" high, 24" wide, and 12" deep. The magnetic field in the air gap averages about 8000 gauss and is produced by a direct current of 800 amperes at 110 volts. The magnet is water cooled and is provided with safety circuits which de-energize it in case of a failure in the cooling system. Fig. 1 is a photograph looking down on the magnet, also showing the rack of electronic chamber control circuits.

The cloud chambers are similar in design, but larger in scale, to the 18" magnet cloud chambers which were designed and operated by Dr. R. B. Leighton. They are rigid-piston, rectangular body, expansion-type chambers with an illuminated volume measuring 22" wide, 8" high, and 8" deep. The tracks are photographed through a plate glass front window against a background of black velvet fastened to the piston. The other inner surfaces are polished and chromium plated except for an illumination window in one side. The geometry of the chambers, geiger counters, absorbers, and cameras is shown in Fig. 2. Four chambers, each above an other, were used until July 1953, at which time the upper two chambers were replaced by one double-height chamber.

The cloud chambers, as well as the air and metal surfaces touching them, are held at an accurately controlled temperature by passing water through copper tubes
Fig. 1. Cosmic Ray Laboratory

This photograph of the cosmic ray laboratory shows the 48" magnet from above. The iron flux return path can be seen in the bottom foreground. The rear current coils and cooling jackets are barely visible at the right edge of the photograph. The lead blocks above the chambers can just be seen in the open thermostat box at the right. The magnet cooling water exhaust manifolds are in the foreground. The electronic control circuits are shown at the left edge. The cameras are in the box which protrudes from the side of the magnet facing the electronics rack.
Fig. 2. Chamber Geometry

In July 1953 the top two cloud chambers were replaced by a single double-height chamber.
soldered in convenient locations. The temperature of the water is then held constant by an electrically regulated control system.

The cloud chambers are filled with Argon gas at an absolute pressure of 105 cm. Hg., and the vapor from a 60% ethyl alcohol, 40% water mixture. The resultant expansion ratio was usually near 1.075.

The entire operation is automatic. Geiger counters function as detectors of penetrating showers, and electronic control systems initiate the various cycling operations.

The stereoscopic photographs are taken on two strips of 70mm. Kodak Linagraph Pan film. They can be reprojected onto a plane at full size through an optical system similar to that used in the photography. The graphical constructions described in Section I-D are then performed on a sketch made of the projected views.

B. Magnetic Field Measurement

The large holes required in the front pole piece of the magnet for observation of the cloud chambers introduce appreciable nonuniformities in the magnetic field in the air gap. In addition, the depth of the air gap is comparable to its width so that there is appreciable bulging of the lines of magnetic flux. In order to make accurate measurements of the momentum of a cosmic ray track it is
necessary to know the value, as well as the direction, of the magnetic induction along the track. Measurements were carried out to determine $B_z$ (the component of $B$ normal to the pole faces) and the component ratios, $p = \frac{B_x}{B_z}$, $q = \frac{B_y}{B_z}$.

(The $X$ direction is horizontal, the $Y$ direction vertical, so as to form a right handed co-ordinate system with the $Z$ co-ordinate increasing toward the camera).

The normal component, $B_z$, was measured using a flip coil and a G. E. Model 248 Fluxmeter. The flip coil was constructed of one turn of Formvar coated copper wire wound around a $3/4''$ diameter bakelite disk, which was connected to a bakelite handle by means of a hollow cylindrical bearing. The leads from the coil were twisted and led through the bearing and along the handle. The coil was flipped by a coil spring around the bearing upon the release of a trigger. This design was adopted to avoid errors caused by the flux that is cut by the leads when a rigid coil assembly is flipped as a unit. The absolute value of the magnetic field was determined at a standard $(X,Y)$ location for various $Z$ co-ordinates in the gap by comparing the fluxmeter deflection with that when the coil was flipped in a standard permanent magnet. This magnet was in turn checked against a proton resonance frequency controlled magnet using the same fluxmeter and the same flip coil. The probable error in the determination of the absolute value of the magnetic field is less than
±50 gauss. Furthermore, the difference between $B_z$ at various $(X,Y)$ co-ordinates and that at the standard $(X,Y)$ location for the same $Z$ depth was measured using a coil of many turns and correspondingly higher accuracy. Therefore, accurate, repeated fluxmeter readings needed to be taken for only the few absolute measurements.

The component ratios, $p$ and $q$, were measured using a brass holder which supported a steel needle freely at its center. A small piece of graph paper was mounted parallel to the bottom surface of the holder so that the deflection of the needle from a perpendicular position can be determined by reading the $X$ and $Y$ co-ordinates subtended by the end of the needle on the graph paper.

The values of $B_z$, $p$, and $q$ were then plotted in the form of contour maps for the planes $Z=1, 11, 16, \text{and } 21 \text{ cm}$. (The $Z=0$ plane is the plane of the front surface of the chamber piston in the compressed position). The extra plane between the center and the front was measured because the hole in the front pole piece causes the field to change rapidly in this region. The contour interval and the distances between planes were chosen so that linear interpolations would not make errors of greater than $\frac{1}{4}\%$. Since the uncertainty in the momentum of a single track due to multiple scattering in the Argon gas used in the chambers is never less than about 2%, the error in determination of the magnetic field is usually negligible. However, it was
found that it is necessary to keep the temperature of the magnet coils reasonably constant since an increase in the average cooling water temperature from $35^\circ$C to $60^\circ$C produced magnetic field changes due to redistribution of current in the coils of the order of 3%.

C. Curvature Measurement

In using the momentum calculation formula derived in Section II, the quantity which characterizes the apparent curvature of the track is the sagitta at the center of the conically projected track. Therefore, it is necessary to measure the location of the track at three points only. However, in order to check on convection distortions or to detect a possible single scattering along the track, it is convenient to check that the track is circular within the perturbations introduced due to a changing magnetic field and the $\delta_z$ correction (see Section II). The following techniques have been used for measuring the curvature of tracks.

The most accurate technique is to determine co-ordinates of points on the track using a precision comparator and to plot these co-ordinates on graph paper, magnifying transverse dimensions by a factor of 10 or 20 times more than the longitudinal dimensions. The resulting elliptical track can be approximated closely by standard parabolas. The sagitta of the track can also be measured.
This procedure is time consuming and is difficult to apply when faint tracks are involved, since some contrast is lost in the optical system of the comparator microscope. Extreme caution must also be taken not to emphasize large clusters of droplets which appear along the track, since these are often due to low energy delta rays which might have been ejected over to one side of the main track. In order to avoid these difficulties, Dr. Leighton designed and constructed an instrument which mechanically magnifies the lateral dimension by about 10 times. It consists of two round aluminum rails on which a carriage rides. On this carriage a fine pointer and a pencil, provided with ways for lateral motion, are driven by wires. Each wire passes over one of two pulleys which are coupled together and whose diameters differ by a factor of about 10. In this manner the pencil is constrained to move at a rate about 10 times that of the pointer. By following a track accurately with the pointer, the pencil traces the corresponding laterally magnified image. By going over the track a number of times in both directions, small random errors in setting the pointer on the track are cancelled out. It is also possible to sight carefully along the track to enable one to see a faint image. The resulting plot is then used similarly to a comparator plot: namely, the consistency of the plot with a parabola is checked and the sagitta measured. Possible sources of error
in this method lie in the subjective nature of the track following procedure and the reliance upon the linearity of the reprojection system.

Another still faster technique is to compare the track directly with circles scribed on lucite sheets. Since a solid line on the sheet obscures the track beneath it, it is convenient to have two comparison circles of the same radius separated by a distance slightly greater than a track width. This method is not as accurate nor as objective as the previously described methods but does take much less time. It can therefore be profitably used on noncritical curvature measurements.

When the track is extremely curved and has a large sagitta, it may be sufficient to make a direct measurement of the sagitta, without any artificial magnification, using a straight-edge and an accurate scale. This method is perhaps less subjective than a comparison with circles and the result may correspond more closely to the desired sagitta measurement, but it overemphasizes very short sections of the track.

D. Co-ordinate Measurement

The positions of pertinent points in the cloud chamber are determined from a full-size reprojection of the stereoscopic photographs. The two conically projected views, hereafter called apparent views, are projected
onto a horizontal plate glass on which a sheet of vellum is placed. The two apparent views of fiducial marks on the front surface of the chamber piston are accurately superimposed and the separation between marks normalized to full size by adjusting the projector magnification. Many adjustments are needed on the projector to make the superposition very accurate over the whole extent of the piston.

Once the images of the piston are in coincidence, the $Z$ co-ordinate of any point in space (corrected for the expansion of the chamber) can be calculated from the horizontal separation between the two apparent views of the point. Thus $Z = \frac{1}{1 + \epsilon} \frac{D\delta X}{S + \delta X}$, where $D$ is the distance from the front surface of the piston to the front principal plane of the lens ($D = 125$ cm. for the 48" magnet cloud chambers); $S$ is the separation between the camera lenses ($S = 17.8$ cm.); $1 + \epsilon$ is the expansion ratio ($\epsilon \approx 0.075$); and $\delta X$ is the measured horizontal separation.

In order to measure the other co-ordinates, $X$ and $Y$, of a point, it is convenient to construct the orthogonal projection of the point on the piston surface. This is done from the sketched apparent views as shown in Fig. 3. The apparent views of a point ($A_L$, $A_R$) and the corresponding lens axes ($L_L$, $L_R$) are joined by straight lines. Each of these lines is the orthogonal projection of a line
Fig. 3. Graphical Constructions

This figure demonstrates the graphical constructions applied to the analysis of a $\nu^0$ decay event. The construction of the orthogonal view is described in Section I-D; the construction of the tangent to the track in Section II-G.
passing through the lens center and the corresponding conical projection of the point. Thus each line must contain the orthogonal projection of the point, which must therefore lie at their intersection (A).

It should be noted that the above construction introduces a large possible construction uncertainty in the X co-ordinate if the point has a Y co-ordinate near zero; e.g. point 0 of Fig. 3. However, since the X co-ordinate is related to a given pair of apparent co-ordinates, X_{aL} and X_{aR}, independently of Y, a correct procedure is to reproduce the interval δX at some distance directly below or above O_L-O_R. Then the X co-ordinate can be reprojected back to the original Y distance after it has been determined by the prescribed graphical procedure.

When locating a possible origin for a V-particle it is necessary to extrapolate tracks into the regions beyond the chambers. In order to do this most accurately, it is desirable to continue the curvature measured in the chamber along the extrapolation, provided the distance is not so great that the magnetic field has decreased appreciably. To do this quickly a rough measurement of the radius of curvature is made using standard circular arcs scribed on sheets of lucite. The circular arc is then allowed to extend slightly beyond the track and a mark is made at the end of the arc. This procedure is continued, locating a number of points on the extrapolation, thus extending it
as far as is needed. The extrapolated curve is then approximated by straight line segments drawn between the constructed points, and the curve in the chamber is sketched. Often, when the tracks are almost straight, only one point near the location of the origin has to be constructed. After having extrapolated many tracks one should weight the straightest ones most heavily in deciding the exact location of the origin because errors due to multiple scattering of the particles in the solid material have less effect on particles of large $P\beta$. Using the above constructions, and the method for constructing tangents to tracks derived in Section II-C and illustrated in Fig. 3, it is possible to measure $\Delta X$, $\Delta Y$, and $\Delta Z$ for all pertinent lines. We thus have available all the data needed to compute the momenta of the tracks as well as the angles between them.

The angles can be computed using the scalar product of the vectors: $\cos \theta_{12} = \frac{L_1 L_2}{L_1 L_2}$. However, when $\theta_{12}$ is small, the cosine is a very slowly varying function. Since it is desirable to use a slide rule for these computations, the following formula is more satisfactory for small angles:

$$2\sin^2 \frac{\theta_{12}}{2} = 1 - \cos \theta_{12} = \frac{L_{12}^2 - (L_1 - L_2)^2}{2L_1 L_2},$$

where $L_{12}$ is the distance between the end points of $L_1$ and $L_2$. It is often convenient to renormalize co-ordinates so that $L_1 = L_2$.

An important quantity to calculate for many $V^0$ decay
events is the coplanarity angle, "δ". This is the angle between the line of flight of the neutral primary and the plane defined by the tangents to the lines of flight of the secondaries at the apex. It can be computed from the vectors by using the formula

\[ \sin \delta = \frac{\mathbf{L}_0 \cdot \mathbf{L}_+ x \mathbf{L}_-}{L_0 L_+ L_- \sin \theta_+} \]

However, the evaluation of the determinant which is involved in calculating the numerator is a tedious procedure. If the angles \( \theta_+, \theta_-, \) and \( \theta_+^- \) have been calculated accurately, the following formula, which can be derived from the corresponding spherical triangle relations, can be used:

\[ \sin^2 \delta = \frac{4 \sin \Delta/2 \sin(\theta_+ + \Delta/2) \sin(\theta_+ - \Delta/2) \sin(\theta_- + \Delta/2)}{\sin^2 \theta_-} \]

where \( \Delta = \theta_+ + \theta_- - \theta_+^- \)

Only when there is a long primary path and an accurately located origin for the \( V^0 \)-particle is the above formula, together with slide rule accuracy, not sufficient. Then the determinant form should be calculated using great care to avoid subtracting large, but almost equal, numbers.

E. Estimation of Errors

The important errors in the calculated results on \( V \)-particle decays can come from the following sources:

1. convection current distortions in curvatures
2. measurement errors in curvatures
3. lens and film distortion in curvatures
4. magnetic field calibration errors
5. errors in the measurement of the positions of points in the chamber
6. errors in extrapolation of tracks to an interaction
7. errors in estimating specific ionizations
8. possible errors in other parameters

The effects of each of these errors will now be estimated.

1. The most important error in the measurements performed to date has been due to the convection currents in the cloud chamber gas prior to the expansion. Unfortunately, it has not been possible to make a systematic quantitative analysis of the distortions by taking pictures with no magnetic field, for two reasons. Firstly, there are some leaks in the magnet cooling jackets which are aggravated by repeatedly turning the magnet on and off. The cause lies in the slight motions of the conductors and cooling jackets due to the changing magnetic forces. Secondly, the thermostat system has undergone numerous changes during the first year of operation and the temperature conditions have varied greatly. Thus any quantitative analysis would have to have been done many times with different results each time.

