

A Theory of Diffusion Cloud Chambers*

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An attempt is made to understand quantitatively the effects of a given gas, gas pressure, vapor, ionization density, and temperature distribution on the operation of diffusion chambers. An integral equation is set up which takes into account diffusion of the vapor through the gas, removal of vapor by condensation, drop growth, and the motion of the drops due to gravity. Temperature distributions necessary to produce tracks through a given depth of a chamber are calculated, applying to a wide variety of different conditions. For example, the average temperature gradient for operation with methanol and air at 1 atmos or hydrogen at 10 atmos exposed to sea level cosmic radiation amounts to 3.6°C/cm while for air at 3 atmos or hydrogen at 30 atmos a gradient of 6.3°C/cm would be required. Because of the formation of uncharged drops at higher operating temperatures and because of the onset of semi-stable convection currents in the light gases operation is expected to become difficult with air at a pressure >3 atmos or hydrogen at a pressure >30 atmos, or with ionization densities >4 times sea level cosmic radiation at 1 atmos of air.

I. INTRODUCTION

IN his article on continuously sensitive cloud chambers Langsdorf¹ showed that a cloud chamber can be constructed making use of diffusion of a vapor through a gas from a hot to a cold surface. Recently Cowan² and Needels and Nielsen³ have revived this idea which has now been taken up widely for demonstrations as well as for research. There are many factors influencing the operation of such chambers and at this point it appeared desirable to understand more clearly, and if possible, more quantitatively how a chosen gas, gas pressure, vapor, ionization density due to radiation, and temperature distribution will affect their behavior.

Constructions of diffusion chambers for different applications have been described in some detail in the given references. Chambers operated at high gas pressures have also been described.⁴ In all of these cases vapor diffuses downward from the warm surface until it has cooled off sufficiently to produce the supersaturation necessary for dropwise condensation on ions. The drops grow and fall to the bottom of the chamber thus removing vapor throughout the region where condensation occurs. The removal of vapor by condensation is one of the most important effects taking place in diffusion chambers and therefore must be taken into account in the theory if quantitative results are desired.

FUNDAMENTAL RELATIONS

The following equations are set up.

1. A one-dimensional diffusion equation shall relate the downward flux of vapor to the vapor density at all points except near liquid drops.

2. Spherically symmetrical vapor and heat diffusion equations determine the growth of the drops.

3. Stokes' equation determines the rate of fall of the drops through the regions at varying temperatures.

4. A relation is set up between the gradient of the vapor flux and the rate at which vapor is removed by condensation of drops on ions.

5. An energy equation shall make it possible to calculate the convection in the gas necessary to establish proper heat exchange with the walls.

f (g/cm² sec) will stand for the vapor flux, ρ^5 for the vapor density, ρ_t for the total density of the gas-vapor mixture, D for the diffusivity of the vapor through the gas-vapor mixture, T for the absolute temperature, and x for the height above the bottom of the chamber of a layer of thickness dx . f , ρ , ρ_t , and T are functions of x . Then

$$f = f\rho/\rho_t - Dd\rho/dx. \quad (1)$$

This equation has been given by Kuusinen,⁶ the first term on the right side taking into account the vapor flux by convection which in most cases considered here will be quite small since $\rho \ll \rho_t$.⁷ Since T varies with x , ρ is not necessarily constant when $f=0$, as (1) would indicate, but rather follows the ideal gas law in first approximation. However, it turns out that this effect is negligible compared to the large variations of ρ when $f \neq 0$.

As boundary conditions we have

$$\begin{aligned} \rho &= \rho(0) & \text{at } x=0 \\ \rho &= \rho(h) & \text{at } x=h, \end{aligned} \quad (2)$$

where h is the distance of the vapor source from the bottom.

Near a liquid drop ρ and T become dependent on

⁵ Cgs units are used unless stated otherwise, °K for temperatures and cal for heat.

⁶ J. Kuusinen, *Ann. Physik* **24**, 445, 447 (1935).

⁷ The significance of the first term can be illustrated by letting ρ approach ρ_t and also letting $d\rho/dx$ go to zero lest large pressure differences be set up in the mixture. Then (1) shows that f approaches pure convection as one should expect. The first term does *not* take into account convection caused by gravitational forces which will be considered later.

