WINTER 2005 - PHYSICS 6A FINAL

(1) Conservation of momentum $\quad M_1 V_1 = M_2 V_2$

$\boxed{C}$

$$V_1 = \frac{M_2 V_2}{M_1} = \frac{1.8 \cdot 2}{1.2} = 3.0 \text{ m/sec}$$

(2) Let t = time taken to reach the top of the trajectory. $t = 6.3$ sec

one half of 12.6 sec

$$\underset{=0}{V} = V_0 - gt \Rightarrow \cancel{t = V_0/g} \quad V_0 = gt = 9.8 \cdot 6.3 \text{ m/sec}$$

$$V_0 = 61.7 \text{ m/sec}$$

$\boxed{C}$

(3) Change of momentum is the same, so impulse is the same.

Air mattress takes longer time to bring the high jumper to rest $\Rightarrow$ smaller net force on average

$\boxed{C}$

(4) $\omega = \omega_0 + \alpha t \qquad \alpha = \frac{\omega - \omega_0}{t} = \frac{9-5}{3} \frac{\text{rad}}{s^2} = \frac{4}{3} \frac{\text{rad}}{s^2}$

$$\theta = \underset{=0}{\theta_0} + \omega_0 t + \frac{1}{2} \alpha t^2$$

$\boxed{B}$

$$\theta = 5 \cdot 3 \text{ rad} + \frac{1}{2} \frac{4}{3} 9 \text{ rad} = 21 \text{ rad}$$

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$V = \omega r \qquad V_1 = \omega r_1 \Rightarrow \omega = V_1/r_1$

$V_2 = \omega r_2 \Rightarrow V_2 = V_1 \frac{r_2}{r_1} = 2.2 \frac{2.1}{1.4} \, m/sec = 3.3 \, m/sec$

$\boxed{B}$

6 Take +ve sign as counterclockwise

$\boxed{B}$

$\tau_1 = -20 \cdot 0.5 \, Nm = -10 \, Nm$

$\tau_2 = +35 \cdot 1.10 \cdot \sin 60 = +33.3 \, Nm$

$\Rightarrow \tau = \tau_1 + \tau_2 = +23.3 \, Nm$

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$I_1 \omega_1 = I_2 \omega_2 \qquad I_2 < I_1$

$\omega_2 = \frac{I_1}{I_2} \omega_1$

$E_1 = \frac{1}{2} I_1 \omega_1^2 \qquad E_2 = \frac{1}{2} I_2 \omega_2^2 = \frac{1}{2} I_2 \frac{I_1^2}{I_2^2} \omega_1^2 = \frac{1}{2} \left(\frac{I_1}{I_2}\right) I_1 \omega_1^2$

$\boxed{B}$

$E_2 = \left(\frac{I_1}{I_2}\right) E_1$

$I_1 > I_2 \Rightarrow E_2 > E_1$

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$I_A = 10 \, m_0 r_0^2$

$I_B = 2 m_0 (2 r_0)^2 = 8 \, m_0 r_0^2$

$I_C = m_0 (3 r_0)^2 = 9 \, m_0 r_0^2$

$\boxed{B}$

(9) Minimum speed at the top when vertical velocity = 0
Horizontal velocity is constant, = 30 m/sec

$\boxed{C}$

(10) $f_s = mg\sin 22 = 8 \cdot 9.8 \cdot \sin 22 \text{ N} = 29 \text{ N}$

$\boxed{D}$

(11) Conservation of momentum

$\boxed{C}$ $M_1 V_1 = (M_1 + M_2) V_2 \qquad V_2 = \frac{M_1}{M_1 + M_2} V_1 = \frac{12{,}000}{18{,}000} 10 \text{ m/s} = 6.7 \text{ m/sec}$

(12) FBD:

$\boxed{E}$ A: $\uparrow T$, $\downarrow mg$ $\Rightarrow$ $T = mg$

B: $T\uparrow$ $2T\uparrow$, $\downarrow mg$ $\Rightarrow$ $T = \frac{1}{3} mg$

C: $T\uparrow$ $\uparrow T$, $\downarrow mg$ $\Rightarrow$ $T = \frac{1}{2} mg$ $\Rightarrow$ B, C, A

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(a) Constant speed $\Rightarrow$ no net force on B $\Rightarrow$ $\boxed{T = W_B}$

(b) Normal force $N = W_A$

Force of kinetic friction $f_K = \mu_K N = \mu_K W_A$

This force must equal the tension if no acceleration on A

Thus

$$f_K = T$$

$$\mu_K W_A = W_B \qquad \boxed{\mu_K = \frac{W_B}{W_A}}$$

(c) Kinetical frictional force $f_K = 2\mu_K W_A = 2W_B$

$\Rightarrow$ $\boxed{\text{Acceleration} = g \text{ UPWARDS}}$

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(a) Work energy theorem

$$W = \Delta K = \frac{1}{2} m V_2^2 - \frac{1}{2} m V_1^2 = \frac{1}{2} m (V_2^2 - V_1^2)$$

$$W = \frac{1}{2} 80 \; (1.5^2 - 5^2) \text{ J} \qquad \boxed{W = -910 \text{ J}}$$