For these reasons, the estimated errors have been assigned more or less subjectively. This procedure is not unreasonable since there are certain clear evidences of the presence of convection distortions. Experience with
similar geometry in the 18" magnet cloud chambers, as well as the 48" chambers, has indicated that slight kinks near the chamber walls invariably precede any large scale distortions. Furthermore, primaries of very energetic interactions can be checked for straightness, and the curvatures of a single track above and below an absorber plate can be measured and compared. During most of the time the data described in this thesis were taken, the maximum detectable momentum for a 20 cm. long track was estimated to be about 3 Bev/c. When in doubt, conservativeness prevailed and larger errors were assigned.

2. Errors in measurement of curvatures played no essential part in the data presented in this thesis. The convection current distortions were always appreciably larger whenever the curvatures were measured by the comparator method.

3. Lens and film distortion played no important part in these measurements. They have not appeared on the film to any recognizable degree. The only indications of photographic distortions have appeared when reprojecting with the lenses opened up to f/4. Then the curvatures measured with Dr. Leighton's plotter are sometimes misleading due to either a lens distortion or a misjudgement in the location of the image caused by the excess light. For the first ten months of operation there was also a slight effect due to buckling of the film. Flat pressure
plates were installed in the cameras to eliminate such difficulties. A slight suction applied to holes in the pressure plate forces the film to lie against the flat surface.

4. The main error in calibrating the magnetic field is due to the fact that the standard permanent magnet has a magnetic induction of only 2300 gauss or about 2/7 that of the large magnet. Thus the reading of the fluxmeter when the coil is flipped in the standard field has to be taken repeatedly and very carefully to avoid a large percentage error. It is estimated that due to this cause, the absolute value of the magnetic induction is uncertain by a probable error of ±50 gauss. On an individual measurement of a momentum this corresponds to an error of about ±.7%, and is therefore negligible compared to multiple scattering errors. However, this magnetic field error is systematic and can contribute a slight additional error to a statistically averaged number: e.g. the average Q-value from a number of similar V-particle decay events.

Errors in the component ratios, p and q, are negligible in all cases. The uncertainty is about ±.01, but the errors are statistical and the component ratios enter into the momentum only when multiplied by Z₀', which is usually less than .5.

5. The main error in location of points in the cham-
ber arises from the Z depth measurement. Using the best
techniques available with the existing projectors, the
estimated error in measurement of the horizontal separa-
tion between the two apparent views of a point is ± 0.016 cm.
This corresponds to an uncertainty in Z depth of ± 0.1 cm.
For tracks which lie near the XY plane the measurements
of angles depend mostly on errors in ΔX and ΔY, which can
be measured to ± 0.03 cm. In the cases of the V⁰-particle
decays that have been analyzed, the errors in angle measure-
ments contributed negligibly to the Q-value, but were some-
times important for transverse momentum balance and co-
planarity calculations.

6. Usually the most uncertain point used in a trans-
verse momentum balance or coplanarity calculation is the
origin of the V⁰-particle. The accuracy of location of
the origin must be judged independently for each case from
the following factors:

a) the number of particles extrapolated

b) the length of the extrapolation

c) the curvatures of the tracks involved

d) possible multiple scattering errors

e) the average value of Z₀ for the extrapolated
tracks (X and Y co-ordinates can be measured
about six times more accurately than Z co-
oordinates on extrapolated tracks).

7. Since the technique of operation of the 48''
magnet cloud chambers has not yet been developed to the
stage where the individual droplets can be resolved on
the photographs, the specific ionizations have to be estimated visually. Naturally large errors are assigned to these estimates, usually corresponding to a factor of two between the extreme values allowed. However, in the data analysis that has been done to date, the ionization estimates have only been used to distinguish between protons and π-mesons; namely, to decide whether a particular \( V^0 \) decay was a \( \Lambda^0 \) decay or a \( \theta^0 \) decay. Having made such a decision, or having decided such a decision was uncertain, the Q-value was calculated assuming the secondaries to be \( P^+\pi^- \) or \( P^+\pi^- \).

8. A few of the other geometrical parameters which enter into the measurement of positions and calculation of momenta are: \( D \), the distance from the front surface of the chamber piston to the front principal plane of the camera lens; \( S \), the separation between the camera lenses; and also the distance between the fiducial marks on the piston which are used to normalize the magnification of the re-projection. These quantities have been carefully checked, either directly or by indirect experiments (e.g. the calculation of the Z co-ordinate of the back surface of the front glass from a measurement of the horizontal separation between the two apparent views of a dust speck). Thus these parameters are known accurately enough that they do not contribute any significant errors.
II. MOMENTUM CALCULATION

A. Formulation of the Problem

In the calculation of the momentum of an ionizing particle traversing a cloud chamber in a magnetic field, a number of corrections need to be made. Firstly, the magnetic field is in general neither uniform nor directed parallel to the camera axis. Secondly, in order to measure the curvature of the track it is not convenient to make an orthogonal projection of the track on any plane. The photograph, or a reprojection thereof, represents a conical projection of the track through the center of the camera lens onto a convenient plane. The purpose of the following calculation is to make an analysis of the resulting track.

We set up the following co-ordinate system in space: the Z axis is perpendicular to the rear cloud chamber wall and coincides with the lens axis. The XY plane coincides with the rear chamber wall; the Y axis is vertical and the X axis horizontal so as to form a right-handed co-ordinate system with the Z co-ordinate increasing toward the camera.

The position of the track in the chamber will be specified by the co-ordinates of a reference point on the track near its center, \((X_o, Y_o, Z_o)\), and the direction cosines of the track at this point, \((X_o', Y_o', Z_o')\). The magnetic field will be specified by its Z component, \(B_z\),
and the component ratios, \( p = \frac{B_x}{B_z} \) and \( q = \frac{B_y}{B_z} \). The momentum, \( P \), of the particle which produced the track will enter into the calculation only through the quantity \( a = \frac{eB_z}{P} \). We shall solve the equations of the path parametrically using as the independent parameter, \( s \), the distance along the track from the reference point.

B. Exact Equation of the Track in Space in A Uniform Magnetic Field

Let us first assume the magnetic field to be uniform in magnitude and direction along the track. The differential equations of the track are then

\[
\begin{align*}
X'' &= a(Y' - qZ') \\
Y'' &= -a(X' - pZ') \\
Z'' &= a(qX' - pY')
\end{align*}
\]

where the prime denotes differentiation with respect to \( s \). These can be integrated once immediately to give

\[
\begin{align*}
X' - X_0' &= a[(Y - Y_0) - q(Z - Z_0)] \\
Y' - Y_0' &= -a[(X - X_0) - p(Z - Z_0)] \\
Z' - Z_0' &= a[q(X - X_0) - p(Y - Y_0)]
\end{align*}
\]

In order to solve these easily let us make the substitution

\[
\begin{align*}
\xi &= (X - X_0) - p(Z - Z_0) \\
\eta &= (Y - Y_0) - q(Z - Z_0) \\
\zeta &= Z - Z_0
\end{align*}
\]
The equations then become
\[
\begin{align*}
\xi' + p \zeta' &= a \eta + x_0' \\
\eta' + q \zeta' &= -a \xi + y_0' \\
\zeta' &= a(q \xi - p \eta) + z_0'
\end{align*}
\]
from which \( \zeta' \) can be eliminated. The resulting equations in \( \xi, \eta, \xi', \eta' \) can be solved by assuming a solution of the form \( \xi = A \exp(i \alpha s), \eta = B \exp(i \alpha s) \) as a homogeneous solution and adding constants as particular solutions. The secular determinant of the equations determines the two possible values of \( \beta \) to be \( \pm \sqrt{1 + p^2 + q^2} \). The equations and the initial conditions determine all the constants, and the equation for \( \zeta \) can be integrated directly.

The solution is
\[
\begin{align*}
\xi &= \frac{1}{\alpha \beta^2} \left[ pq x_0' - (1 + p^2) y_0' + q z_0' \right] \cos(\alpha \beta s) - 1 + \\
&\quad + \frac{1}{\alpha \beta}(x_0' - p z_0') \sin(\alpha \beta s) \\
\eta &= \frac{1}{\alpha \beta^2} \left[ (1 + q^2) x_0' - pq y_0' - p z_0' \right] \cos(\alpha \beta s) - 1 + \\
&\quad + \frac{1}{\alpha \beta}(y_0' - q z_0') \sin(\alpha \beta s) \\
\zeta &= z_0' + \frac{1}{\alpha \beta^2} \left[ - q x_0' + p y_0' \right] \cos(\alpha \beta s) - 1 + \\
&\quad - \frac{1}{\alpha \beta^2} \left[ p x_0' + q y_0' - (p^2 + q^2) z_0' \right] \sin(\alpha \beta s) - (\alpha \beta s)
\end{align*}
\]
Changing back to the original co-ordinates this becomes
\[
\begin{align*}
X &= x_0 + \frac{1}{\alpha \beta^2} \left[ (1 - \cos(\alpha \beta s)) + \frac{p}{\beta^2} (px_0' + qy_0' + z_0') \right] s + \\
&\quad + \frac{1}{\alpha \beta^2} \left[ (1 + q^2) x_0' - pq y_0' - p z_0' \right] \sin(\alpha \beta s) \\
Y &= y_0 - \frac{1}{\alpha \beta^2} \left[ (1 + q^2) x_0' - pq y_0' - p z_0' \right] \cos(\alpha \beta s) + \\
&\quad + \frac{1}{\alpha \beta^2} \left[ - pq x_0' + (1 + p^2) y_0' - q z_0' \right] \sin(\alpha \beta s) \\
Z &= z_0 + \frac{1}{\alpha \beta^2} \left[ p x_0' + q y_0' - (p^2 + q^2) z_0' \right] \left[ (\alpha \beta s) - \sin(\alpha \beta s) \right]
\end{align*}
\]
At this point the only approximation that has been made is that the magnetic field is uniform. This is the exact parametric equation of the helical track in space referred to convenient co-ordinate axes.

C. Projection of the Track on the Rear Chamber Wall

In order to reduce to a useful form the equation we have derived we now perform two operations. Firstly, we must make a conical projection of this curve through the center of the lens (0,0,D) onto the plane Z=0. This projection is the curve that is usually observed in an actual cloud chamber photograph. Secondly, since the parameter s has no direct significance on the projection, we shall transform to the parameter $s_a$, which is the arc length measured on the projection.

In general we can consider a track segment whose length is shorter than its radius. Thus, in order to make our formulas manageable, we shall expand them in powers of $as$ or $as_a$. The convergence is very rapid, as will be seen by noting the magnitude of the correction terms which we shall derive.

Thus the parametric equations of the track in space become the following:
\( X = X_0 + X_0's + \frac{1}{2}a(Y_0' - qZ_0')s^2 - \frac{1}{6}a^2[(1 + q^2)X_0' - pqY_0' - pZ_0']s^3 - \frac{1}{4}a^3\beta^2(Y_0' - qZ_0')s^4 + \ldots \)

\( Y = Y_0 + Y_0's + \frac{1}{2}a(X_0' - pZ_0')s^2 - \frac{1}{6}a^2[-pqX_0' + (1 + p^2)Y_0' - qZ_0']s^3 + \frac{1}{4}a^3\beta^2(X_0' - pZ_0')s^4 + \ldots \)

\( Z = Z_0 + Z_0's + \frac{1}{2}a(qX_0' - pY_0')s^2 + \frac{1}{6}a^2[pX_0' + qY_0' + (p^2 + q^2)Z_0']s^3 - \frac{1}{4}a^3\beta^2(qX_0' - pY_0')s^4 + \ldots \)

Performing the indicated conical projection and defining co-ordinates on that projection as \((X_a, Y_a)\) we get,

\[
X_a = X_0\frac{D}{D - Z} ; \quad Y_a = Y_0\frac{D}{D - Z} ; \quad \text{Define} \quad \gamma = \frac{D}{D - Z} 
\]

Then

\[
X_a = X_0 + X_0's + \frac{1}{2}X_0''s + \frac{1}{3}X_0'''s^3 + \frac{1}{4}X_0''''s^4 + \ldots
\]

\[
Y_a = Y_0 + Y_0's + \frac{1}{2}Y_0''s + \frac{1}{3}Y_0'''s^3 + \frac{1}{4}Y_0''''s^4 + \ldots
\]

where

\[
X_0 = \gamma X_0 \quad \quad \quad \quad \quad \quad \quad \quad \quad Y_0 = \gamma Y_0
\]

\[
X_0' = \gamma(X_0' + \frac{X_0Z_0'}{D}) \quad \quad \quad \quad \quad \quad Y_0' = \gamma(Y_0' + \frac{Y_0Z_0'}{D})
\]

\[
X_0'' = a\gamma\left\{Y_0' + \frac{2Y_0'Z_0'}{aD} - qZ_0' + \frac{2\gamma^2X_0'Z_0'^2}{aD^3} + \frac{\gamma X_0'}{D}(qX_0' - pY_0')\right\}
\]

\[
Y_0'' = -a\gamma\left\{X_0' - \frac{2Y_0'Z_0'}{aD} - pZ_0' - \frac{2\gamma^2Y_0'Z_0'^2}{aD^3} - \frac{\gamma Y_0'}{D}(qX_0' - pY_0')\right\}
\]

\[
X_0''' = -a^2\gamma\left\{X_0' - \frac{3Y_0'Z_0'}{aD} - \frac{6\gamma^2X_0'Z_0'^2}{a^2D^2} - pZ_0' + \frac{3\gamma qZ_0'^2}{aD}
\right. \left. - \frac{3\gamma X_0'}{aD}(qX_0' - pY_0') - \frac{6\gamma^2X_0'Z_0'^2}{a^2D^3}\right\}
\]

\[
Y_0''' = -a^2\gamma\left\{Y_0' + \frac{3Y_0'Z_0'}{aD} - \frac{6\gamma^2Y_0'Z_0'^2}{a^2D^2} - qZ_0' - \frac{3\gamma pZ_0'^2}{aD}
\right. \left. - \frac{3\gamma Y_0'}{aD}(qX_0' - pY_0') - \frac{6\gamma^2Y_0'Z_0'^2}{a^2D^3}\right\}
\]

\[
X_0'''' = -a^3\gamma\left\{Y_0' + \frac{4\gamma Y_0'Z_0'}{aD} - 12\frac{\gamma^2Y_0'Z_0'^2}{a^2D^2}
\right. \left. - 12\frac{\gamma X_0'Z_0'^2}{a^2D^3}\right\}
\]

\[
Y_0'''' = a^3\gamma\left\{X_0' - \frac{4\gamma Y_0'Z_0'}{aD} - 12\frac{\gamma^2Y_0'Z_0'^2}{a^2D^2}
\right. \left. - 12\frac{\gamma X_0'Z_0'^2}{a^2D^3}\right\}
\]
The terms $X_{ao}^{\mu\nu}$ and $Y_{ao}^{\mu\nu}$ are evaluated including only terms of first order in $p, q, \frac{X_o}{D}, \frac{Y_o}{D}, \frac{Z_o}{D}$. The terms $X_{ao}^{(w)}$ and $Y_{ao}^{(w)}$ are evaluated including only zero order terms.