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¹ A. Langsdorf, *Rev. Sci. Instr.* **10**, 91 (1939).

² E. W. Cowan, *Rev. Sci. Instr.* **21**, 991 (1950).

³ T. S. Needels and C. E. Nielsen, *Rev. Sci. Instr.* **21**, 976 (1950).

⁴ Miller, Fowler, and Shutt, *Rev. Sci. Instr.* **22**, 280 (1951).

the distance r from the drop. The diffusion equation for spherical symmetry is

$$\partial F/\partial t = (\alpha/r^2)(\partial/\partial r)(r^2\partial F/\partial r), \quad (3)$$

where F stands either for ρ or for T , and α either for the vapor diffusivity D or for the heat diffusivity $K/\rho c_p$, respectively, with K for the heat conductivity and c_p for the specific heat of the mixture. The variation of α with r due to the dependence of T on r has been neglected in the derivation of (3).

A boundary condition for $\partial\rho/\partial r$, derived by means of (1), is

$$dm/dt = 4\pi r_0^2 [(D/(1-\rho/\rho_l))\partial\rho/\partial r]_{r=r_0}, \quad (4)$$

where dm/dt is the time rate of change in mass of the drop and r_0 is the drop radius. Of course $m = (4\pi/3)r_0^3\rho_d$ with ρ_d for the density of the liquid.

The time rate of change of the heat content of a drop is determined by the amounts of heat freed by condensation and conducted away from the drop. Therefore a boundary condition for $\partial T/\partial r$ is

$$mc_d\partial T_d/\partial t = Ldm/dt + 4\pi r_0^2 K(\partial T/\partial r)_{r=r_0}. \quad (5)$$

Here T_d is the drop temperature, c_d the specific heat, and L (cal/g) the latent heat of the liquid.

At the surface of the drop ρ depends only on the vapor pressure of the liquid at the temperature T_d while at a distance $r \gg r_0$, ρ and T depend on x alone. Thus

$$\begin{aligned} T &= T_d, & \rho &= \rho'(T_d) & \text{for } r &= r_0 \\ T &= T(x), & \rho &= \rho(x) & \text{for } r &= \infty. \end{aligned} \quad (6)$$

If v is the velocity of the falling drop, g the gravitational acceleration, and μ the viscosity of the gas-vapor mixture, one has, according to Stokes,

$$d(mv)/dt = -6\pi\mu vr_0 - mg. \quad (7)$$

Designating the mass of a drop started at the height ξ and observed at x by $m(\xi, x)$, and the number of condensation nuclei formed per cm^3 per sec at ξ by $n(\xi)$, we can write for the gradient of the vapor flux

$$\begin{aligned} df/dx &= \int_x^{\infty} [dm(\xi, x)/dx] n(\xi) d\xi \\ &+ [dm(x_0, x)/dx] \int_{x_0}^h n'(\xi) d\xi. \end{aligned} \quad (8)$$

The first term at the right side stands for the removal of vapor by drops formed between x and x_0 , the point

⁸ For chambers employing upward diffusion the only modification necessary in the theory would be to write instead of (8)

$$df/dx = \int_x^h [dm(\xi, x)/dx] n(\xi) d\xi \text{ for } x > x_0,$$

and

$$\begin{aligned} df/dx &= \int_{x_0}^h [dm(\xi, x)/dx] n(\xi) d\xi \\ &+ [dm(x_0, x)/dx] \int_0^{x_0} n'(\xi) d\xi \text{ for } x < x_0. \end{aligned}$$

where condensation first starts, and the second term takes into account that fraction $n'(\xi)$ of the condensation nuclei formed in the space $x_0 \leq x \leq h$ which drifts downward finally reaching $x = x_0$.

In order to maintain temperature equilibrium the amount of heat $H(x)$ (cal/cm³ sec) must be supplied to the gas-vapor mixture. We have

$$\begin{aligned} H(x) &= (df/dx)[L - c_p'(T_d - T(x))] \\ &+ fc_p'dT/dx - Kd^2T/dx^2, \end{aligned} \quad (9)$$

where c_p' is the specific heat of the vapor. The first term at the right side accounts for the heat due to condensation at the temperature T_d , the second term for the transport of heat due to the vapor flux, and the third term for the conduction of heat through the mixture.

