# Physics 6A Winter 2006 FINAL

- The test has 16 multiple choice questions and 3 problems.
- Scoring:

| Question 1-16 | 55 points total |
|:---:|:---:|
| Problem 1 | 20 points |
| Problem 2 | 15 points |
| Problem 3 | 10 points |

- Enter the solution to the problems in the blue book.
- Enter the answer to the multiple choice questions in the pink scantron sheet. Use a pencil, not a pen.
- No penalty for wrong answers to multiple choice questions.
- Write your name and perm number on **both** the scantron sheet and the blue book.
- The scantron sheet has an entry box labeled "TEST FORM". There are 4 slightly different sets of multiple choice questions, different students get questions in different orders. Make sure to enter the appropriate "TEST FORM" (A, B, C, or D) on your scantron sheet.
- Moments of inertia for some solid bodies are tabulated on the next page.

# YOUR "TEST FORM" IS A

# Moments of inertia of solid bodies

You may find some of these useful.  M is the mass of the object.

| Shape | Axis of rotation | Moment of inertia |
|---|---|---|
| Solid cylinder or disk, radius R | Axis of cylinder | $\frac{1}{2}MR^2$ |
| Thin hollow cylinder shell, radius R | Axis of cylinder | $MR^2$ |
| Rectangular plate, sides a & b | Perpendicular to plate through center | $1/12\ M(a^2+b^2)$ |
| Solid sphere, radius R | Axis through center | $2/5\ MR^2$ |
| Thin hollow spherical shell, radius R | Axis through center | $2/3\ MR^2$ |
| Thin uniform rod, length L | Perpendicular to rod, passing through one of the ends | $1/3\ ML^2$ |
| Thin uniform rod, length L | Perpendicular to the rod, passing through the center | $1/12\ ML^2$ |

## Question 1

The vector $\vec{A}$ has a magnitude of 25 m and makes an angle of 30° with the positive x-axis. Then, the vector $-\vec{A}$

- A:  has a magnitude of 25 m
- B:  has the same direction as $\vec{A}$ but a magnitude of –25 m
- C:  makes an angle of -30° with the positive x axis
- D:  has the opposite direction as $\vec{A}$ and a magnitude of -25 m
- E:  both A and C

**Correct answer is A.**
Answer B is wrong because the magnitude of a vector is always positive. Also, the negative of a vector points in the opposite direction, not the same direction.
Answer C is wrong because in this case the x-component would be unchanged.
Answer D is wrong because the magnitude of a vector is always positive.
Answer E is wrong because answer C is wrong.

## Question 2

A hockey puck of mass 0.16 kg is slapped so that its velocity is 50 m/sec. It slides 40 meters across the ice before coming to rest. How much work is done by friction on the puck?

- A:  4 J
- B:  -60 J
- C:  -200 J
- D:  -340 J

$W = \Delta K = -\frac{1}{2} m v^2 = -\frac{1}{2} \, 0.16 \, 50^2 \text{ J} = -200 \text{ J}$ →**Correct answer is C**

## Question 3

A car moving at 80 mph takes 400 ft to stop with uniform acceleration after its brakes are applied. How far will it take it to stop under the same conditions (i.e., with the same acceleration) if its initial velocity is 40 mph?

- A:  20 ft
- B:  50 ft
- C:  100 ft
- D:  200 ft
- E:  150 ft

Motion with constant acceleration: $v^2 = v_0^2 + 2a(x\text{-}x_0)$ → stopping distance $d = v_0^2/2a$.
If $v_0$ is halved (from 80 mph to 40 mph), then d changes by a factor of ¼ .
But ¼ 400 ft = 100 ft → **Correct answer is C**

## Question 4

A ball thrown horizontally from the top of a building hits level ground in 0.50 seconds. If it had been thrown with twice the speed in the same direction, it would have hit the ground in

  A:   4.0 sec
  B:   1.0 sec
  C:   0.50 sec
  D:   0.25 sec
  E:   0.125 sec

Motions in the horizontal and vertical directions decouple. The vertical velocity is the same (=0) in both cases. Therefore the time taken to travel the same vertical distance and hit the ground is the same in both cases. → **Correct answer is C**