The parameter $s_a$ is now defined by the following relation:

$$\left(\frac{ds_a}{ds}\right)^2 = \left(\frac{dx_a}{ds}\right)^2 + \left(\frac{dy_a}{ds}\right)^2$$

$$= K^2(1+a_1 s + a_2 s^2 + a_3 s^3 + \ldots)$$

where $K = X_{ao}^{(w)} + Y_{ao}^{(w)}$

$$a_1 = \frac{2}{K}(X_{ao}^{(w)} + Y_{ao}^{(w)})$$

$$a_2 = \frac{2}{K}(X_{ao}^{(w)} + Y_{ao}^{(w)} + X_{ao}^{(w)} + Y_{ao}^{(w)})$$

$$a_3 = \frac{6}{K}(X_{ao}^{(w)} + Y_{ao}^{(w)} + X_{ao}^{(w)} + Y_{ao}^{(w)})$$

Extracting the square root and integrating:

$$\frac{s_a}{s} = K[1 + \frac{3}{2}a_1 s + \frac{1}{2}(a_2 - \frac{1}{2}a_1^2)s^2 + \frac{3}{17}(a_3 - \frac{1}{2}a_1 a_2)s^3 + \ldots]$$

We must solve this equation for $s$ in terms of $s_a$:

$$s = \frac{s_a}{K}[1 + b_1 s_a + b_2 s_a^2 + b_3 s_a^3 + \ldots]$$

We shall see that it is only necessary to solve for $b_1$ and $b_2$.

$$b_1 = -\frac{a_1}{4K}$$

$$b_2 = \frac{1}{3K}(a_1^2 - \frac{1}{2}a_2)$$

The substitution for $s$ in the equations we have derived enables us to express them in terms of the parameter $s_a$:

$$X_a = X_{bo} + X_{bo}^{(w)} s_a + \frac{3}{2} X_{bo}^{(w)} s_a^2 + \frac{5}{2} X_{bo}^{(w)} s_a^3 + \frac{7}{2} X_{bo}^{(w)} s_a^4$$

$$Y_a = Y_{bo} + Y_{bo}^{(w)} s_a + \frac{3}{2} Y_{bo}^{(w)} s_a^2 + \frac{5}{2} Y_{bo}^{(w)} s_a^3 + \frac{7}{2} Y_{bo}^{(w)} s_a^4$$

where $X_{bo} = X_{ao}$

$$X_{bo}^{(w)} = \frac{1}{K}(X_{ao}^{(w)} + 2Kb_1 X_{ao}^{(w)})$$

$$X_{bo}^{(w)} = \frac{1}{K}(X_{ao}^{(w)} + 6Kb_1 X_{ao}^{(w)} + 3K^2 b_2 X_{ao}^{(w)})$$

$$X_{bo}^{(w)} = \frac{1}{K}[X_{ao}^{(w)} + 12Kb_1 X_{ao}^{(w)} + 12K^2 (b_1^2 + b_2) X_{ao}^{(w)} + 4K^3 b_3 X_{ao}^{(w)}]$$

There are similar expressions for $Y_{bo}^{(w)}$. 
D. Calculation of the Sagitta of the Projected Track

At this point we wish to calculate a quantity which characterizes the curvature. The quantity we shall use is the sagitta of the projected track at its center point. Let us measure $s_a$ from the center point of the projected track. Then the perpendicular distance from a point on the track to the line tangent at $s_a = 0$ is given by

$$\delta = \delta_1 + \delta_2 + \delta_3$$

where

$$\delta_1 = \frac{1}{3} s_a^2 (X_{bo} Y_{bo}' - Y_{bo} X_{bo}')$$

$$\delta_2 = \frac{1}{3} s_a^3 (X_{bo} Y_{bo}' - Y_{bo} X_{bo}')$$

$$\delta_3 = \frac{1}{24} s_a^4 (X_{bo} Y_{bo}' - Y_{bo} X_{bo}')$$

It is convenient to define quantities which represent the values that the $\delta$'s would have if the relation between $s$ and $s_a$ were linear:

$$\delta_{10} = \frac{s_a^2}{2K^2} (X_{ao} Y_{ao}' - Y_{ao} X_{ao}')$$

$$\delta_{20} = \frac{s_a^3}{6K^4} (X_{ao} Y_{ao}' - Y_{ao} X_{ao}')$$

$$\delta_{30} = \frac{s_a^4}{24K^6} (X_{ao} Y_{ao}' - Y_{ao} X_{ao}')$$

In terms of these quantities,

$$\delta_1 = \delta_{10}$$

$$\delta_2 = \delta_{20} + 2b_1 s_a \delta_{10}$$

$$\delta_3 = \delta_{30} + 3b_1 s_a \delta_{20} + (b_1^2 + b_2) s_a^2 \delta_{10}$$

We notice that $b_2$ does not enter into these equations. Also $b_1$ and $b_2$ enter only into the $\delta_2$ and $\delta_3$ correction terms, thus we only need to evaluate them approximately.
We can see from the figure that

\[ \delta_a = \frac{\delta_1 + \delta_3}{\cos \varphi} \]

where

\[ \varphi = \tan^{-1} \frac{\delta_a}{\delta_2} \]

In actual measurements \( p, q \ll 1 \) and also \( \frac{X}{D}, \frac{Y}{D}, \frac{Z}{D} \ll 1 \). Thus we shall neglect high powers of these quantities in the \( \delta_2 \) and \( \delta_3 \) correction terms.

In order to organize our computations, we shall define the following quantities:

\[
A = \frac{\gamma Z_0}{(1-Z_0^1)^2} \left\{ \frac{X_0}{D} X_0' + \frac{Y_0}{D} Y_0' \right\}
\]

\[
B = \frac{Z_0}{(1-Z_0^1)^2} (pX_0' + qY_0')
\]

\[
C = \frac{\gamma^2 Z_0^1}{(1-Z_0^1)^2} \left\{ \frac{X_0}{D}^2 + \left( \frac{Y_0}{D} \right)^2 \right\}
\]

\[
D' = \frac{\gamma(pX_0' - qY_0')}{(1-Z_0^1)^2} \left\{ \frac{Y_0}{D} X_0' - \frac{X_0}{D} Y_0' \right\}
\]

\[
E = \frac{\gamma Z_0^1}{(1-Z_0^1)^2} \left\{ pX_0' + qY_0' \right\}
\]

In terms of these quantities,

\[
K^2 = \gamma^2 (1-Z_0^1)^2 (1+2A+C)
\]

\[
\delta_1 = \delta_{10} = \frac{eB_0 \beta_0^2 (1+A-B-D'-E)}{2 \gamma^2 (1-Z_0^1)^2 (1+2A+C)^{3/2}}
\]

This latter equation can now be solved for the one unknown quantity \( F \).

The correction terms, \( \delta_2 \) and \( \delta_3 \), depend on higher powers of \( a = \frac{eB_0}{F} \). However, we can substitute into them an approximate value of \( a \) given by neglecting all correc-
tion terms and approximating the track by a circle of radius 
\[ \rho_a = \frac{s_a^2}{20a}. \]

The next step will be to evaluate these correction terms. As we have seen, \( \delta_3 \) is added to \( \delta_1 \) in the measured sagitta whereas \( \delta_2 \) only serves to rotate the chord by an angle \( \phi \). The correction factor \( \cos \phi \) is negligible and will be ignored. However, \( \delta_2 \) will serve to distort the projection from a symmetrical form, and thus a direct comparison of the photograph of the track with a circular arc is likely to give a misleading result. It should be emphasized that the pertinent quantity for the present calculation is the apparent sagitta.

We shall now evaluate the \( a' \)s and \( b' \)s, in the process neglecting high powers of \( p,q, \frac{X}{D}, \frac{Y}{D'}, \frac{Z}{D} \).

\[ a_1 = \frac{3Y_0^2}{D} - 2aZ_0^1(F+G) \]
\[ a_2 = \frac{20Y^2Z_0^{12}}{D^2} \]

where we have defined 
\[ F = \frac{qX_0^1 - pY_0^1}{1-Z_0^{12}} \]
\[ G = \frac{\gamma}{(1-Z_0^{12})} \left\{ \frac{Y_0 Y_0^1}{D} - \frac{X_0 X_0^1}{D} \right\} \]

Considering \( s_a \) to be measured to the ends of the track from the apparent center we get,

\[ b_1 s_a = -\frac{YAZ}{2D} - \frac{\gamma^2(\Delta Z)^2}{4D^2} + \frac{5\alpha^2(\Delta Z)^2}{4}(F+G) \]
\[ b_2 s_a^2 = \frac{Y^2(\Delta Z)^2}{2D^2} - \frac{4\alpha Y(\Delta Z)^2}{3D}(F+G) + \frac{1}{2}\alpha^2(\Delta Z)^2(F+G)^2 \]

The approximate values for the sagittas which neglect
variations in magnification along the track become:

\[
\delta_{10} = \delta_{10} \left\{ \frac{\gamma\Delta Z}{2D} (1 - A + \frac{\gamma\Delta Z}{2D}) - \frac{a\Delta Z}{2} (F+G) \right\}
\]

\[
\delta_{30} = \delta_{10} \left\{ \frac{\gamma^2 (\Delta Z)^2}{4D} - \frac{1}{4} a^2 L^2 \right\}
\]

where \( L \) is the true length of the chord of the track in space and \( \Delta Z \) is the change in \( Z \) co-ordinate between its ends. Using the previously expressed relations we find,

\[
\delta_{2} = \delta_{10} \left\{ \frac{\gamma\Delta Z}{2D} (1 + A - \frac{\gamma\Delta Z}{2D}) - \frac{3a\Delta Z (F+G)}{2} \right\}
\]

\[
\delta_{3} = \delta_{10} \left\{ \frac{\gamma^2 (\Delta Z)^2}{4D} - \frac{1}{4} a^2 L^2 - \frac{23a\gamma (\Delta Z)^2 (F+G)}{24D} + \frac{13}{4} a^2 (\Delta Z)^2 (F+G) \right\}
\]

We notice that the contribution to \( \delta_{2} \) and \( \delta_{3} \) from the variation in the magnification is of the same order as that from the nonlinearity of the conical projection. In other words, \( \delta_{20} \) and \( \delta_{2} \) are of the same order of magnitude, but the coefficients have been changed appreciably.

In making measurements on the projection it is convenient to measure \( L_a \), the length of the chord, rather than \( s_a \), the half length of the arc. In applying a correction factor it is sufficiently accurate to assume the curve to be a circular arc of radius \( \rho_a \). In this case

\[
s_a^2 \approx \frac{L_a^2}{4} \left( 1 + \frac{L_a^2}{12\rho_a^2} \right)
\]

It is also convenient to factor out some of the terms into one experimentally measured factor \( L/L_a \), where \( L \) is the true chord length in space. For the case of a straight
line \((a=0)\)

\[
\frac{s}{s_a} = \frac{1+b_1 s_a + b_2 s_a^2}{\gamma \sqrt{(1-Z_0)^2}} (1+2A+C)
\]

where, when \(a=0\),

\[
b_1 s_a = -\frac{\gamma A Z}{2D}
\]

\[
b_2 s_a^2 = \frac{\gamma^2 (\Delta Z)^2}{2 D^2}
\]

Remembering that \(s_a\) is measured from the center of the projected track,

\[
\frac{L}{L_a} = \frac{1 + \frac{\gamma^2 (\Delta Z)^2}{4 D^2}}{\gamma \sqrt{(1-Z_0)^2}} (1+2A+C)
\]

In many cloud chambers used for accurate momentum measurements the rear wall serves as the movable chamber piston. Thus the photograph, as well as its reprojection, is actually the projection of the track on the piston in its expanded position, whereas we should insert into our formula quantities measured when the chamber is compressed. The correction is very small so that the only appreciable effect is to divide all quantities measured directly on the conical projection by \((1+\frac{\epsilon Z_0}{D Z_0})\), where \((1+\epsilon)\) is the expansion ratio. However, since our formula will have an equal number of directly measured quantities in numerator and denominator this correction cancels out.

The final result of these calculations is the following formula:

\[
P = e B_2 \rho a \frac{L}{L_a} (\epsilon) (1+f)
\]
where: \[ \rho_a = \frac{L_a^2}{C \delta_a} \]

\[ \Sigma = \frac{1 + A - B - D' - E}{1 + 2A + C} \approx 1 - A - B \]

\[ f = \frac{3L_a^2}{4\rho_a} - \frac{23YZ_0' \Delta Z L_a}{24D\rho_a} (F+G) + \frac{13Z_0^2 L_a^2}{48\rho_a^2} (F+G)^2 \]

This latter term incorporates the correction due to \( \delta_3 \), the correction term in \( \text{L/L}_a \), and the conversion from \( s_a \) to \( L_a \). The A term is the first order correction for the effect of the conical projection. The B term is the first order correction for the X and Y components of the magnetic induction.

For \( Z_0' < 0.4 \) we make an error of less than 1% by neglecting C and E. D' is appreciable only in extreme cases of a track near the front edge of the chamber where \( \frac{X}{D} \) and also \( p, q \) become large.

E. Measurement of the Direction of the Track

Since it is difficult to construct accurately a tangent at the center of the projected track, it is convenient to assume that the direction cosines of the chord are a good approximation to the values of \( X_0', Y_0', Z_0' \). To check that this procedure introduces only negligible errors we evaluate approximately the \( \Delta X, \Delta Y, \Delta Z \) of the chord.