The range of temperatures considered here is so wide that the variation of D , L , K , and μ with T must be taken into account. This matter will be considered below.

APPROXIMATIONS AND SOLUTION

The procedure from here on can be summarized as follows. A criterion is set up which shows that the stationary solution of (3) can be used. With this considerable simplification (1) to (8) can be combined into two simultaneous equations one of which (14) determines the temperature of a growing drop under given conditions. The other equation (18) is an integral equation permitting calculation of the temperature distributions necessary to maintain proper supersaturation throughout a given depth. Finally physical relations must be assumed including the vapor pressure over electrically charged drops, vapor density as a function of vapor pressure, the physical properties of different gases and liquids, and ionization densities.

We proceed to show that it is not necessary to find an exact solution of (3) with the given nonstationary boundary conditions.

A particular transient solution of (3) is

$$F = F_0(r, t) \exp(-r^2/4\alpha t), \quad (10)$$

where the exponential function represents by far the most important variations with r and t . A general solution can be composed of particular solutions of this type. One would therefore expect that at a given distance r from the drop the transients become negligible at a time $t \gg r^2/4\alpha$. At that time $\partial F/\partial t = 0$ and, making use of (6), the stationary solution becomes

$$F = F(x) + (F_d - F(x))r_0/r, \quad (11)$$

which is about equal to $F(x)$ at a distance $r = \omega r_0$, where $\omega \gg 1$. Since r_0 is very small we assume that ωr_0 is still small and the question arises whether a drop remains for a time $t \gg \omega^2 r_0^2/4\alpha$ in a region of depth ωr_0 , to permit the transients to wear off. The change in momentum of a drop in the small region ωr_0 must be negligibly small.

Therefore we have from (7)

$$|v| = \omega r_0 / t = mg / 6\pi\mu r_0. \quad (12)$$

Substituting $m = (4\pi/3)r_0^3\rho_d$ and $t \gg \omega^2 r_0^2 / 4\alpha$ we obtain the criterion that

$$18\alpha\mu / g\rho_d\omega r_0^3 \gg 1, \quad (13)$$

if we are to be permitted to use the stationary solution (11). Typical values (Table II) are for $\alpha = 0.1$, $\mu = 2 \times 10^{-4}$, $g = 10^3$, $\rho_d = 0.8$, and $r_0 = 10^{-3}$, and for $\omega = 10$ one finds a value of 50 for the left side of (13). Thus our criterion is well satisfied in this case and in all others to be considered later. We thus conclude that the stationary solution (11) of (3) represents a good approximation. Further estimates making use of the final results show that one is well justified also to neglect the left sides of (5) and (7), because $mc_d dT_a/dt \ll Ldm/dt$, and $d(mv)/dt \ll mg$. Thus, with (4) and the right side of (7) representing the only time dependence we restate the problem in the following semistationary form.

Substituting (11) into (4) and (5) and combining (4) and (5) we obtain

$$\rho'(T_a) - \rho(x) = -\zeta \Delta T_a \quad (14)$$

where

$$\Delta T_a = T_a - T(x)$$

$$dT/dx = \frac{\int_{T_1}^T \frac{D}{1-\rho/\rho_t} \frac{d\rho(T)}{dT} dT - \int_0^x x \frac{d}{dx} \left[\int_x^{x_0} \left(\int_x^\xi \eta \Delta T_a dx \right)^{\frac{1}{2}} n(\xi) d\xi + \left(\int_x^{x_0} \eta \Delta T_a dx \right)^{\frac{1}{2}} \left(\int_{x_0}^h n'(\xi) d\xi \right) \right] dx}{x[D/(1-\rho/\rho_t)]d\rho(T)/dT} \quad (18)$$

where T_1 is the temperature at $x=0$.