## Question 5

You shoot a bullet at an apple high up in a tree some distance in front of you, hoping to knock it loose so you can catch it and eat it. Ignoring air resistance, in order to hit this apple you must aim

  A:   directly at it
  B:   above it
  C:   below it

You must aim above hit because gravity will cause the bullet to acquire a downward component of velocity during its flight. To compensate for it, aim high! → **Correct answer is B**

## Question 6

A frictional force is

  A:   a force that acts parallel to two surfaces in contact
  B:   a force that acts perpendicular to two surfaces in contact
  C:   a scalar quantity since it can act in any direction along a surface
  D:   always proportional to the weight of an object
  E:   always equal to the normal force between two objects

**Correct answer is A**

**Question 7**
Which car has a westward acceleration?

    A:        a car traveling westward at constant speed
    B:        a car traveling eastward and speeding up
    C:        a car traveling westward and slowing down
    D:        a car traveling eastward and slowing down
    E:        a car starting from rest and moving towards the east

A:  constant speed → a=0
B:  acceleration is eastward
C:  acceleration is eastward (equivalent to deceleration westward)
D:  acceleration is westward (equivalent to deceleration eastward)
E:  acceleration is eastward
    → **Correct answer is D**

**Question 8**
A diver in midair pulls her legs inwards towards her chest to rotate faster. Doing so changes which of these quantities: her angular momentum L, her moment of inertia I, and her <u>rotational</u> kinetic energy $K_{rot}$:

    A:        L only
    B:        I only
    C:        $K_{rot}$ only
    D:        L and I only
    E:        $K_{rot}$ and I only

The moment of inertia changes because the distribution of mass around the axis changes. The angular momentum is constant because there is no external torque. Thus

$L = I_{old}\omega_{old} = I_{new}\omega_{new}$ → $\omega_{new} = (I_{old}/I_{new})\,\omega_{old}$

$K_{old} = \frac{1}{2}\,I_{old}\,\omega^2_{old}$

$K_{new} = \frac{1}{2}\,I_{new}\,\omega^2_{new} = \frac{1}{2}\,I_{new}\,(I_{old}/I_{new})^2\,\omega^2_{old} = \frac{1}{2}\,(I^2_{old}/I_{new})\,\omega^2_{old} \neq K_{old}$

Therefore K and I change while L stays the same → **Correct answer is E**

## Question 9

A uniform bar of mass m is supported by a pivot at its top, about which the bar can swing like a pendulum. If a force F is applied perpendicularly to the lower end of the bar as in the figure, how big must F be in order to hold the bar in equilibrium at an angle $\theta$ from the vertical?

 A:   $(mg/2) \sin\theta$
 B:   $(mg/2) \cos\theta$
 C:   $2mg \sin\theta$
 D:   $2mg \cos\theta$
 E:   $mg \sin\theta$
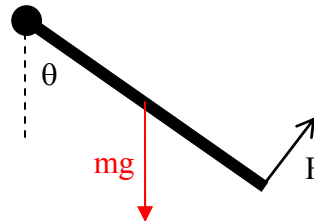
The weight force is applied at the center of mass, which is at the midpoint of the bar. Let L be the length of the bar. The torque due to F is +FL; the torque due to the weight is $-mg(L/2)\sin\theta$. The sum of the torques must be zero. Thus:
$+FL -mg(L/2)\sin\theta = 0$
$F = (mg/2) \sin\theta$ → **Correct answer is A**

## Question 10

You are standing on a bathroom scale in an elevator. In which of these situations does the scale read the same as when the elevator is at rest?