If \( s_1 \) and \( s_2 \) are the values of \( s \) corresponding to the two ends of the track, then

\[ \Delta X = X_0' (s_2 - s_1) + \frac{1}{2} \Delta Y_0' (s_2^2 - s_1^2) \]

\[ \Delta Y = Y_0' (s_2 - s_1) - \frac{1}{2} \Delta X_0' (s_2^2 - s_1^2) \]
\[ \Delta Z = Z'_0(s_2 - s_1) + \frac{1}{4}a(qX'_0 - pY'_0)(s_2^2 - s_1^2) \]

But \( s_2 - s_1 \approx L \)

and \( s_2^2 - s_1^2 \approx \frac{4bL}{K^2} \)

\( sa^2 = -\frac{\gamma\Delta Z L}{2D} \)

Thus, \[ \frac{\Delta X}{L} = X'_0 - \frac{LaV'_0 \gamma\Delta Z}{\rho a D} \]

\[ \frac{\Delta Y}{L} = Y'_0 + \frac{La}{\rho a X'_0} \frac{\gamma\Delta Z}{D} \]

\[ \frac{\Delta Z}{L} = Z'_0 - \frac{La}{\rho a} (qX'_0 - pY'_0) \frac{\gamma\Delta Z}{D} \]

Remembering that \( X'_0 \) and \( Y'_0 \) enter into the correction terms only when multiplied by first order terms like \( \frac{X'_0}{D} \), \( p \), etc., the error made by replacing them by \( \frac{\Delta X}{L} \) and \( \frac{\Delta Y}{L} \) is always negligible. The correction to \( \frac{\Delta Z}{L} \) is already of the second order and thus has even smaller effect.

F. Effect of an Inhomogeneous Magnetic Field

In order to analyze the effect of a magnetic field that varies over the length of the track, it is not convenient to use the rigorous solution to the equations of motion. However, since we are interested only in a correction term, the following less exact procedure is justified.

Consider that for the purpose of evaluating the magnetic force, the velocity vector is fixed, having direction cosines \((X'_0, Y'_0, Z'_0)\). We can then determine the deflection of the track from the tangent line by evaluating the
\[ \vec{\delta} = \frac{e}{p} \int \int \int_{0}^{l} (\vec{V} \times \vec{B}) d\vec{s} d\vec{\ell} = \frac{e\vec{V}}{p\vec{V}} \times \int \int \int_{0}^{l} \vec{B} d\vec{s} d\vec{\ell} \]

To be more exact, the above integral should be carried out from the center to each end of the track, and the results averaged, to give a value for the true sagitta in space, which can then be projected onto the rear wall of the chamber. Thus we should insert into our previously derived equations an average value of \( \vec{B} \) obtained from:

\[ \vec{\overline{B}} = \frac{4}{l^2} \int \int \int_{0}^{l} \vec{B} d\vec{s} d\vec{\ell} + \frac{4}{l^2} \int \int \int_{0}^{\frac{l}{2}} \vec{B} d\vec{s} d\vec{\ell} \]

If we assume \( B_z, p, \) and \( q \) to vary linearly over the half lengths of the track, the values of \( \vec{B} \) at points one-third of the distance from the center to the ends should be determined and then averaged. The result will be a good approximation to \( \vec{\overline{B}} \). It should be noted that in a cloud chamber magnet the hole through the front pole piece for viewing the chamber decreases the field at its center. Thus, if a single track traverses a chamber from a high magnetic field at the top edge to a minimum at the center and back to a high value at the bottom, the above two linear approximations will be quite satisfactory; whereas, the magnetic field at the center of the track will be quite inappropriate as an average value.
G. Construction of the Tangent At One End of the Track

Consider a track whose conical projection is described by the equations:

\[
\begin{align*}
X_a &= X_{b0} + \frac{1}{2}X_{b0}'s_a + \frac{1}{6}X_{b0}''s_a^2 + \cdots \\
Y_a &= Y_{b0} + \frac{1}{2}Y_{b0}'s_a + \frac{1}{6}Y_{b0}''s_a^2 + \cdots
\end{align*}
\]

If \(s_a\) is measured from the center of the apparent view, the ends will be represented by \(s_a = \pm s_{a1}\). We define the notation \([i,j] = X_{b0}^{(i)}X_{b0}^{(j)} + Y_{b0}^{(i)}Y_{b0}^{(j)}\). Then the length of the chord will be given by

\[
L_a = 2s_{a1}\left(1 + \frac{1}{3}[1,3]s_{a1}^2 + 0^2\left(\frac{s_{a1}}{p_a}\right)\right)
\]

We construct a point on the line tangent at \(s_a = -s_{a1}\), at a distance \(L_a\) from this end.

\[
\begin{align*}
X_a &= X_{b0} - \frac{1}{2}X_{b0}'(s_{a1} - L_a) + \frac{1}{6}X_{b0}''(s_{a1} - 2L_a)s_{a1} + \cdots \\
Y_a &= Y_{b0} - \frac{1}{2}Y_{b0}'(s_{a1} - L_a) + \frac{1}{6}Y_{b0}''(s_{a1} - 2L_a)s_{a1} + \cdots
\end{align*}
\]

The distance from this point to the end of the chord is given by

\[
\Delta a = \sqrt{(\Delta X_a)^2 + (\Delta Y_a)^2}
\]

\[
= \frac{1}{2}L_a^2[2,2]^{1/2}\left(1 - \frac{L_a}{12[2,2]}(2[2,3] + [1,2][1,3] + 0^2\left(\frac{L_a}{p_a}\right))\right)
\]

Using the expressions derived in the momentum calculation, it is possible to derive

\[
[2,2] = \frac{a^2(1-2B)}{r^2(1-Z_0^2)(1+2A)^2}
\]
to zero order:  \[ [2, 3]^2 = \frac{-3a^2Z_0'z}{D\gamma^2(1-Z_0'^2)^2} \]

exactly:  \[ [1, 2] = 0 \]

(This equation re-emphasizes the fact that the acceleration is perpendicular to the velocity).

Combining these results we find

\[ \Delta a = \frac{L_a^2}{2D\rho a^2}(1 + \gamma \Delta Z) \]

The method used in applying this quantity to the construction of the tangents is demonstrated in Fig. 3 (page 11).

Another correction that must be applied in some extreme cases is due to the fact that nonaxial components of the magnetic field induce changes in \( Z' \) along the track. Thus the \( Z \) co-ordinate of the end of the tangent is slightly different from that of the end of the chord. The change can be seen directly from the expansion,

\[ Z = Z_0 + Z_0's + \frac{3}{2}a(qX_0' - pY_0')s^2 + \ldots \]

If we make the approximation that \( s \) is measured from the center of the track we find that

\[ \Delta Z = Z_T - Z_C = -\frac{L_a}{D\rho a^2}(qX_0' - pY_0') \]
III. DECAY DYNAMICS

A. Introduction

The purpose of this section is to analyze various modes of decay which might be applicable to neutral \( V \)-particles. The dynamics of two-body decays are the simplest, and most of the results listed herein have been used by investigators in analyzing their data.\(^{1-4}\) The reasons for including the two-body decay dynamics here are two-fold: firstly, it is convenient to have the results in a single unified presentation; secondly, the two-body decay considerations form a useful introduction to the analysis of the more complicated three-body dynamics which follows.

It is important to analyze closely the dynamics of a two-body decay for two reasons. Firstly, many of the cloud chamber observations are far from ideal and the information gathered from such data may only be statistical in nature. It may be possible to set limits on the \( Q \)-value and other calculated quantities from limited information on a particular case. It is necessary to maximize information from even the poorest cases, because they have to be included in any statistical analysis, e.g., a calculation of the mean lifetime. Secondly, it is important to check on any bias which might be introduced by the chamber geometry and the selection of cases. A convenient technique to perform such checks is to repre-
sent the data on one of the plots described in the analysis of the two-body decay dynamics.

The decay of charged particles will not be discussed. However, a similar analysis can be carried out for charged V-particles and the results are analogous to those derived here.

The emphasis in the analysis of three-body decay dynamics will be on the properties of the charged decay products. For the purpose of comparison with experimental results it is important to consider the manifestations of a three-body decay when the charged products are analyzed as if they originated in a two-body decay. For example, the measured energy release, Q, for a single two-body decay should be unique within experimental errors. Therefore, it is useful to analyze the significance of the distribution of calculated Q'-values if the observed products actually originate in a three-body decay.

B. Notation

In an ideal cloud chamber observation of the decay of a neutral V-particle the following quantities are measured:

\( P_+ \), \( P_- \) : the magnitude of the momenta of the charged products

\( \theta \) : the angle between the momentum vectors of the charged products
\( I^+, I^- \): the specific ionization of the charged products. (Due to the difficulty of estimation of the ionizations they are in general known only roughly).

If an interaction in the solid material near the cloud chamber projects charged particles into the chamber, and if the \( V \)-particle is assumed to originate in that interaction, the following additional quantities can be measured:

\( \theta^+, \theta^- \): the angles between the momentum vectors of the charged products and the line of flight of the \( V^0 \).

From the above data the following quantities can be calculated directly:

\( M^+, M^- \): the rest masses of the charged products. (Due to the large errors in most estimates of ionizations, the calculated masses have large errors. The usual procedure is to identify the masses with those of known particles for the remaining calculations).

\( Q' \): the energy release calculated assuming no neutral secondaries were emitted, from \( P^+, P^- \) and \( \theta \).

\( \delta \): the angle between the plane of the charged decay products and the line of flight of the \( V^0 \).

\( P_{T^+} = P_+ \sin \theta^+ \): the transverse momentum of the positive secondary

\( P_{T^-} = P_- \sin \theta^- \): the transverse momentum of the negative secondary

\[ P_T = \frac{P_+ P^- \sin \theta}{\sqrt{P_+^2 + P_-^2 + 2P_+P_- \cos \theta}} \]: the transverse momentum if a two-body decay is assumed.

\( \Delta P_T \): the total unbalanced momentum of the charged products perpendicular to the line of flight of the \( V^0 \).

\( \Delta P_T = P_{T^+} - P_{T^-} \): this quantity is approximately the component of the unbalanced transverse momentum in the plane of the secondaries.
\[ a = \frac{p_+ \cos \theta_+ - p_- \cos \theta_-}{p_+ \cos \theta_+ + p_- \cos \theta_-} \]

: the distribution of this quantity gives an indication of the relative masses of the charged decay products.

\[ a' = \frac{p_+^2 - p_-^2}{p_+^2 + p_-^2 + 2p_+ p_- \cos \theta} \]

: for a two-body decay this quantity is equal to \( a \), but it can be calculated independently of the location of a possible origin for the \( V \)-particle.

\[ a'' = \frac{\sin(\theta_2 - \theta_1)}{\sin \theta} \]

: for a two-body decay this is also equal to \( a \), but it can be calculated independently of the momenta of the secondaries.

In the following analysis quantities referred to the co-ordinate system in which the \( V^0 \)-particle is at rest shall be denoted by primes. The direction of flight of the \( V^0 \) shall be the \( X \) direction. All momenta, masses, and energies shall be given in energy units, so that the velocity of light is considered as unity. Momenta shall be designated by \( P \), total energies by \( W \), and kinetic energies by \( T \).

C. Two-Body Decay Dynamics

Laboratory System

Center of Mass System
Let us consider the two-body decay: \( M_0 \rightarrow M^+_1 + M^-_1 + Q \).
The pertinent quantities are defined in the figure above. Conservation of energy and momentum imply the following relations:

\[
W_0 = W^+_1 + W^-_1 \\
P'_0 = P^+_1 + P^-_1 + 2P^+_1 P^-_1 \cos \theta \\
\frac{P^+_1 \sin \theta_1}{P^-_1 \sin \theta_1} = \frac{P^+_1}{P^-_1} \\
P' = \sqrt{\left(1 - \frac{Q}{2M_0}\right)^2 + Q \left(1 - \frac{Q}{2M_0}\right) \frac{2M^+_1 M^-_1}{M_0}} \\
W^+_1 = M^+_1 + Q \left(1 - \frac{M^+_1}{M_0} - \frac{Q}{2M_0}\right) ; \quad W^-_1 = M^-_1 + Q \left(1 - \frac{M^-_1}{M_0} - \frac{Q}{2M_0}\right)
\]

In a cloud chamber observation of a neutral \( V \)-particle decay, the most accurately measurable quantities are usually \( P^+_1, P^-_1, \) and \( \theta \). If we assume the masses of the secondary particles, the Q-value can be calculated,

\[
Q = (M^+_1 + M^-_1) \left[\sqrt{1 + \frac{2Q^2}{M^+_1 M^-_1}} - 1\right]
\]

where \( Q_1 = \frac{1}{M^+_1 M^-_1} [W^+_1 W^-_1 - M^+_1 M^-_1 - P^+_1 P^-_1 \cos \theta] \)

If \( Q_1 \ll M^+_1 + M^-_1 \) the following expansion can be used conveniently:

\[
Q = Q_1 - \frac{Q_1^2}{2(M^+_1 + M^-_1)} + \frac{Q_1^3}{2(M^+_1 + M^-_1)^2} - \ldots
\]

It is convenient to calculate approximately the effect of errors in the momenta, angles and masses on the Q-value by using the relationships:

\[
\frac{\partial Q}{\partial P^+_1} = \frac{1}{M_0} \frac{W^-_1 P^+_1 - P^-_1 \cos \theta}{W^+_1 P^+_1 - P^-_1 \cos \theta} ; \quad \frac{\partial Q}{\partial P^-_1} = \frac{1}{M_0} \frac{W^+_1 P^-_1 - P^+_1 \cos \theta}{W^-_1 P^-_1 - P^+_1 \cos \theta}
\]
\[
\frac{\partial Q}{\partial \theta} = \frac{P_+ P_\text{sin}\theta}{57.3M_0} \text{ Mev/deg.}
\]

\[
\frac{\partial Q}{\partial M_+} = \frac{\frac{M_+}{M_0}(1 + \frac{W_-}{W_+}) - 1}{M_0} ; \quad \frac{\partial Q}{\partial M_-} = \frac{\frac{M_-}{M_0}(1 + \frac{W_+}{W_-}) - 1}{M_0}
\]

Many photographs of V-particle disintegrations cannot yield accurate values for all the quantities we have assumed. Furthermore, it is inconvenient to carry out a complete computation of all these quantities for the relatively poor cases. In particular, it is more convenient to represent the data in a fashion which demonstrates their compatibility or incompatibility with certain assumed decay schemes and which checks the effects of observational bias on the isotropy of emission in the center of mass system. For this purpose it is convenient to make use of the following quantities:

\[
a = \frac{P_+ \cos\theta_+ - P_- \cos\theta_-}{P_+ \cos\theta_+ - P_- \cos\theta_-} ; \quad a_0 = \frac{M_+^2 - M_-^2}{M_0^2}
\]

\[
a' = \frac{P_+^2 - P_-^2}{P_+^2 + P_-^2 + 2P_+P_- \cos\theta} ; \quad a'' = \frac{\sin(\theta_- - \theta_+)}{\sin\theta}
\]

\[
P_{T+} = P_+ \sin\theta_+ ; \quad P_{T-} = P_- \sin\theta_-
\]

\[
P_T = \frac{P_+P_\text{sin}\theta}{\sqrt{P_+^2 + P_-^2 + 2P_+P_- \cos\theta}}
\]

For an ideal two-body decay observation \(a = a' = a''\) and \(P_{T+} = P_{T-} = P_T\). The values may differ due to errors in measurement or due to a wrong identification of the origin of the V-particle. It should be noted that \(a'\) and \(P_T\) can be determined by measurements on the secondaries only.