For $x > x_0$, where no condensation takes place, a solution for $\rho(x)$ can easily be found by integration of (1) joining its value at $x=x_0$ to that found during the course of solving (18). Thus one finds for the required vapor density at the top ($x=h$)

$$\rho(h) = \rho(x_0) + \left(\frac{D}{1-\rho/\rho_t} \frac{d\rho}{dx} \right)_{x=x_0} \int_{x_0}^h \left(\frac{1-\rho/\rho_t}{D} \right) dx. \quad (19)$$

Since ΔT_a is a function of T (18) is a relatively complicated integral equation, a general solution for which we may not hope to obtain. However, after assuming values for T_1 and $(dT/dx)_{x=0}$ one can assume as a first approximation some reasonable continuation of $T(x)$ and thus calculate the right side of (18) which then represents a second approximation for $T(x)$. This process can be continued to find higher approximations. In our case this method of iteration converges very fast, and often the second approximation is quite sufficient for our purpose. Because of a weak singularity at $x=x_0$ it is advantageous to carry out the operations under the second integral in the indicated succession when numerical computations are performed.

and

$$\zeta = \frac{K(1-\rho/\rho_t)_{r=r_0}}{LD}. \quad (15)$$

Remembering that $v = dx/dt$, (4) combined with (7), (11), and (14), gives

$$d(m^{4/3})/dx = -\eta \Delta T_a \quad (16)$$

where

$$\eta = (4\pi/3)^{4/3} 18K\mu / Lg\rho_d^{2/3}. \quad (17)$$

We now must solve (1), (8), (14), and (16). We are interested to know under what conditions a diffusion chamber will show track formation through a given depth. We therefore demand that a given critical supersaturation exists throughout that depth. Thus at all x the vapor density $\rho(x)$ becomes a prescribed function of T , namely $\rho(T)$, and since the function $\rho'(T_a)$ will also be known (14) can be solved for ΔT_a as a function of T . In order to produce the prescribed $\rho(T)$ a certain temperature distribution $T(x)$ is necessary which we shall attempt to determine.

We integrate (16) and substitute the result for m into (8). Next we differentiate both sides of (1) with respect to x and then multiply both sides by x . Subsequent partial integration of the left side and substitution of (8) for df/dx results in the equation for $T(x)$

Before proceeding several physical relations must be determined. For the vapor pressure $p(T, r)$ exerted by a singly charged drop of radius r and at a temperature T Thomson⁹ has derived the formula

$$p(T, r) = p'(T) \exp[(M/RT\rho_d)(2\sigma/r - e^2/8\pi r^4)], \quad (20)$$

where $p'(T)$ is the vapor pressure over a plane surface, M the molecular weight, and σ the surface tension of the liquid, R the gas constant, and e the electronic charge. Equation (20) has a maximum for $r=r_c$ whose value is

$$p(T, r_c) = p'(T) \exp[(M/RT\rho_d)(3/2)(4\pi\sigma^4/e^2)^{\frac{1}{3}}]. \quad (21)$$

This maximum vapor pressure must at least be maintained where condensation on ions is desired so that drops may grow spontaneously. $p'(T)$ (atmos) is given by

$$p'(T) = \exp(B - E/T), \quad (22)$$

where B and E are constants characteristic of the liquid used.

σ is a function of T found from tables^{10,11} namely

$$\sigma(T) = \sigma_0 [1 - \gamma(T - 273)]. \quad (23)$$

⁹ J. J. Thomson, *Application of Dynamics to Physics and Chemistry* (MacMillan Company, Ltd. London, 1888).

¹⁰ International Critical Tables.

¹¹ *Handbook of Chemistry and Physics*, 30th Edition, 1947.

While one can use mixtures of liquids, as is often done with expansion cloud chambers, the process of diffusion tends to vary their compositions. Therefore, pure liquids are more desirable for diffusion chambers. Perhaps the best suited liquid is methyl alcohol, requiring a low critical vapor pressure $p(T, r_c)$. The latent heat is given by

$$L(T) = L_0[1 - \delta(T - 273)]. \quad (24)$$

The pertinent data for methyl alcohol are tabulated in Table I.