 A:   Moving up at constant speed
 B:   Moving up with increasing speed
 C:   In free fall, i.e., after the elevator cable has snapped
 D:   None of the above

In cases B and C your body is accelerating. Therefore the normal force that the scale exerts on you is not equal (in magnitude) to your weight. On the other hand, in case A the acceleration is zero, so the normal force is equal (in magnitude) to the weight.
→ **Correct answer is A**

## Question 11

An object moving in a circle at constant speed is

 A:   Accelerating in the direction of motion
 B:   Accelerating toward the center of the circle
 C:   Accelerating away from the center of the circle
 D:   Not accelerating because the speed is constant

Uniform circular motion: acceleration is towards the center → **Correct answer is B**

## Question 12
Two blocks are connected by a light string passing over a <u>massless</u> pulley (see figure). The block of mass $m_1$ slides on a frictionless horizontal surface, while the block of mass $m_2$ hangs vertically ($m_1>m_2$). The tension in the string is
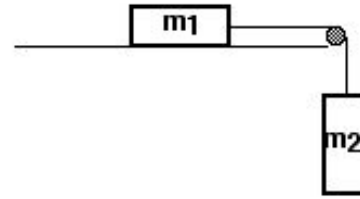


A:       zero
B:       $< m_2g$
C:       $= m_2g$
D:       $> m_2g$, but $< m_1g$
E:       $> m_1g$

Net force on the two blocks is $F = m_2g$. But $F=(m_1+m_2)a \rightarrow a = m_2g/(m_1+m_2)$.
Consider the mass $m_1$: it has the acceleration given above, and the net force on it is the tension of the string. Thus $T = m_1a = m_1m_2g/(m_1+m_2)$.
Then, since $m_1/(m_1+m_2) < 1$, it follows that $T< m_2g$ → **Correct answer is B**
(We could have gotten to this answer without doing any work. Clearly, the tension is not zero, but also the tension cannot be $> m_2g$ otherwise mass $m_2$ would be moving up instead of down)

## Question 13
Two small objects of equal weight are attached to the ends of a thin weightless bar that spins about an axis perpendicular to the bar at its center. Each mass is at a distance d from the axis of rotation. The kinetic energy of rotation is K.
If both masses are now moved closer in so that each is a distance d/2 from the axis of rotation, while the angular velocity does not change, the kinetic energy of rotation becomes

A:       ¼ K
B:       ½ K
C:       2 K
D:       4 K

$$K = \tfrac{1}{2} I \,\omega^2.$$
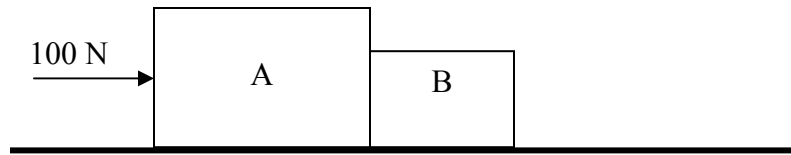$$I_{old} = md^2 + md^2 = 2md^2$$
$$I_{new} = m(\tfrac{1}{2}d)^2 + m(\tfrac{1}{2}d)^2 = \tfrac{1}{2}\, md^2 = \tfrac{1}{4}\, I_{old}.$$
Thus, if $\omega$ stays the same, $K_{new} = \tfrac{1}{4}\, K_{old}$ → **Correct answer is A**

## Question 14
A person pushes two boxes with a horizontal force of 100 N on frictionless floor:



Box A is heavier than box B.  Which of the following statements is correct?
  A:  Box A pushes on box B with a force of 100 N, and box B pushes on box A with a force of 100 N
  B:  Box A pushes on box B harder than box B pushes on box A
  C:  Boxes A and B push on each other with equal forces less than 100 N
  D:  The two boxes will not begin to move unless the total weight of the two boxes is less than 100 N
  E:  None of the above

$F = 100$ N.  Acceleration $a = F/(m_A+m_B)$.
Force between the boxes is the only force on B.  Call it $F_B$.
Newton's $2^{nd}$ law: $F_B = m_B\, a = F\, m_B\,/(m_A+m_B) < F$
→ **Correct answer is C**