In terms of the parameters which specify the decay
in the center of mass system,

\[ p_T = P'\sin \theta' \]
\[ \alpha = \alpha' = \alpha'' = \alpha_0 + \frac{2P'\cos \theta'}{M_0\beta_0} \]

where \( \beta_0c \) is the velocity of the \( v^0 \). If the secondaries are emitted isotropically in the center of mass system, these quantities should have the following distributions:

\[
\text{Prob}(p_T)dp_T = \frac{p_T dp_T}{\sqrt{P'_{12} - p_T^2}} \quad \text{for} \quad |\alpha - \alpha_0| \leq \frac{2P'}{M_0} \\
\text{Prob}(\alpha) d\alpha = \frac{M_0}{4P'} d\alpha \quad \left( \int_{\beta_0 \text{Prob}(\beta_0) d\beta_0} \right)
\]

The distribution of \( p_T \) is sharply peaked at \( P' \). Its average is \( \frac{P'}{4} \) and its standard deviation is \( .22P' \). The peaked nature is due to the large solid angle per unit angle near \( 90^\circ \). Experimental errors will make the peak smoother but should not affect the average value appreciably.

The distribution of \( \alpha \) is symmetric about \( \alpha_0 \). This value gives us an estimate of the relative magnitude of the secondary masses by using the inequality:

\[
\frac{M_+}{M_-} \geq \frac{1 + \alpha_0 (1 + \frac{2P'}{M_0})}{1 - \alpha_0 (1 + \frac{2P'}{M_0})} > 1 + \alpha_0 \quad (M_+ \geq M_-)
\]

The central region of the distribution \( |\alpha - \alpha_0| \leq \frac{2P'}{M_0} \) is flat and has a height \( \frac{M_0}{4P'} \beta_0 \). For a distribution of \( \beta_0 \) which is peaked near \( \beta_0 = 1 \), the standard deviation of the distribution of \( \alpha \) is of the order of the width of this flat region. For example, if we assume \( \text{Prob}(\beta_0) d\beta_0 = 3\beta_0^2 d\beta_0 \),
then $\sigma(a) = \frac{2P^1}{M_o}$. In actual cloud chamber observations such a distribution would not be surprising since the very slow $V^0$'s decay before reaching the illuminated volume, and many fast cases are observed. It is important that no bias toward certain angles in the center of mass system be introduced by the selection of cases. For example, the requirement that both momenta be well measurable tends to favor cases in which the heavier secondary is emitted backwards because it receives a larger fraction of the primary momentum than the lighter secondary.

A number of plots have been found useful for representing $V$-particle data and checking their consistency with various decay modes. One of the most important has been the "$Q$-plot":\(^{(3,4)}\) a plot of $P_T$ vs. $a$ (Fig. 4). In terms of the constants of the decay the theoretical relation between $P_T$ and $a$ for a given primary momentum is represented by

$$(a-a_o)^2 + \left(\frac{2P_T}{M_oP_o}\right)^2 = \left(\frac{2P^1}{M_o\beta_o}\right)^2$$

Thus the curves of $P_T$ vs. $a$ for constant $P_o$ are ellipses, centered about $a=a_o$, $P_T=0$, which approach a limiting ellipse as $\beta_o \to 1$. This representation is very useful for cases in which the primary is of high momentum so that $\beta_o \approx 1$. In this case the secondaries are usually also very fast so that their ionization is almost minimum and they cannot be identified. However, by observing the
Fig. 4. $\eta$-Plot

The dashed lines are curves of $P_T$ vs. $\alpha$ as $P_+$ is allowed to vary, if $P_-$ and $\theta$ are fixed. The three curves all correspond to $P_\text{sin}\theta = 100$ Mev/c. The curve for $\theta = 30^o$ is almost a straight line, as are the curves for smaller angles. In general, the intercept on the $P_T$ axis ($\alpha = 0$) is $P_\text{sin}\theta/2$. 
distribution of points on the Q-plot, the nature of the decay modes present can be guessed.

Another useful application of the Q-plot is for cases in which the negative momentum and the total angle, $\theta$, are accurately known, but only rough limits can be set on the positive momentum. If we calculate the behavior of $P_T$ vs. $\alpha$ as $P_+$ varies we find that the curve passes through $(1, P_-\sin\theta)$ with slope $\frac{3}{2} P_-\sin\theta$ when $P_+\to\infty$, through $(0, P_-\sin\theta/2)$ with slope $P_-\sin\theta/2$ when $P_+ = P_-$, and through $(-1, 0)$ with slope $\frac{3}{2} P_-\tan\theta$ when $P_+ = 0$. Thus for small $\theta$, the curve of $P_T$ vs. $\alpha$ as $P_+$ varies is approximately a straight line passing through $(1, P_-\sin\theta)$ and $(-1, 0)$. This fact can be seen from the exact relation*

$$\frac{2P_T}{1+\alpha} = P_-\sin\theta \sqrt{1 + \left(\frac{P_-\sin\theta}{P_+ + P_-\cos\theta}\right)^2}$$

This relation is also useful in assigning error vectors on the $\alpha$ vs. $P_T$ plot. If $\theta$ is small, or $P_- \ll P_+$, the error vector due to an error in $P_+$ must point to $(-1, 0)$. If $\theta$ is small a similar argument predicts that the error vector due to an error in $P_-$ must point to $(1, 0)$. Since $\alpha$ is dimensionless in the momenta it can be seen that an error in calibration of the magnetic field would produce an error vector parallel to the $P_T$ axis because it would multiply all momenta by a constant factor.

*This relation was pointed out to the author by Dr. R. V. Adams.
As a check on the isotropy of emission in the center of mass system, one can use the relation \( P_T = P' \sin \theta' \), to draw lines at equal intervals of \( \cos \theta' \) on the Q-plot. If the emission is indeed isotropic, there should be equal numbers of points in each of the intervals on each branch of the ellipse. This procedure is equivalent to checking the distribution of \( P_T \).

It should be noted, in applying the Q-plot to cases for which \( P_+ \) is uncertain, that each part of the \( P_+ \) error vector may correspond to a slightly different value of \( \beta_0 \). Thus in checking the consistency of a case with a particular decay scheme, the variation of \( \beta_0 \) along the error vector must be included. However, for those cases in which \( P_+ \) is uncertain, \( \beta_0 \) is usually very nearly unity so that the variation of \( \beta_0 \) is small. This convenience does not apply to the other plots to be discussed here.

When dealing with data taken from a cloud chamber with no magnetic field, the momenta cannot be found directly. However, in a multiplate chamber the origin can often be located accurately. Thus the angles \( \theta_+ \) and \( \theta_- \) can be measured and \( \alpha \) can be calculated. Another quantity which can be calculated from the angles is

\[
\frac{2P_T}{P_0} = \frac{2 \sin \theta_+ \sin \theta_-}{\sin \theta}
\]

Therefore, it is convenient to plot multiplate chamber data on a \( \frac{2P_T}{P_0} \) vs. \( \alpha \) plot. On such a plot the lines of
constant $\theta$ are universal circles, independent of the
decay mode, given by the equation

$$a^2 + \left(\frac{2P_T}{P_0} + \cot \theta\right)^2 = \csc^2 \theta$$

Thus a calculation of $a$ or $\frac{2P_T}{P_0}$, together with the knowledge
of the total angle, is sufficient to locate a point on the plot.

To check the consistency of events with a particular
mode of decay, further information is needed. If one of
the secondaries is stopped in one of the absorbing plates
in the chamber by Coulomb interactions only, its residual
range at the point of emission can be measured. In addi-
tion, if its mass can be inferred by combining its residual
range with ionization estimates or scattering measurements,
its initial momentum can be calculated, and therefore its
momentum transverse to the line of flight of the $V^0$ is
known. The transverse momentum can then be compared with
the value predicted by a particular decay scheme using
the hyperbolas for constant $P_T$:

$$(a-a_0)^2 - \left(\frac{2P_T}{P_0}\right)^2 \left(\frac{P_{1T}^2 - P_T^2}{P_T^2}\right) = \frac{4(P_{1T}^2 - P_T^2)}{M_o^2}$$
or the ellipses for constant $P_0$:

$$(a-a_0)^2 + \left(\frac{2P_T}{P_0}\right)^2 \left(\frac{P_T^2 + M_o^2}{M_o^2}\right) = \left(\frac{2P_{1T}}{P_0}\right)^2 \left(\frac{P_T^2 + M_o^2}{M_o^2}\right)$$

Isotropy of emission in the center of mass can be checked
using the hyperbolas of constant $\cos \theta'$:

$$(a-a_0)^2 - \left(\frac{2P_T}{P_0}\right)^2 \cot^2 \theta' = \left(\frac{2P_{1T}}{M_o}\right)^2 \cos^2 \theta'$$
It is also useful to study the behavior of the total angle between the secondaries. It can be expressed in terms of the primary momentum and the constants of the decay:

$$\tan \theta = \frac{M_0^2 P_0 P' \sin \theta'}{P_0 \sqrt{1 + W_0^2 \cos \theta'} \left(P_0 \sqrt{1 - W_0^2 \cos \theta'} - M_0^2 P_0 \cos \theta' \sin \theta' \right)}$$

The behavior of this angle can be visualized best by considering the following diagram representing the relativistic vector addition of momenta: \( (M_+ > M_-) \)

A study of the Lorentz transformation of momenta will convince one that the vectors \( \vec{P}_+ \) and \( \vec{P}_- \) are indeed the laboratory momenta of the secondaries. At very low momenta \( (P_0 \ll M_0 \frac{P_0'}{M_+}) \) the point \( O \) lies within both ellipses and low angles are excluded. The minimum value of \( \theta \) decreases with increasing \( P_0 \) until just below the transition \( P_0 = M_0 \frac{P'_0}{M_+} \), the minimum value of \( \theta \) is \( \leq 90^\circ \). At a value of the momentum larger than \( M_0 \frac{P'_0}{M_+} \) but smaller than \( M_0 \frac{P'_0}{M_-} \) all angles, \( \theta \), are allowed. But at \( P_0 \) just larger than \( M_0 \frac{P'_0}{M_-} \) angles above \( 90^\circ \) are forbidden. This upper limit on \( \theta \) then decreases for increasing momenta.

For many high energy \( V \)-particle decays, the total
angle can give a rough estimate of the primary momentum if the nature of the decay is assumed. For high energy \(V\)-particles the distribution of the angle, \(\theta\), is highly peaked near the value corresponding to \(\theta' = 90^\circ\). Also neglecting \(M_o^2 P^1\) compared to \(P_o^2 W_+ W^-\) we get the estimate:

\[ P_o \propto \frac{M_o^2 P^1}{W_+ W^- \theta} \]

This value will in general be close to a lower limit, but will still be a good estimate of \(P_o\).

D. Relation Between Two-Body and Three-Body Decays

If all or part of the observed \(V^0\)-particle decays have three secondaries, one of which is neutral, calculations based on the assumption of a two-body decay will give inconsistent results. However, it will still be useful to calculate the \(Q'\)-value, based on the assumption of two-body decay, and correlate the observed statistical distribution of \(Q'\) with that expected from a three-body decay. Likewise, it should be the aim of an analysis of a three-body decay scheme to predict the distribution of quantities which can be measured by making observations on the charged secondaries and the origin of the \(V^0\)-particle: e. g. \(Q', a, \Delta E_T, \Delta E_L, \delta, P_T\).

Firstly we will derive the significance of the
distribution of \( Q' \). If \( M_o' \) is the "primary mass" calculated assuming a two-body decay, we see that the following equations serve to define the procedures for calculating \( M_o \) and \( M_o' \): (\( M_n \) is the mass of the neutral particle in this writing).

\[
M_o^2 = (W_n + W_+ + W_-)^2 - (P_n^+ + P_+ + P_-)^2
\]

\[
M_o'^2 = (W_+ + W_-)^2 - (P_+^+ + P_-)^2
\]

Thus it follows:

\[
M_o'^2 = (M_o - M_n)^2 - 2(M_o T_n + M_n T_0) - 2T_o T_n + 2P_o \cdot P_n
\]

Since we have made no assumptions concerning the Lorentz system in which the above equation has been derived; or, alternatively, since the basic equations are in a relativistically invariant form, we can evaluate the expressions in the center of mass system, \( (T_o' = P_o' = 0) \).

Thus \( M_o'^2 = (M_o - M_n)^2 - 2M_o T_n' \)

or, in terms of the \( Q \)-value,

\[
Q' = \frac{Q + 2(M_+ + M_-)}{Q' + 2(M_+ + M_-)} \left(1 - \frac{T_n'}{T_n \text{ max}}\right)
\]

where \( T_n \text{ max} = Q \left(1 - \frac{M_n}{M_o} - \frac{Q}{2M_o}\right) \)

If \( T_n \text{ max} \ll M_o - M_n \)

Then \( Q' = Q - \frac{T_n'}{1 - \frac{M_n}{M_o}} - \frac{1}{2} \frac{T_n'^2}{M_o(1 - \frac{M_n}{M_o})^2} - \cdots \)

or, in a different form, if \( Q \ll M_+ + M_- \)

Then \( Q' = Q \left(1 - \frac{T_n'}{T_n \text{ max}}\right) \left(1 + \frac{Q T_n'}{2(M_+ + M_-)T_n \text{ max}} \right) + \cdots \)

*The following derivation and result was first pointed out by Dr. R. P. Feynman.*
$T_{\text{max}}'$ is the value of $T_{\text{n}}'$ corresponding to $M_+$ and $M_-$ being emitted in the center of mass system with the same velocity. In any case it is clear that the quantity which determines the "two-body energy release", $Q'$, is the energy of the neutral particle in the center of mass system, and its distribution between 0 and $Q$ is determined by the distribution of $T_{\text{n}}'$.