A relation between the vapor densities ρ , ρ' and the vapor pressures p , p' given in (21) and (22) must now be established. If the vapor were an ideal gas we would have

$$\rho = pM/RT \quad \text{and} \quad \rho' = p'M/RT. \quad (25)$$

Making use of Clapeyron's equation for the vapor pressure

$$d\rho'/dT = L\rho'/T \quad (26)$$

and of (22) one finds

$$\rho' \approx p'E/LT \quad (27)$$

instead of (25). Thus if $E/L \approx M/R$ (25) can be used with sufficient approximation. A similar condition can be derived for the supersaturated vapor described by (21), and one finds that (25) is accurate within 15 percent in most cases except for the lowest temperatures of interest where a deviation of 25 percent is found. Since the final result is not sensitive to this error (25) will be used.¹² According to (21), (22), and (25) the required vapor density in the supersaturated layer is

$$\rho(x) = \rho(T) = (M/RT) \exp[B - E/T + (M/RT\rho_a)(3/2)(4\pi\sigma^4/e^2)^{1/3}] \quad (28)$$

while the vapor density at the surface of a drop is

$$\rho'(T_a) = (M/RT_a) \exp(B - E/T_a). \quad (29)$$

According to (20) the drop radius has an effect on the vapor pressure. However, one sees that this is important only near $r = r_c \approx 10^{-7}$ cm and becomes negligible at $r = 10^{-6}$ cm while we are interested only in drops whose radius is $\gg 10^{-6}$ cm when we apply (29).

D , K , and μ are functions of T and of the composition of the gas-vapor mixture. However, in practically all cases of interest to us the vapor density is much smaller than the gas density, even when the light gases such as hydrogen or helium are considered, because the latter can be used only at high pressures for downward diffusion. Thus D , K , and μ for pure gases have been used. From the tables^{10,11} one finds

$$D(T) = D_0/P(T/273)^\nu, \quad (30)$$

where P (atmos) is the total gas pressure, and ν an

¹² The Thomson formula (20) is derived with the same approximation. Experimental agreement with this formula is further evidence that (25) is sufficient even in a supersaturated gas.

empirical constant. Furthermore we find

$$K(T) = K_0[(273 + C')/(T + C')](T/273)^{3/2} \quad (31)$$

and

$$\mu(T) = \mu_0[(273 + C'')/(T + C'')](T/273)^{3/2} \quad (32)$$

where C' and C'' are constants characteristic of the gas.

Numerical calculations shall be carried out for a radiation producing n_0 ions per cm^3 per sec in air at N.T.P. $n_0 = 2$ is nearly the rate of ionization of the cosmic radiation at sea level. Then

$$n(\xi) = n_0 P(273/T(\xi))(\tau Z/2Z_{\text{air}}) \quad (33)$$

is the number of ions produced at the pressure P , temperature $T(\xi)$, in a gas of atomic number Z with τ atoms per molecule. Next $n'(\xi)$ can be assumed as proportional to the ratio of the cross-sectional area A of the chamber to the total surface $(2A + W)$ of the volume between x_0 and h , where W stands for the surface of the side walls. Hence

$$\int_{x_0}^h n'(\xi) d\xi = 273 \frac{A}{2A + W} n_0 P \left(\frac{\tau Z}{2Z_{\text{air}}} \right) \int_{x_0}^h d\xi/T(\xi). \quad (34)$$

Table II shows all the pertinent data for five important gases that might be used in diffusion chambers.

TABLE I. Data of interest for methyl alcohol.

M	ρ_d	B	E	σ_0	γ	L_0	δ
32	0.793	13.9	4700	24.5	3.6×10^{-3}	280	1.36×10^{-3}

For some gases no data could be found in the tables but values were obtained by interpolation making use of formulas derived by methods of statistical mechanics.

RESULTS

After substituting (28) and (29) into (14), the resulting transcendental equation can be solved numerically giving ΔT_a as a function of T . dT/dx can now be calculated from (18) by successive approximations as described above. Upon solving (14) for different cases one finds that ΔT_a for a given T is mostly proportional to ζ^{-1} , defined by (15), at temperatures below about 260°K, while at temperatures above 260°K ΔT_a becomes rather proportional to $\zeta^{-2/3}$. It follows that dT/dx as a function of x depends with good approximation on one single parameter β only. By substituting (28), (30), (31), (32), (33), and (34) into (18) we obtain for the parameter

$$\beta_a = \mu_0 P^{5/3} (n_0 \tau Z)^{4/3} D_0^{-1/3} \quad (35a)$$

if temperatures $< 260^\circ\text{K}$ are involved, while at temperatures $> 260^\circ\text{K}$

$$\beta_b = \mu_0 P^2 (n_0 \tau Z)^{4/3} D_0^{-2/3} K_0^{1/3} \quad (35b)$$

where only the small variation of ν in the temperature dependence of D for different gases has been neglected.