## Question 15
A 100 N weight is supported by two weightless wires as shown in the figure.  What can you conclude about the tensions in these wires
  A:  The tensions are equal to 50 N each
  B:  The tensions are equal and < 50 N each
  C:  The tensions are equal and > 50 N each
  D:  The tensions are not equal
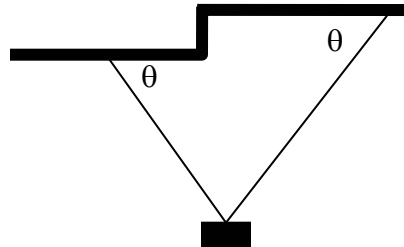


By symmetry, the tensions are obviously equal.  The two vertical components must add up to 100 N, so they are 50 N each.  But the two tensions also have non-zero horizontal components.  Therefore their magnitudes must be > 50 N (which is the value of the vertical components alone).  → **Correct answer is C**

**Question 16**

A uniform cylinder is released from rest from the top of an incline, and rolls down without slipping. At the bottom of the incline the speed of the cylinder is V.
Next, a hole is drilled through the cylinder along its axis, and the experiment is repeated. This time the speed at the bottom of the incline is measured to be U. How does U compare with V?

    A: $U = V$
    B: $U > V$
    C: $U < V$
    D: Depends on the radius of the hole that was drilled.

$M$ = mass of cylinder before hole is drilled
$M'$ = mass of cylinder after hole is drilled
$I$ = moment of inertia before
$I'$ = moment of inertia after
$R$ = radius of cylinder
$r$ = radius of hole
$\omega$ = angular velocity at the bottom, no hole = $VR$
$\omega'$ = angular velocity at the bottom, with hole = $UR$
$h$ = height of incline
Conservation of energy
$Mgh = \frac{1}{2} MV^2 + \frac{1}{2} I\omega^2 = \frac{1}{2} MV^2 + \frac{1}{2} IV^2R^2 \rightarrow V^2 = 2gh/(1+IR^2/M)$
Similarly, $U^2 = 2gh/(1+I'R^2/M')$
The question now is whether $I/M$ is larger or smaller than $I'/M'$.
Obviously, $I$ and $I'$ must be proportional to $M$ and $M'$ respectively.
Let's write them as $I=aM$ and $I'=a'M'$
In the case of the hole, the mass is more concentrated away from the axis of rotation, thus
$a' > a \rightarrow I'/M' > I/M \rightarrow U < V \rightarrow$ **Correct answer is C**

Also, more explicitly, $I = \frac{1}{2} MR^2$ from the tables of moments of inertia $\rightarrow$ **$a = \frac{1}{2} R^2$**
The moment of inertia of a cylinder with a hole is equal to the moment of inertia of the cylinder with the hole minus the moment of inertia of a 2nd cylinder, of equal material, but with radius equal to the radius of the hole. Let m be the mass of the 2nd cylinder.
$I' = \frac{1}{2} MR^2 - \frac{1}{2} mr^2$
Let L be the length of the cylinder and $\rho$ the density. Then $M=\pi\rho LR^2$ and $m=\pi\rho LRr^2$
$I' = \frac{1}{2} \pi\rho LR^4 - \frac{1}{2} \pi\rho LRr^4 = \frac{1}{2}\pi\rho L(R^4 - r^4) = \frac{1}{2}\pi\rho L(R^2 - r^2) (R^2 + r^2)$
But $\pi\rho LR^2=M$ and $\pi\rho LRr^2=m$, so I can write
$I' = \frac{1}{2} (M - m) (R^2 + r^2)$     but $M' = M - m$
$I' = \frac{1}{2} M' (R^2 + r^2)$
And since we had said that $I'=a'M' \rightarrow$ **$a' = \frac{1}{2} (R^2 + r^2)$**
Thus, $a'>a$, just as we had argued above!