The total unbalanced momentum perpendicular to the line of flight of the $V^0$-particle is equal in magnitude to the component of $P_{\text{n}}'$ perpendicular to the incident direction. If there is no coupling between the direction of motion of the $V^0$-particle and the decay in the center of mass system we can see that the distribution of $\Delta P_\perp$ should be:

$$\text{Prob}(\Delta P_\perp) d(\Delta P_\perp) = \frac{P_{\text{n}}'_{\text{max}}}{\sqrt{\frac{P_{\text{n}}' \cdot P_{\text{n}}'}{P_{\text{n}}' - \Delta P_\perp^2}}} \text{Prob}(P_{\text{n}}') dP_{\text{n}}' d(\Delta P_\perp)$$

The distributions of $a$ and other quantities are much more difficult to evaluate. Only with specific assumptions regarding the decay scheme and the energies involved can we get an idea of their behavior. Therefore we will discuss them together with the specific assumed schemes.

E. General Three-Body Decay

We shall consider the general three-body decay in the center of mass system. Thus the constraints on the
system are that \( \vec{F}_1' + \vec{F}_2' + \vec{F}_3' = 0 \), \( W_1' + W_2' + W_3' = M_0 \). In general we shall be interested in the distribution in energy or momentum of the neutral secondary, which will be chosen as \( M_1 \). As we have seen, the distribution of \( T_1' \) determines the distribution of "two-body" \( Q' \)-values. Since we shall assume no correlation between the direction of motion of the primary particle and the decay in the center of mass system, the distribution of \( P_1' \) will also determine the distribution of unbalanced momentum. In general,

\[
\text{Prob}(P_1')dP_1'd\Omega_1' = \int_{\Omega_2'} f(P_1', P_2', \theta_2') \frac{dn}{dM_0} \]

where the density of states is given by

\[
\frac{dn}{dM_0} = K \frac{P_1'^2 P_2'^2 dP_1'dP_2'd\Omega_1'd\Omega_2'}{dM_0}
\]

\[
= K \frac{P_1'^2 P_2'^2 W_1' W_2' d\Omega_1'd\Omega_2'dP_1'}{P_2'^2 (M_0 - W_1') + W_2' P_1'^2 \cos \theta_2'}
\]

where \( K \) is a normalization constant and \( f(P_1', P_2', \theta_2') \) is determined from the matrix element involved in the transition from the initial to the final state. An alternative form of this formula is sometimes useful for computational purposes:

\[
\text{Prob}(P_1')dP_1' = K P_1'^2 dP_1' \frac{d}{dM_0} \int_{\Omega_2'} f(P_1', P_2', \theta_2', M_0, M_1) \int_{\Pi=0} f(P_1', \Pi', \theta_2') \Pi'^2 d\Pi' d\Omega_2'
\]

This expression is particularly useful if we are going to perform the integration over \( \Omega_2' \), since the integrations are usually much easier to perform before the differentiation.
When the mass of one of the secondaries (for example $M_3$) is very large, so that its kinetic energy is negligible compared to $T_1'$ and $T_2'$, we can see, by analogy with beta decay theory, that $f(P_1', P_2', \theta_2')$ should include a factor $(P_1'^2 + 2P_1'P_2'\cos\theta_2' + P_2'^2)^\ell$, where the product particles are to carry away an orbital angular momentum of $\ell \hbar$. In this case the expression for $P_2'(P_1', \theta_2', M_0, M_1)$ is independent of $\theta_2'$. Thus, if there are no further factors depending on $\theta_2'$ in $f$, the integration over $\Omega_2'$ can be performed first. Furthermore, if there are no other factors in $f$ we can get an expression for the distribution of $P_1'$:

$$
\text{Prob}(P_1') dP_1' = 2\pi M_2' \sum_{\ell=0}^{\infty} \frac{(2\ell+1)!}{(2\ell-2s+1)!(2s+1)!} P_1'^{2s+2} P_2'^{2s-2s+1}
$$

F. Uncoupled Three-Body Decay

If there is no coupling between the emitted particles $f(P_1', P_2', \theta_2') = 1$, we can proceed to evaluate the probability rigorously. We notice that the integral

$$
I = \int_{\Omega_2'} P_2' d\Pi d\Omega_2'
$$

is just the volume of the figure in a three-dimensional momentum space $(\xi, \eta, \zeta)$ whose bounding surface is the locus of possible values of the momentum vector $P_2'$. The constraint is imposed by conservation of energy,
namely:

\[ W_1' + \sqrt{\xi^2 + \eta^2 + \zeta^2 + M_0^2} + \sqrt{(\xi + P_1')^2 + \eta^2 + \zeta^2 + M_0^2} = M_0 \]

This becomes the formula for a prolate spheroid:

\[ \frac{(\xi - h)^2}{a^2} + \frac{\eta^2 + \zeta^2}{b^2} = 1 \]

where

\[ a^2 = \frac{W_1'^2 M_0}{(W_1'^2 - P_1')^2} (W_{1m}' - W_1')(W_{1m}' - W_1') \]

\[ b^2 = \frac{M_0}{(W_1'^2 - P_1')^2} (W_{1m}' - W_1')(W_{1m}' - W_1') \]

\[ h = \frac{P_1'}{2(W_1'^2 - P_1')} (2M_0 W_1' - M_0 a^2 - M_1 a^2 - M_2 a^2 + M_3 a^2) \]

where \( W_1' = M_0 - W_1' = W_2' + W_3' \)

\[ W_{1m}' = \frac{1}{2M_0} [M_0^2 + W_1^2 - (M_2 + M_3)^2] = M_1 + \frac{Q}{2M_0} (Q + 2M_2 + 2M_3) \]

\[ W_{1m}' = \frac{1}{2M_0} [M_0^2 + W_1^2 - (M_2 - M_3)^2] = M_1 + \frac{Q}{2M_0} (Q + 2M_2) (Q + 2M_3) \]

\( W_{1m}' \) is the maximum total energy particle \( M_1 \) can have. Both \( a \) and \( b \) vanish, namely, the spheroid shrinks to a point, when \( W_1' \) attains this maximum value. The root \( W_1' = W_{1m}' \) is unattainable and corresponds to one energy having a negative value; it is always larger than \( W_{1m}' \).

Now we can evaluate \( I \) as the volume of this spheroid, namely \( 4/3\pi ab^2 \).

\[ I = \frac{4}{3} \frac{M_0^3 (W_{1m}' - W_1')^2 (W_{1m}' - W_1')^2 (M_0 - W_1')}{(M_0 - 2M_0 W_1' + M_2 a^2)^2} \]

The only remaining step is the differentiation, which is cumbersome but straightforward:

\[ \text{Prob}(P_1') dP_1' = dP_1' \frac{2\pi KP_1'^2 (W_{1m}' - W_1')^2 (W_{1m}' - W_1')^2}{3(1 - \frac{2W_1'}{M_0} + \frac{M_3}{M_0} a^2)} U(W_1') \]
where, \[ U(W_1') = 3\left(1 - \frac{W_1'}{M_0}\right)^2 \left(1 - \frac{2W_1'}{M_0} + \frac{M_1^2}{M_0^2}\right)(W_{1m'} + W_{1\mu'} - 2W_1') \]

\[ - \frac{2}{M_0}(W_{1m'} - W_1')(W_{1\mu'} - W_1')(3 - \frac{6W_1'}{M_0} + \frac{4W_1'^2}{M_0^2} - \frac{M_1^2}{M_0^2}) \]

It can be noted that it would be a very tedious task to arrive at such an expression from an integration over angles of \( \frac{d\eta}{d\Phi_0} \).

Some approximate expressions are especially useful. Firstly, if \( M_3 \) is very large so that \( T_3' \ll \Theta \), we get the formula, (5)

\[ \text{Prob}(P_1')dP_1' = 4\pi K P_1'^2 P_2' W_2'dP_1' \]

Secondly, if \( W_{1m'} \ll M_0 \),

\[ \text{Prob}(P_1')dP_1' = 2\pi K P_1'^2 (W_{1m'} - W_1') \left(\frac{W_{1\mu'} - W_1'}{W_{1m'} + W_{1\mu'} - 2W_1'}\right) \]

We note that the expression for the probability distribution of \( P_1' \), when \( M_3 \) is very large, can be gotten easily by integrating the expression for \( \frac{d\eta}{d\Phi_0} \) in this approximation, since the latter is then independent of \( \Theta_2' \). Furthermore, in this particular case we can insert any \( f(\Theta_2') \) without affecting the distribution of \( P_1' \).

G. **Cascade Three-Body Decay**

Another type of decay scheme that should be considered is one in which the primary undergoes a two-body decay, but one of the secondaries undergoes another two-body decay so rapidly that only the two charged end products can be observed in the cloud chamber. Consider the decay
scheme: \[ M_0 \rightarrow M_1 + M_* + Q_1 \]
\[ \rightarrow M_2 + M_3 + Q_2 \]

Quantities in the system in which \( M_0 \) is at rest (center of mass system) will be denoted by primes, quantities in the system in which \( M_* \) is at rest by double primes.

Two types of decays should be considered: those in which \( M_1 \) and \( M_* \) are neutral particles (type I) and those in which \( M_1 \) and \( M_* \) are charged particles and \( M_3 \) is a neutral particle (type II). Decays of type I are simpler and will be considered first.

In the center of mass system of cascade decays of type I, the neutral product, \( M_1 \), has a unique energy \( W_1' \) and a unique momentum \( P_1' \) determined by the conservation of energy and momentum in the two-body decay \( M_0 \rightarrow M_1 + M_* + Q_1 \). Calculations from the measurements on the charged products \( M_2 \) and \( M_3 \) will yield a unique value of \( Q' \), namely \( Q_2 \), and the two-body distribution of \( P_T \) and \( \alpha' \). The only evidence for such a decay scheme would come from the location of the origin of the primary particle. The cascade decay scheme would manifest itself by yielding a distribution of \( \Delta P_1 \) determined by the momentum of the neutral secondary, \( M_1 \).

\[
\text{Prob}(\Delta P_1) d(\Delta P_1) = \frac{\Delta P_1 d(\Delta P_1)}{P_1' \sqrt{P_1'^2 - (\Delta P_1)^2}}
\]

If \( Q_1 \) were small compared to \( Q_2 \), such a decay scheme would be very difficult to distinguish from a simple two-
body decay in which errors of measurement produced the observed distribution of $\Delta p_1$.

Cascade decay II has more interesting manifestations. The distribution of the energy of the neutral secondary, $M_3$, in the center of mass system determines the distribution of the "two-body energy release", $Q'$. In terms of the parameters of the decay

$$W_3' = \frac{W_{*}' W_{*}'' + P' P'' \cos \theta''}{M_*}$$

where $W_{*}'$ and $P'$ are the total energy and momentum of $M_*$ in the center of mass system, and $W_{*}''$, $P''$, and $\theta''$ are the energy, momentum and polar angle of emission of $M_3$ in the system in which $M_*$ is at rest. If the second decay is isotropic in its center of mass system, $\cos \theta''$ is distributed uniformly between -1 and 1. Thus, the distribution of $W_3'$ is uniform between the limits

$$\frac{1}{M_*} (W_{*}' W_{*}'' \pm P' P'')$$. If $Q_1 + Q_2 < M_0 - M_3$ this distribution implies an almost uniform distribution of $Q'$ centered about $Q_1 + Q_2 - \frac{1}{M_*} (W_{*}' W_{*}'' - M_* M_3) \left( \frac{M_0}{M_0 - M_3} \right)$ with a half width $\frac{P' P'' M_0}{M_* (M_0 - M_3)}$.

This distribution does not necessarily reach 0 or $Q$, as distinct from the general three-body decay. The reason lies in the fact that $T_3'$ can be zero only if $\frac{P'}{M_*} = \frac{P''}{M_3}$; and that the condition when $M_1$ and $M_2$ are emitted with equal velocities also cannot be realized in general for
cascade decays.

The expression for the transverse component of \( \vec{P}_3 \) can be worked out by a succession of Lorentz transformations and rotations. However, the expression is cumbersome and a rigorous derivation of the distribution of \( \Delta P_1 \) would be very difficult. We can get an estimate of the magnitudes of \( \Delta P_1 \) to be expected by taking the average values of the angles encountered: namely \( \cos \theta \approx 0.5 \), etc. Then the magnitudes of \( \Delta P_1 \) to be expected will be of the order of

\[
\frac{1}{2} P''(1 + \frac{W'}{M'}) + \frac{P'W''}{M''}
\]

Since the observed products come from two different decays, the distribution of \( a' \) will be very complicated and of little value.

H. Three-Body Decay With One Heavy Secondary

If one of the charged products of a decay scheme is very heavy, so that it takes a negligibly small fraction of the \( Q \)-value as kinetic energy in the center of mass system, the dynamics of various three-body decay schemes are particularly simple. This case is a usable approximation if one of the products is a proton and the others have masses no heavier than \( \pi \)-mesons. Of course, the results will then be inexact, but the statistical data from a limited number of observations is also very rough
and only general indications can be inferred from them.

As pointed out in section III-F, the probability distribution for \( P_1' \) for an uncoupled three-body decay becomes

\[
\text{Prob}(P_1') dP_1' = 4\pi K P_1'^2 P_2' W_2' dP_1' \quad (M_3 \text{ large})
\]

If we investigate what this means in terms of the vector triangle in three-dimensional momentum space we find that the locus of \((\xi', \eta', \zeta')\) reduces to a sphere. This fact follows from the conservation of energy, which requires the magnitude of \( P_2' \) to be fixed, when \( P_1' \) is fixed, at a value corresponding to \( T_2' = Q - T_1' \). Furthermore, introducing a weighting factor dependent only on angles has no effect on the distribution of \( P_1' \) since the integration over angles is independent of the magnitude of \( P_2' \), and thus independent of the magnitude of \( P_1' \). In this case it might be possible to get correlations between the direction of emission of the secondaries, but the distribution of \( Q' \) and \( P_1' \) are unaffected.

If in addition to assuming that \( M_1 \) and \( M_2 \) are small compared to \( M_3 \) we also assume \( Q \) to be small we can calculate an approximate value for \( \alpha \):

\[
\alpha \approx \frac{2M_3 - M_0}{M_0^2} + \frac{2}{\beta_0 M_0^2} \left[ M_3 \beta_0 W_1' - (M_0 - M_3) P_1' \cos \theta_1' - M_0 P_2' \cos \theta_2' \right]
\]

where \( \cos \theta_1' \) and \( \cos \theta_2' \) are independently distributed uniformly between -1 and +1. Thus we get a distribution of \( \alpha \), for each value of \( W_1' \), centered about \( \frac{2M_3(M_0 + W_1') - M_0^2}{M_0^2} \).