TABLE II. Constants for different gases.

	Z	τ	D_0	ν	$\frac{K_a}{10^{-5} \times}$	C'	$\frac{\mu_a}{10^{-4} \times}$	C''
Hydrogen	1	2	0.506	1.75	38.0	94	0.84	83
Deuterium	1	2	0.350	1.75	35.0	90	1.20	75
Helium	2	1	0.358	1.75	33.0	33	1.87	70
Air	7.2	2	0.133	2	5.3	125	1.71	120
Argon	18	1	0.096	2	3.8	210	2.09	142

Usually conditions are such that β_a must be used. Every solution of (18) for a given β represents solutions for a large variety of different cases. The relative pressures of different gases requiring identical temperature distributions are given in Table III.

Furthermore one sees that a change of the ionization density n_0 by a given amount has the same effect as if P had been changed by the same amount raised to the $\frac{4}{3}$ th power (35a) or to the $\frac{2}{3}$ rd power (35b), respectively, which is of interest because for an increase of n_0 a decrease of P may result in satisfactory operating conditions.

$T(x)$ has been calculated for several values of β . It is an important result that not only T_1 but also $(dT/dx)_{x=0}$ can be chosen because this explains why a large number of possibilities exist for successfully operating diffusion cloud chambers. Several curves for $T(x)$ have been plotted in Fig. 1 for all of which the values $x_0=10$ cm and $h=25$ cm have been chosen. The temperature gradients dT/dx are not sensitive to these assumptions. Therefore the curves shown are applicable rather generally. In every case considered there exists a satisfactory temperature distribution which is almost linear in the region $0 \leq x \leq x_0$. Experimentally, this condition is most easily obtained where the temperature distribution is to be controlled by supplying heat along the walls of the chamber. In the chambers discussed by Cowan² the temperature distribution is not controlled, and in some cases $T(x)$ is very nonlinear. In the region $x_0 \leq x \leq h$ $T(x)$ may be taken arbitrarily as a linear function. The temperature of the vapor supply, T_2 , has been found from (19) and (29).

It appears that for a given parameter β the temperature distributions tend to become parallel lines from $x \approx 2$ cm on up regardless of the choice of T_1 and $(dT/dx)_{x=0}$. Characteristic average values for the gradients in these parallel sections are

$$(dT/dx)_{av} = 3.6^\circ\text{K/cm for } \beta_a = 1.18 \times 10^{-2}$$

and

$$(dT/dx)_{av} = 6.3^\circ\text{K/cm for } \beta_a = 7.38 \times 10^{-2}.$$

TABLE III. Pressures of several gases, relative to air, requiring identical temperature distributions.

		H ₂	D ₂	He	A
P/P_{air}	from (35a)	9.7	7.7	5.6	0.70
	from (35b)	6.1	4.5	3.7	0.75

The values of β_a chosen correspond, for example, to operation with air at pressures of 1 atmos and 3 atmos, respectively, exposed to cosmic radiation at seal level.

The greater the value of β the closer the temperature distributions lie together for different given conditions at $x=0$, making proper experimental adjustment of $T(x)$ more critical.

LIMITING FACTORS

One sees from Fig. 1 that for $\beta_a = 7.38 \times 10^{-2}$ rather high top temperatures are required. At these temperatures two disturbing phenomena set in. In experiments at this laboratory⁴ it was found that raising the temperature T_2 at the top above a certain point may result in semi-stable convection in the track-sensitive layer. The light gases are particularly sensitive to this effect. One observes that the usually very stable track-sensitive layer breaks up into a large number of vertically circulating currents. Because of the resulting adjacent super- and undersaturated regions the layer assumes an apparently cellular structure. At pressures near one atmosphere with downward vapor diffusion stability cannot be obtained with the light gases simply because $d\rho_i/dx > 0$ at some places. Indications are that the present effect is rather connected with the large heat conductivity of these gases as will be discussed below. Whatever the cause, the limiting top temperature is usually below 290°K for the light gases. The second difficulty consists of the formation of uncharged drops which are formed much more easily at higher temperatures as the following consideration shows. Volmer *et al.*¹³ have derived that the number of uncharged liquid condensation nuclei $N(r)dr$ in the range dr near r present at every instant is