## Problem 1

A skeet (clay target) of mass M=0.25 Kg is fired at an angle $\theta=30°$ to the horizontal with a speed v=30 m/sec. When it reaches its maximum height h, it is hit from below by a pellet of mass m=15 g traveling vertically upward with a speed u=200 m/sec. The pellet is embedded in the skeet. The figure below shows the trajectory of the skeet if the pellet had not hit it (solid line), and the trajectory of the skeet hit by the pellet (solid line until the pellet hits the skeet, then dashed line after the pellet imbeds itself in the skeet).
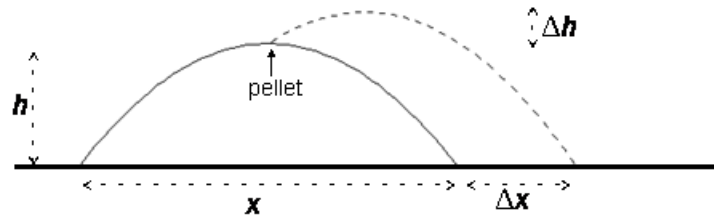


(a) Find the height h where the pellet hits the skeet.
(b) Find the distance x that the skeet would have traveled if the pellet had not hit it.
(c) Find the additional height $\Delta h$ that the skeet reaches because it is hit by the pellet.
(d) Find the additional horizontal distance $\Delta x$ traveled by the skeet.

(a) Conservation of energy: $\frac{1}{2} Mv^2 = Mgh + \frac{1}{2} M(v\cos\theta)^2$. (Remember that the horizontal velocity $v\cos\theta$ does not change.
$h = (1/2g) \, v^2(1-\cos^2\theta) \rightarrow \underline{\mathbf{h = \frac{1}{2} \, v^2 sin^2\theta/g}}$
$h = \frac{1}{2} \, 30^2 \, (\frac{1}{2})^2/9.8$ m  $\underline{\mathbf{h = 11.5 \, m}}$

(b) Time taken to reach the top is t.  $0 = v \sin\theta - gt \rightarrow t = v \sin\theta/g$
Total flight time is 2t. The horizontal velocity is a constant $v\cos\theta$
Thus, $x = v\cos\theta \, (2t) = 2v^2\sin\theta\cos\theta/g \rightarrow \underline{\mathbf{x = \, v^2 sin2\theta/g}}$
$x = 30^2 \, \sin60/9.8$ m  $\underline{\mathbf{x = 79.5 \, m}}$

(c) Conservation of momentum in collision. Take x-axis horizontal, y-axis vertical.
Let w be the velocity of the skeet pellet system after the collision
BEFORE: $P_x=Mv\cos\theta$  and   $P_y=mu$
AFTER: $P_x=(M+m)w_x$  and   $P_y=(M+m)w_y$
Thus:   $w_x = Mv\cos\theta/(M+m)$  and   $w_y=mu/(M+m)$.

Then conservation of energy for trajectory after the collision gives
$\frac{1}{2} (M+m) \, w^2 + (M+m)gh = \frac{1}{2} (M+m) \, w_x^2 + (M+m)g(h+\Delta h)$
$\frac{1}{2} w_x^2 + \frac{1}{2} w_y^2 = \frac{1}{2} w_x^2 + g\Delta h$
$\frac{1}{2} w_y^2 = g\Delta h$
$\frac{1}{2} m^2u^2/(M+m)^2 = g\Delta h$
$\underline{\mathbf{\Delta h = \frac{1}{2} \, m^2u^2/[(M+m)^2 \, g]}}$
$\Delta h = \frac{1}{2} \, 0.015^2 \, 200^2/(0.265^2 \, 9.8)$ m = $\underline{\mathbf{6.5 \, m}}$

(d) Take the origin of the coordinate system where the skeet was launched.
When the skeet-pellet combination hits the ground, $x=x+\Delta x$ and $y=0$.