However, we have assumed \( Q \) to be small compared to \( M_3 \)
and therefore small compared to $M_0$. Therefore the central value of $a$ will be near $\frac{2M_0(M_0+M_1)-M_0^2}{M_0^2}$. For $\beta_0$ near 1 the width of the distribution will be smaller than this central value and the distribution will resemble quite closely the two-body decay $a$ distribution when one secondary is heavy compared to the other.
IV. PRELIMINARY RESULTS ON NEUTRAL V-PARTICLE DECAYS

A. Introduction

During the year 1953 the 48" magnet cloud chamber equipment has produced approximately 16,400 photographs. The geometry of absorbers and cloud chambers has been changed from time to time and many different minimum coincidence requirements have been tried. No systematic enumeration has been made of the number of penetrating showers detected, nor of the average number of shower particles present. Fig. 5 is a tabulation of the number of V-particles detected by various coincidence requirements used with two different chamber geometries and lead or copper absorbers. The data include only the runs in which all three geiger counter trays had lead or copper shielding completely around each counter to absorb most knock-on electrons. The results suggest that the extra density in lead makes it more efficient for the production of V-particles. There is also an indication that a larger fraction of the V-particles produced in lead have measurable secondaries, i.e. have low momenta, but the number of events involved is too small to make such a conclusion definite.

During this year of operation a total of 360 neutral V-particle decay events and 57 charged V-particle decay events were observed in the 48" magnet cloud chambers. Due to the limitation of available time, not all of these
Fig. 5. \( V^0 \) Decay Yields

<table>
<thead>
<tr>
<th>Counters</th>
<th>Running time</th>
<th>Pictures per hour</th>
<th>( V^0)!</th>
<th>( V^0 )!/hr.</th>
<th>( V^0 )</th>
<th>( V^0 )/hr.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lead Absorbers With Four Chamber Geometry</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-2-0+0-0-2-2</td>
<td>775</td>
<td>4.4</td>
<td>21</td>
<td>.027</td>
<td>64</td>
<td>.082</td>
</tr>
<tr>
<td>2-2-1+0-2-2</td>
<td>633</td>
<td>3.3</td>
<td>16</td>
<td>.025</td>
<td>57</td>
<td>.090</td>
</tr>
<tr>
<td>Lead Absorbers With Three Chamber Geometry</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-2-1+0-2-2</td>
<td>572</td>
<td>2.6</td>
<td>21</td>
<td>.037</td>
<td>46</td>
<td>.080</td>
</tr>
<tr>
<td>Copper Absorbers With Four Chamber Geometry</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-2-1+0-2-2</td>
<td>409</td>
<td>4.7</td>
<td>6</td>
<td>.015</td>
<td>32</td>
<td>.078</td>
</tr>
<tr>
<td>Copper Absorbers With Three Chamber Geometry</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2-2-1+0-3-2</td>
<td>749</td>
<td>3.1</td>
<td>19</td>
<td>.025</td>
<td>53</td>
<td>.071</td>
</tr>
<tr>
<td>2-2-1+0-2-2</td>
<td>542</td>
<td>3.9</td>
<td>16</td>
<td>.030</td>
<td>36</td>
<td>.066</td>
</tr>
</tbody>
</table>

This table summarizes the rate of detection of \( V^0 \) events with various counter requirements. The numbers 2-2-0 refer to the minimum number of counters in each tray which are required to fire simultaneously. The numbering sequence is from top to bottom. The three chamber geometry consists of the double-height chamber on top with two regular size chambers below.

*\( V^0 \)! refers to those decays in which the momenta of both secondaries can be measured. \( V^0 \) includes all identified \( V^0 \) decays.
cases have been analyzed and only a fraction of the inherent information has been extracted. The emphasis has been placed on making accurate measurements on the best cases. The definition of the best cases is not precisely formulated, but it involves reasonable measurability on the curvatures of both secondary tracks. The \( \gamma^0 \) decays included in this summary were required to have an estimated error of the \( Q \)-value less than one-third the calculated \( Q \)-value. Such an a posteriori criterion introduces some bias into the selection of cases, but it will not be significant since no important conclusions will be drawn from statistical data which are sensitive to this choice.

The problem of distinguishing \( \Lambda^0 \) decays from \( \Theta^0 \) decays is an important one. The criterion adopted has led to three classifications:

1. The positive product is clearly heavily ionizing and consistent with a proton.

2. The positive secondary is clearly lighter than a proton and its momentum and ionization are consistent with it being an \( L \)-meson.

3. The ionization of the positive secondary is indistinguishable from minimum but its momentum is so large that it could be a proton.

In considering \( \Lambda^0 \) decays only cases in class 1 were used. This criterion is likely to introduce some bias favoring the proton being emitted backwards in the center
of mass system, and caution must be taken in interpreting the data on the isotropy of emission of the secondaries.

In considering $\theta^0$ decays, a number of class 3 particles were included in addition to the class 2 particles, because the $Q$-value calculated assuming the decay to be a $\Lambda^0$ decay was anomalously high. It is true that these cases could be the high-$Q$ $\Lambda^0$ decays which have been reported previously, but considering them as $\theta^0$-particles does not alter any of the conclusions drawn from the data. Again a bias is introduced favoring backward angles of emission in the center of mass system for the positive secondary, because slow positive secondaries are favored. It can also be seen by examining a $P_T$ vs. $\alpha$ diagram that for $\beta_0 \sim .9$, the curve corresponding to a proton being emitted forward in the center of mass system of a $\Lambda^0$ decay overlaps part of the corresponding curve for a $\theta^0$. Thus in this region a number of cases will occur in which the selection cannot be made on the basis of $Q$-value. Furthermore, when $\beta_0 \sim .9$, the ionization of a proton emitted forward in the center of mass system is indistinguishable from minimum when visual ionization estimates are made.

B. $\Lambda^0$ Decays

The question of the uniqueness of the $Q$-value for the $\Lambda^0$ decay mode has long been a subject of controversy.\(^{(3,4,6-8)}\) Therefore, one of the first tasks to be
attacked with the new data was to calculate the $Q$-values for the best $\Lambda^0$ decays. Naturally, the criterion of good measurability will tend to eliminate high-$Q$ cases, on the average. Nevertheless, such a selection of cases should serve to distinguish between one or more unique $Q$-values and a distribution of calculated $Q$-values due to the presence of a three-body decay. The observed distribution would have no quantitative meaning until the bias were removed. Likewise the absence of cases with very high $Q$-values in the sample would not necessarily imply their nonexistence.

The $Q$-values of the $\Lambda^0$ decays (class I only) were calculated from the measured values of $P_+$, $P_-$ and $\theta$, assuming the products to be a proton and a $\pi$-meson. The errors in the momenta were estimated and the corresponding error in the $Q$-value calculated. The cases for which $\Delta Q < Q/3$ were included in the histogram of Fig. 6, wherein the cases for which $\Delta Q \leq 5$ Mev contributed the shaded area. Each case was assigned a block of width 5 Mev, centered about the calculated $Q$-value, and height $\frac{Q^2}{100(\Delta Q)^2}$. The weighted mean and the weighted standard deviation of the best $\Lambda^0$ decays ($\Delta Q \leq 5$) was computed analytically. The Gaussian curve drawn on Fig. 6 has the same area, mean, and standard deviation as this distribution. The weighting factor was $\frac{1}{(\Delta Q)^2}$ as prescribed by a least squares adjustment of the data.
Fig. 6. Q-Values of $\Lambda^0$ Decays

Only cases with $\Delta Q < Q/3$ have been included in this histogram. Each case was assigned a block of width 5 Mev and height $\frac{Q^2}{100(\Delta Q)^2}$ centered about the calculated Q-value. The shaded portion is the contribution of 20 cases which have $\Delta Q \leq 5$ Mev.
The probable error of the distribution of the best Q-values is ±2.5 Mev. The individual cases which contributed to this distribution had estimated errors ranging from ±2.5 Mev to ±5 Mev, with a weighted average of ±3.3 Mev. The fact that the distribution of Q-values has a width less than the estimated errors implies firstly that the errors were slightly overestimated, and secondly that the distribution is indistinguishable from that due to a single two-body decay.

The presence of possible systematic errors in this Q-value determination has been discussed in section I-E. The only appreciable systematic error, which is due to the magnetic field calibration, has a magnitude of about ±.7%. The effect of this error on the Q-value through the proton momentum is negligible because \( \frac{\Delta Q}{\Delta P_+} \) is always small. However, \( \frac{\Delta Q}{\Delta P_-} \) is usually about .5, and the average value of \( P_- \) for these cases is about 150 Mev/c. Thus the estimated effect of such an error on the Q-value is to introduce a systematic error of about ±.5 Mev. Combining this with the statistical error of ±.5 Mev we conclude:

\[ \Lambda^0 \rightarrow p^+ + \pi^- + (34.8 \pm 1) \text{MeV} \]

An examination of the histogram in Fig. 6 reveals that inclusion of all cases for which \( \Delta Q < Q/3 \) does not change the conclusion that the data are consistent with a single two-body decay.

The existence of a two-body decay with a Q-value,
calculated assuming \((\Lambda^0 \rightarrow p^+ + \pi^-)\), near 35 Mev seems well established by these data. Other Q-values, particularly very high ones, cannot be excluded due to the small sample and selection bias. However, it can be stated that such cases probably do not occur at low energy more frequently than once for every ten of the 35 Mev cases.

Photographs of \(\Lambda^0\) decays in a regular size chamber and the double-height chamber are reproduced in Figs. 7 and 8, respectively. These demonstrate the fact that the cloud chambers are designed so that the camera can photograph tracks within .5 cm. of the top and bottom walls. Thus a V-particle can be produced in the walls, or in the absorber lying on the walls, and have its decay observed in the chamber gas after a flight of only a few centimeters. Thus the decay of very slow cases can still be observed in the chamber. Furthermore, the illuminated region of the chambers is quite deep (~20 cm.) so that particles emitted at large angles with the vertical direction can also be observed frequently. These facts account for the large number of low energy \(\Lambda^0\)-particles observed and the accuracy of the resultant data.

A search has been made in all of these photographs for possible origins for the \(\Lambda^0\)'s. When such an origin is located accurately, the coplanarity angle \(\delta\) and the transverse momentum unbalance \(\Delta P_T\) can be computed. In most of these cases the limiting factor in the determin-
Fig. 7. Photograph 14979 ($\Lambda^0$ Decay)

The $\Lambda^0$ decay occurred at the left in the bottom cloud chamber. The momentum of the positive secondary is unusually low (110±20 Mev/c). There is no clear origin in the copper plate between the chambers, but there is evidence of nuclear interactions. The Q-value is calculated to be 35.5±2.5 Mev.
Fig. 8. Photograph 23023 (Λ^0 Decay)

The narrow angle Λ^0 decay occurred at the left of the main shower traversing the double-height cloud chamber. The long tracks available make this a well measurable case despite the high primary momentum (763 ± 35 MeV/c). The plane of decay of the Λ^0 includes the excellent origin and transverse momentum balances within the experimental uncertainties. The Q-value is calculated to be 36 ± 4 MeV.
ation of these quantities was the accuracy of location of the origin. However, an upper limit on the magnitude of the momentum of a possible neutral third secondary can be estimated by observing the prevailing magnitudes of $\Delta P_T$ and $P_0\delta$. For a unique value of $P_n'$, the distribution of $\sqrt{(\Delta P_T)^2 + (P_0\delta)^2}$ should be peaked near $P_n'$. Thus, the radius of a circle which encloses most of the $\Lambda^0$ points on the $\Delta P_T$ vs. $P_0\delta$ plot of Fig. 9 should be an estimate of the maximum value of $P_n'$ which could be present. The maximum value of 70 Mev/c estimated from Fig. 9 is probably high since most of the values of $\Delta P_T$ and $P_0\delta$ plotted in Fig. 9 can be explained by the experimental inaccuracy in locating the origin. The only origins that were discarded as being obviously inappropriate were ones for which the unbalanced momentum would have been many times this value.

C. $\theta^0$ Decays

Two photographs of $\theta^0$ decays are reproduced in Figs. 10 and 11. The $Q$-values, calculated assuming two $\pi$-meson secondaries, for all cases in class 2 (light positive secondary), as well as for those in class 3 which cannot be ordinary $\Lambda^0$ decays, have been plotted in Fig. 12. Again, the momenta of the secondaries and the angle between them were used to calculate the $Q$-value. The data represent a large range of $Q$-values and therefore it is desirable to indicate the significance of the percentage
Fig. 9. $\Delta P_T$ vs. $P_0\delta$ for $\Lambda^0$'s and $\theta^0$'s.

Most of the $\Lambda^0$ values of $\Delta P_T$ and $P_0\delta$ are consistent with the estimated experimental uncertainties. A circle of radius 70 Mev/c encloses most of the $\Lambda^0$ points indicating that these data cannot rule out a possible neutral third product whose momentum is usually less than this number. Many $\theta^0$ points lie well outside this circle, but there are not enough cases of $\theta^0$ decays to draw any conclusions.
Fig. 10. Photograph 9630 (θ^0 Decay)

This photograph illustrates a θ^0 decay observed in the second chamber of the four chamber geometry. The line of flight of the θ^0 (assuming a two-body decay) comes forward in the chamber, so that the origin may lie behind the top chamber. The positive momentum of 265 ± 19 Mev/c makes its ionization inconsistent with that of a ξ-meson. The Q-value, calculated assuming two π-mesons as secondaries, is 186 ± 38 Mev.
Fig. 11. Photograph 22332 ($\theta^0$ Decay)

This is the best example of a $\theta^0$ decay observed with the 48" magnet cloud chambers. The positive secondary appears to undergo a single scattering or a $\pi-\mu$ decay (deflection $\sim 7^0$) at a point one-third of its length from the apex. The Q-value is calculated to be $215 \pm 17$ Mev.
Fig. 12. Q-Values of $\theta^0$ Decays

Only cases with $\Delta Q < Q/3$ have been included in this histogram. Each case was assigned a block of width $Q/5$ and height $\frac{Q^2}{100(\Delta Q)^3}$ centered about the calculated Q-value. The statistics are very poor since most of the peak is contributed by one case (see Fig. 11). The two cases near 40 Mev are "anomalous $\theta^0$" decays described in Section IV-D.
error rather than the absolute error in each case. Therefore the cases were assigned a block of width $= \sqrt{Q}$ and a height $\frac{Q^2}{100(\Delta Q)^2}$.

At once it is clear from Fig. 12 that the quality of these data is not comparable to the $\Lambda^O$ data. The explanation for this fact lies in the observed phenomenon that the slowest $\theta^O$-particles are observed at much higher momentum than the slow $\Lambda^O$-particles.