$$N(r)dr \sim (1/r^2 T) \exp[-(1/T)(E + 4\pi r^2 \sigma(T)/3k)] \quad (36)$$

where the proportionality constant must be determined experimentally. k is the Boltzmann constant. None of these nuclei can, of course, grow below supersaturation but in a supersaturated atmosphere all the nuclei above a certain radius will grow into drops. The amount of supersaturation necessary for condensation on ions can be found from (21). After substituting this supersaturation into (20), omitting the second term in the brackets which applies to ions only, one can solve (20) for the smallest radius of uncharged nuclei that may still grow. Integrating (36) between this radius and ∞ one estimates that for every increase of the temperature of 6°K the total number of uncharged drops doubles. By applying an electric sweeping field one can show that at temperatures between 273°K and 283°K at the vapor supply there is created of the order of 1 uncharged drop per cm³ per sec. Thus near room temperature at the vapor supply the amount of background fog may become serious. Most important, however, is the fact that

¹³ M. Volmer and A. Weber, *Z. physik. Chem.* **119**, 277 (1926); L. Farkas, *Z. physik. Chem.* **125**, 236 (1927).

vapor is used up in considerable amount to form these drops. Thus a temperature increase may now result only in an increase of the background fog and not in an increase of the depth of the track-sensitive layer.

For the two reasons explained, with dry ice temperatures at the bottom, best operation can be obtained when the top temperature is kept fairly low, say below 293°K. But then, as β is increased, the depth of the available track-sensitive layer may soon become too small for most purposes. For instance, since operation with 3 atmospheres of air requires temperatures above 293°K for best operation, a limit for reasonably good operation with air is to be expected somewhere between 2 and 3 atmospheres. From Table III one concludes that hydrogen still might be used at pressures of 20–30 atmospheres making diffusion chambers particularly well suited for studies of fundamental interactions with protons and also with deuterons and α -particles. We furthermore conclude from (35) that at 1 atm of air a continuous radiation amounting to perhaps 3 to 4 times the sea level cosmic radiation will result in difficulties of operation because again the temperature at the vapor supply cannot be raised sufficiently without producing uncharged droplets in great numbers or lack of stability.

If the proper temperature distribution is impressed upon the side walls of the chamber, stratification of the gas will take place due to gravitational forces. Even a slight difference in the gas temperatures at the center and near the walls will result in convection currents counteracting the cause of the temperature difference. A very convenient mechanism is thus provided to remove the heat due to condensation and conduction from the gas and to produce the proper temperature distribution throughout the chamber volume.

The amount of heat produced at the height x is given by (9). All terms in this equation have become known during the process of solving (18). Unless $H(x)$ turns out to be zero heat must be exchanged at the walls of the chamber by continuous convection of the gas. Where (9) indicates that heat is supplied by the gas ($H(x) < 0$) the latter will rise in the chamber, move outward toward the walls. The motion takes place in the opposite direction if $H(x) > 0$. The second term at the right side of (9) is negligible in all practical cases while the first and last terms are of equal order of magnitude. Due to their large heat conductivity the last term is usually most important for the light gases. One finds that usually $-3 \times 10^{-5} < H(x) < +3 \times 10^{-5}$ with an average of $H(x) \approx 10^{-5}$. We proceed to calculate the velocity of motion of the gas. The change in heat content per second of a thin layer of gas of volume $A dx$ moving with a vertical velocity V is given by

$$c_p \rho_t A dx (dT/dt) = H(x) A dx,$$

or with $V = dx/dt$

$$V = H(x)/c_p \rho_t dT/dx. \quad (37)$$

For $H \approx 10^{-5}$ cal, $dT/dx = 4$, and at a pressure of 1 atmosphere $V \approx 10^{-2}$ cm/sec which is almost independent of the gas used and decreases further with increasing pressure. This small amount of motion required explains the excellent stability of the gas in diffusion cloud chambers. However, this motion may have an important effect on the depth of the track-sensitive layer in the following manner. One can calculate that the average velocity of diffusion of the vapor through the gas $w = f/\rho$ is only several times as large as V . As far as vapor transport is concerned w and V must be superimposed. Further estimates show that as a result some contraction and vertical shifting of the track-sensitive layer takes place, and also some additional background fog is produced.