Time t=0 is the time when the pellet hits the skeet.  Time t=t is the time when the pellet-skeet combination hits the ground.
Equation of y-motion, starting from where the pellet hits the skeet
$0 = h + w_y t - \frac{1}{2} gt^2$
This can be solved to give $t = (w_y + SQRT(w^2_y + 2gh))/g$.
$w_y = mu/(M+m) = 0.015 \; 200 / 0.265$ m/sec $= 11.32$ m/sec
$h = 11.5$ m
then $t = (11.32 + SQRT(11.32^2 + 2 \; 9.8 \; 11.5)/9.8$ sec $= 3.07$ sec

Equation of x-motion, starting from where the pellet hits the skeet
$x + \Delta x = \frac{1}{2} x + w_x t$
$\Delta x = w_x t - \frac{1}{2} x$
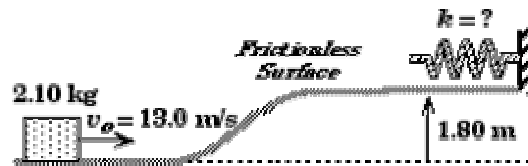But $w_x = Mv\cos\theta/(M+m) = 0.25 \; 30 \cos30 / 0.265$ m/sec $= 24.51$ m/sec
$\Delta x = 24.51 \; 3.07$ m $- \frac{1}{2} \; 79.5$ m $= \underline{\textbf{35.5 m}}$

## Problem 2
A 2.10 kg block slides along a frictionless surface at 13.0 m/s. The block goes up a 1.80 m high ramp, collides with a spring, and compresses the spring 11.0 cm before coming to a halt.



(a) What is the mechanical energy of the block when it is half way up the ramp?
   (take the block to have 0 gravitational potential energy at the bottom of the ramp)
(b) What impulse does the block impart to the spring in compressing the spring 11.0 cm?
(c) What is the value of the spring constant k?

(a) By conservation of energy, same as what it was at the bottom, i.e. $\underline{\textbf{E} = \frac{1}{2}\textbf{Mv}_0^2}$
   $E = \frac{1}{2} \; 2.10 \; 13^2$ J $= 177$ J

(b) Let v = velocity at the top.  Impulse $= I = mv$.
Conservation of energy $\frac{1}{2} Mv_0^2 = \frac{1}{2} Mv^2 + Mgh \rightarrow v^2 = v_0^2 - 2gh$
$\underline{\textbf{I} = \textbf{m SQRT(v}_0^2 - \textbf{2gh})}$
$I = 2.1 \; SQRT(13^2 - 2 \; 9.8 \; 1.8)$ N sec $= \underline{\textbf{24.3 N sec}}$

(c) Conservation of energy while compressing the spring
$\frac{1}{2} Mv^2 = \frac{1}{2} k \Delta x^2$
$k = Mv^2/\Delta x^2 \qquad \rightarrow \underline{\textbf{k} = \textbf{M (v}_0^2 - \textbf{2gh)}/\Delta \textbf{x}^2}$
$k = 2.1 \; (13^2 - 2 \; 9.8 \; 1.8)/0.11^2 = \underline{\textbf{23,200 N/m} = \textbf{232 N/cm}}$

**Problem 3**

A 2.0 kg block is tied to a string that is wrapped around the rim of a pulley of radius 10 cm. The pulley is attached to the edge of a table and can rotate freely around its axis. The block is released from rest and falls vertically towards the floor.

(a) Assuming that the pulley is a uniform disk with a mass of 0.3 Kg, find the speed of the block after it has fallen through a height of 0.5 m.

(b) If a small lead weight is attached near the rim of the pulley and this experiment is repeated, will the speed of the block increase, decrease, or stay the same? (Correct answer with no explanation or correct answer with wrong explanation gets no credit).

(a) M=mass of block and m=mass of pulley. I = moment of inertia.

Conservation of energy: $Mgh = \frac{1}{2}Mv^2 + \frac{1}{2}I\omega^2$

But $I = \frac{1}{2}mR^2$ and $\omega = v/R$, therefore

$Mgh = \frac{1}{2}Mv^2 + \frac{1}{4}mv^2$.

$\underline{\mathbf{v^2 = 2gh/(1 + \frac{1}{2}\ m/M)}}$

$v = SQRT(2 \cdot 9.8 \cdot 0.5/(1 + 0.3/2)) = \underline{\mathbf{2.9\ m/sec}}$

(b) Decrease, because I increases, and so more of the energy is taken up in the rotation of the disk.