The distribution of $Q$-values near 200 Mev is consistent with a unique two-body decay, although these data alone are not good enough to establish such a conclusion. $(3,4)$ However, there exists an excellent case, no. 19143B, whose calculated $Q$-value is $41\pm5$ Mev. This case forms the basis of the discussion of "anomalous $\theta^O" decays in section IV-D.

The possible origins have been located and Fig. 9 presents the resulting values of $\Delta p_T$ and $p_0\delta$. The number of cases is too small to draw any conclusions from these data. The only indications observed subjectively are that the origins for the $\theta^O$-particles are not as clearly located, nor do the values of $\Delta p_T$ and $p_0\delta$ fall within the estimated errors as often as for the $\Lambda^O$s. The explanation for these observations, if true, may be connected with the high average primary momentum: namely, the particles travel farther before decaying and thus are not clearly associated with a single origin.
D. "Anomalous $\theta^0$ Decays*

1. Introduction

It has been recognized for some time by many laboratories that decay events exist which cannot be explained by the $\Lambda^0(\pi, \pi, 35\text{ Mev})$ or $\theta^0[\pi, L, 214(\pi, \pi)]$ decay modes. (1, 3, 4, 8, 12) One of these anomalous modes is the so-called high-$Q$ $\Lambda^0$-particle. Another is the $V_s^0(L^+, \pi^-)$ mode suggested by one photograph from the 18" magnet cloud chambers, (8) which could also be a charged $V$ decay with an anomalously large energy release. The 48" magnet cloud chambers have to date furnished no evidence for either of these decay modes. However, a case of a decay into two light secondaries with a very small energy release has been observed. Similar cases have been observed in other laboratories, but many of these could have been explained by the $V_s^0$ decay scheme. The following discussion will present in detail the data from the three best cases observed in the three Caltech magnet cloud chambers. Four more cases will be added to the tabulated data, and an attempt will be made to enumerate some of the possible interpretations.

2. Photograph 8796 (18" Magnet)

Fig. 13 is a reproduction of the first of these

---

*The following discussion is essentially that published in reference number 14.*
Fig. 13. Photograph 8796 ("Anomalous $\theta^0$" Decay) (18" magnet)

The $V_0$ decay is observed just below the lead plate. The $V_0$-particle probably originated at the interaction in the lead plate. A positive particle corresponds to a clockwise curvature.
unusual $V^0$ decays observed in this laboratory.\textsuperscript{(11)} The $V^0$-particle decayed in the lower chamber and was undoubtedly produced in the interaction in the lead plate between the two chambers. The energy release calculated assuming the products to be two $\pi$-mesons is $116 \pm 30$ Mev. The momentum of the positive particle (curved clockwise) would have to be doubled in order to make the calculated $Q$-value of this event $214$ Mev, or to make it consistent with the decay of a $\Lambda^0$-particle with a $Q$-value of $35$ Mev. The plane of the decay products contains the interaction in the lead plate within the large errors of measurement introduced by the shortness of the line of flight of the unstable particle. Also the components of the momenta perpendicular to the assumed line of flight of the $V^0$-particle balance within experimental accuracy. This statement would not be true if the positive momentum were doubled.

3. Photograph 12590 (21" Magnet)

Fig. 14 shows the decay of a $V^0$-particle into two light secondaries. On the original film the individual droplets are clearly resolved along all tracks in the chamber.\textsuperscript{(13)} Therefore the relative ionization of the $V$-particle secondaries and comparison tracks can be determined. If the identity and momenta of the particles producing the comparison tracks are known, then through
Fig. 14. Photograph 12590 ("Anomalous $\theta^0$ Decay) (21" magnet)

The $V^0$ decay is in the upper left corner and the secondaries both leave the chamber through the rear wall. Individual droplets can be counted on the original photograph permitting a comparison of the ionizations of the secondaries and the two curved electron tracks. A negative particle corresponds to a clockwise curvature.
the application of a theoretical expression for the dependence of ionization on velocity, the specific ionization of the V-particle secondaries can be determined. In the present case the two highly curved comparison tracks are identified as electrons, and the resulting values for the ionizations of the positive and negative secondaries of the V-particle are $1.4 \pm 0.2$ and $1.3 \pm 0.2$, respectively. These values are probably too high, since a preliminary study of the ionization of electrons in the velocity range of the comparison tracks has indicated the actual ionization to be somewhat less than that given by the theoretical formula.

The measured momentum of the negative secondary is $249 \pm 12 \text{ Mev/c}$ and if it is assumed to be a π or μ-meson, its specific ionization is $1.1$. Using this track as a comparison track the specific ionization of the positive secondary is $1.2 \pm 0.2$. The momentum of the positive track is $493 \pm 33 \text{ Mev/c}$. If it is assumed to be a κ-meson its specific ionization is $1.6$. Therefore, the present measurements make unlikely the possibility that the secondaries are a π or μ-meson and a κ-meson, although the possibility that either or both might be electrons cannot be excluded from ionization measurements.

The energy release calculated assuming the secondaries to be two π-mesons is $79 \pm 10 \text{ Mev}$. The short straight track and the longer electron track appear to intersect
in the lead above the chamber, although the exact intersection is uncertain due to the large possible multiple scattering of the electron. This possible origin lies in the plane of decay of the $V^0$-particle and the components of momentum transverse to the assumed line of flight of the $V^0$-particle balance within experimental error.

4. Photograph 19143 (48" Magnet)

Fig. 15 shows two decay events which occurred at points A and B in the large upper cloud chamber of the 48" magnet. The decay at point A is consistent with the usual $\theta^0$ decay scheme. Its energy release, assuming the products to be two $\pi$-mesons, is $238 \pm 30$ Mev.

The decay which appears at point B in this photograph has a Q-value of $41 \pm 5$ Mev, calculated assuming two $\pi$-mesons as the only decay products. If this event actually represents a two-body decay, momentum balance requires that the point of production of the parent particle lie on the extension of the line BP which lies within the illuminated region of the chamber. Extensions of the lines of flight of all charged particles in the chamber other than electrons and heavily ionizing tracks locate two intersections in the lead plate above the chambers at $O_1$ and $O_2$. In addition there are two more nuclear interactions at $O_3$ and $O_4$ in the upper chamber wall from which only heavily ionizing tracks emanate. Whereas event A
shows good coplanarity and transverse momentum balance if $Q_2$ is assumed to be its origin, it is clear for event B that there is no evidence for any interaction along the extension BP. Furthermore, the locations of the interactions above the chamber indicate that the penetrating shower was probably initiated by a single primary interacting at $Q_1$, with secondaries producing further interactions at $Q_2$, $Q_3$, and $Q_4$. It seems quite improbable that a secondary from one of these events was emitted at nearly $90^\circ$ to the primary direction, interacted at the right of the lead plate and emitted an energetic unstable particle in a backward direction along the line PB, and furthermore produced no charged secondaries which entered the chambers.

We thus conclude that this event is probably not a two-body decay. The possibility that it is a three-body decay will now be discussed.

If one assumes the origin of production of the parent particle to be any of the four observed interactions, the component of momentum of the neutral secondary perpendicular to the primary line of flight lies in the range $170 \pm 20$ Mev/c to $210 \pm 25$ Mev/c. This information, together with the assumption of the mass of the neutral secondary, determines a lower limit to the $Q$-value of the three-body decay.

If one assumes the neutral secondary to be a $\pi^0$-
meson, by analogy with the charged $\tau$ decay scheme, the lower limit on the $Q$-value is $165 \pm 20$ Mev. The $Q$-value of the charged $\tau$-meson is known to be $75 \pm 1$ Mev, and thus the interpretation of this event as a $\tau^0(\pi^0, \pi^+, \pi^-, \pi^0)$ decay is inconsistent with the assumption that the particle originated in one of the four observed interactions.

5. Summary of Cases

Fig. 16 tabulates the important data from the events described and a few others. The ionization of all tracks except those of #12590 were estimated visually.

Three of the four cases that have been added to this tabulation have no clear origin for the parent particle. Nevertheless, the estimated ionizations and the orientations make such possible alternatives as the decay of a $\kappa$-meson unlikely. Some of these events could also be interpreted as the decay of a neutral particle into a $\pi$ or $\mu$-meson and a $\kappa$-meson. However, no simplification would result from such an assumption since the best cases cannot be so interpreted.

6. Discussion

It is tempting to interpret the cases which have a very low $Q$-value as representing the decay of a neutral $\tau$-meson into three $\pi$-mesons. However, it has been shown that such an interpretation is very unlikely for at least
Fig. 16. Data on "Anomalous $\theta^0$" Decays

<table>
<thead>
<tr>
<th>Event</th>
<th>Secondary</th>
<th>Momentum</th>
<th>Specific Ionization</th>
<th>Mass</th>
<th>Angle Between Secondaries</th>
<th>$\varphi(\pi,\pi)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8796 (18&quot;)</td>
<td>+</td>
<td>570 ± 130</td>
<td>1 - 1.7</td>
<td>&lt;1200Me</td>
<td>44°</td>
<td>116 ± 30</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>120 ± 5</td>
<td>1.5- 3</td>
<td>220-420</td>
<td></td>
<td></td>
</tr>
<tr>
<td>56337 (18&quot;)</td>
<td>+</td>
<td>240 ± 65</td>
<td>1 - 1.7</td>
<td>&lt;600</td>
<td>63°</td>
<td>148 ± 30</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>390 ± 30</td>
<td>1 - 1.7</td>
<td>&lt;300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>59824 (18&quot;)</td>
<td>+</td>
<td>124 ± 10</td>
<td>1 - 2</td>
<td>&lt;310</td>
<td>61°</td>
<td>96 ± 16</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>365 ± 55</td>
<td>1 - 1.7</td>
<td>&lt;780</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12590 (21&quot;)</td>
<td>+</td>
<td>493 ± 33</td>
<td>1.9 ± 1.2*</td>
<td>840 ± 150*</td>
<td>34°</td>
<td>79 ± 10</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>249 ± 12</td>
<td>1.3 ± 1.2*</td>
<td>385 ± 130*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>15329 (48&quot;)</td>
<td>+</td>
<td>940 ± 100</td>
<td>1 - 1.7</td>
<td>&lt;2000</td>
<td>8°</td>
<td>45 ± 25</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>1420 ± 300</td>
<td>1 - 2</td>
<td>&lt;3500</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19143A (48&quot;)</td>
<td>+</td>
<td>375 ± 30</td>
<td>1 - 2</td>
<td>&lt;970</td>
<td>46°</td>
<td>238 ± 30</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>820 ± 70</td>
<td>1 - 1.7</td>
<td>&lt;1700</td>
<td></td>
<td></td>
</tr>
<tr>
<td>19143B (48&quot;)</td>
<td>+</td>
<td>260 ± 20</td>
<td>1 - 2</td>
<td>&lt;640</td>
<td>40°</td>
<td>41 ± 5</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>87 ± 4.5</td>
<td>1.5- 3</td>
<td>160-300</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20411 (48&quot;)</td>
<td>+</td>
<td>880 ± 150</td>
<td>1 - 1.7</td>
<td>&lt;1600</td>
<td>15°</td>
<td>57 ± 15</td>
</tr>
<tr>
<td></td>
<td>-</td>
<td>525 ± 150</td>
<td>1 - 2</td>
<td>&lt;1300</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*These values are calculated assuming the dubious theoretical electron ionization and are probably too high.

†This case is included here because $\varphi(p,\pi)$ would be 343 ± 80.

‡The positive secondary traverses a 2" lead plate with no apparent increase in ionization. Also $\varphi(p,\pi)$ would be 104 ± 50.
one such case (19143B). In addition there are a number of cases whose two-body Q-values are already too high to be interpreted as such a $\tau^0$ decay.

If one wishes to retain the simplifying assumption that most of these cases are decays of the same particle, a scheme with a much higher energy release is required. One possible decay scheme which satisfies this requirement is an alternate mode of decay of the $\theta^0$-particle into $\pi^+\mu^+\nu$. The computed energy release for this decay is about 244 Mev.

Calculation of the minimum Q-value assuming $\theta^0 \rightarrow \pi^+\mu^-\nu$ for case 19143B yields $245 \pm 30$ Mev. Thus such an hypothesis would explain all of these unusual cases. It should be pointed out, however, that this decay scheme is certainly not a unique one. For example, another decay scheme which can explain all of these cases is $\theta^0$ or $\tau^0 \rightarrow \pi^+\mu^+\mu^-\nu$ ($\sim 148$ Mev). Both of these hypotheses are so broad that they can explain almost any individual event of the type considered here. To confirm or to disprove either of them would require a large amount of accurate data on these rare events.

It is possible that the $\theta^0(\pi,\pi)$ decays having Q-values near 214 Mev represent a part of the same three-body decay distribution. However, if this were the case it is unlikely that the observed Q($\pi,\pi$)-value distribution should be as sharply peaked as present experimental measurements appear to indicate.
7. Summary

There is evidence for the existence of a neutral particle which decays into two charged \( L \)-mesons which cannot be explained by the scheme \( \theta^0 \to \pi^+ + \pi^- (214 \text{ Mev}) \). If only two-body decays are considered, at least two different new particles have to be postulated with \( Q(\pi, \pi) \) near 41 Mev and 90 Mev respectively, and under this assumption a question as to the point of production in at least one case still remains. Not all of these cases can be explained by the decay of a neutral \( \tau \)-meson into three \( \pi \)-mesons. Hence a decay scheme with a higher energy release is indicated. One scheme which might explain these cases in terms of an alternate mode of decay of a known particle is \( \theta \to \pi^+ + \mu^- + \nu + (\sim 244 \text{ Mev}) \). Of course, many other decay schemes might be postulated, but there is no direct experimental evidence for any particular one of these.
BIBLIOGRAPHY

1. Armenteros, Barker, Butler, Cachon; Phil. Mag. 42, 1113, (1951)

2. Podalanski, Armenteros; Phil. Mag. 45, 13, (1954)

3. Thompson, Buskirk, Etter, Karzmark, Rediker; Phys. Rev. 90, 1122(L) (1953)

4. Thompson, Buskirk, Cohn, Karzmark, Rediker; Report B-2 at Bagneres Conference (1953)

5. Brueckner, Thompson; Phys. Rev. 87, 390 (1952)


7. Armenteros; Report A-8 at Bagneres Conference (1953)


10. Reynolds; Report B-5 at Bagneres Conference (1953)

11. Leighton, Wanlass, Alford; Phys. Rev. 83, 843(L) (1951)

12. Barker; Report B-3 at Bagneres Conference (1953)