The velocity V' at the walls can also be found approximately. For continuity one must have

$$AV = -by_0V' \quad (38)$$

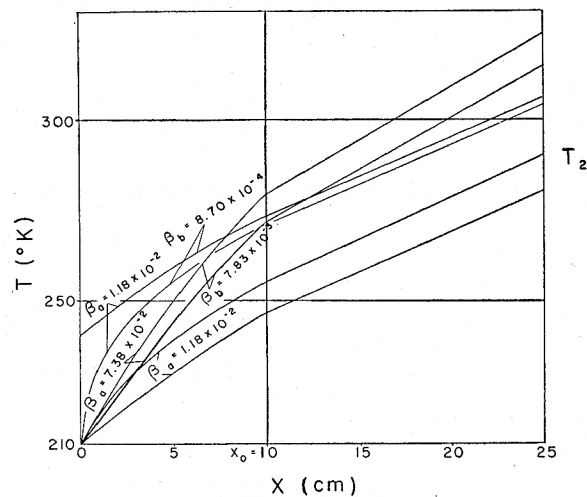


FIG. 1. Calculated temperature distributions for different parameters, and for different temperatures and temperature gradients at the bottom of a diffusion chamber.

where b is the circumference of the chamber and y_0 the thickness, measured from the wall, of a layer of gas at an average velocity V' . To estimate y_0 we set up the heat conduction equation

$$\partial T'/\partial t = V' \partial T'/\partial x = (K/c_p \rho_t) \partial^2 T'/\partial y^2, \quad (39)$$

where y is measured inward from the walls and $T'(x, y)$ is the temperature near the walls. Furthermore, the equation of motion of the gas in equilibrium is given by

$$-(\rho_{1t} - \rho_t)g + \mu \partial^2 V'/\partial y^2 = 0, \quad (40)$$

where ρ_{1t} is the gas density near the walls.

Applying (25) one obtains

$$1 - T'/T = (\mu/g\rho_{1t}) \partial^2 V'/\partial y^2. \quad (41)$$

Differentiating (41) twice and substituting into (39)

one has

$$V' = -\frac{K\mu T}{c_p g \rho_{1t}^2 d T/dx} \frac{\partial^4 V'}{\partial y^4} = -\epsilon \frac{\partial^4 V'}{\partial y^4}, \quad (42)$$

since $\partial T'/\partial x \approx dT/dx$ and $\rho_{1t} \approx \rho_{1t}$. The interesting part of a particular solution of (42) is $V' \sim \exp(-y\epsilon^{-1/4})$. A general solution is composed of solutions of this type and therefore V' becomes relatively small at $y = \epsilon^{1/4}$. We thus can set $y_0 \approx \epsilon^{1/4}$. In all cases one obtains values of the order of 0.2 cm for y_0 . Thus from (38) $V' \approx 1$ cm/sec for a chamber of 30 cm diameter.

The critical velocity of a gas moving through a tube of diameter d is given by the expression $1000 \mu/\rho_{1t}d$ which is equal to a few cm/sec in our case. Since at velocities of this order of magnitude straight line motion breaks down into turbulent motion, and since V' is just of such a magnitude one may well conceive that under certain conditions the above described semi-stable mode of turbulent motion of the gas is possible.

The effect of radiation from the top plate through the gas to the bottom plate should finally be mentioned. For the amount of radiation from a black top plate at $T_2 = 300^\circ\text{K}$ to a black bottom plate at 200°K one has $5.73 \times 10^{-12} (300^4 - 200^4) = 3.4 \times 10^{-2}$ watt/cm² which is of the same order as the amount of heat necessary for evaporation of the liquid at the top. By applying Planck's radiation formula one finds that the maximum of the radiation spectrum occurs near a wavelength of 10^5A . There are very few absorption lines for gases in this region and a few more for vapors, and the total radiation energy is spread out over such a wide range of wavelengths that the amount of energy available for excitation of an absorption line is completely negligible. Therefore, the effect of radiation on the gas-vapor mixture can be neglected.

All of the theoretical results stated have been verified experimentally as will be described in a later paper.

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