(b) With no pedalling, conservation of energy says that we should have a velocity V_3 such that

$$\frac{1}{2} m V_3^2 + mgh = \frac{1}{2} m V_1^2 \qquad (V_1 = 5 \text{ m/sec})$$

$$\frac{1}{2} m V_3^2 = \frac{1}{2} m v_1^2 - mgh$$

Instead, we have a velocity $V_2 = 1.5$ m/sec

The difference of kinetic energy (no pedelling vs pedalling) is the work done by cyclist

$$W = \frac{1}{2} m V_2^2 - \frac{1}{2} m V_3^2$$

$$W = \underbrace{\frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2}_{\Delta K = -910 \text{ J}} + mgh$$

$W = \Delta K + mgh \qquad W = -910 \text{ J} + 80 \cdot 9.8 \, 5.2 \text{ J} \quad \boxed{W = 3167 \text{ J}}$

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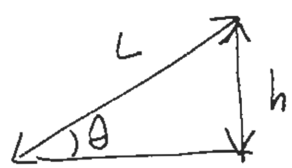
(a) Conseroation of energy

$$\frac{1}{2}kx^2 = \frac{1}{2}mv^2 \qquad \boxed{V = \sqrt{\frac{k}{m}}\, x}$$

$$V = \sqrt{\frac{400}{2}}\,(0.22)^2 \ \text{m/sec} \qquad \boxed{V = 0.68 \text{ m/sec}}$$

(b)



$$h = L\sin\theta$$

$$\rightarrow L = \frac{h}{\sin\theta}$$

Conseroation of energy

$$\frac{1}{2}kx^2 = mgh \qquad \text{or} \qquad \frac{1}{2}mv^2 = mgh$$

$$h = \frac{kx^2}{2mg} \qquad \text{or} \qquad h = \frac{mv^2}{2mg} = \frac{v^2}{g}$$

$$\boxed{L = \frac{kx^2}{2mg\sin\theta}} \qquad \text{or} \qquad \boxed{L = \frac{v^2}{g\sin\theta}}$$

SAME BECAUSE $v^2 = \frac{k}{m}x^2$

$$L = \frac{400\,(0.22)^2}{2\cdot 2\cdot 9.8\,\sin 37}\ \text{m} \qquad \boxed{L = 82\text{ cm}}$$

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(a) Conservation of energy

$$(2m)\,g\,h = \frac{1}{2}(2m)V_1^2 \qquad \boxed{V_1 = \sqrt{2gh}}$$

(b) Before collision After collision

Conservation of energy and momentum

$$\begin{cases} 2mu_1 + mu_2 = 2mV_1 \\ \frac{1}{2}(2m)u_1^2 + \frac{1}{2}mu_2^2 = \frac{1}{2}(2m)V_1^2 \end{cases} \qquad \begin{cases} 2u_1 + u_2 = 2V_1 & (1) \\ 2u_1^2 + u_2^2 = 2V_1^2 & (2) \end{cases}$$

From eqtn (1) : $u_2 = 2V_1 - 2u_1 = 2(V_1 - u_1)$ (3)

Plug (3) into (2) :

$$2u_1^2 + 4(V_1 - u_1)^2 = 2V_1^2$$

$$2u_1^2 + 4V_1^2 - 8V_1u_1 + 4u_1^2 = 2V_1^2$$

$$6u_1^2 - 8V_1u_1 + 2V_1^2 = 0$$

$$3u_1^2 - 4V_1u_1 + V_1^2 = 0$$

$$u_1 = \frac{4V_1 \pm \sqrt{16V_1^2 - 12V_1^2}}{6} = \frac{4V_1 \pm 2V_1}{6}$$

$= V_1$ ← This corresponds to no collision

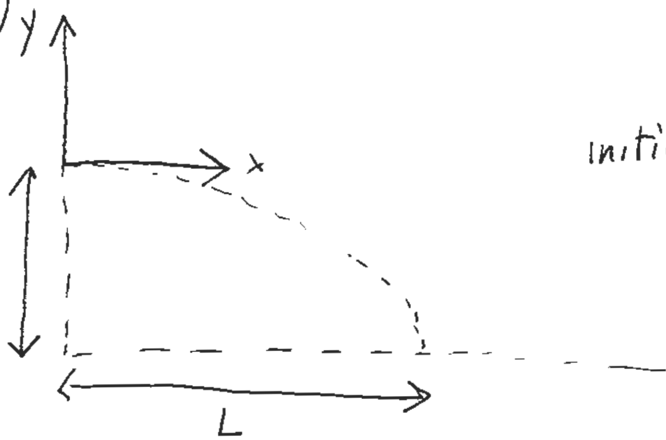
$= \frac{1}{3}V_1$ ← OK

$\Rightarrow$ $\boxed{u_1 = \frac{1}{3}V_1}$ Then using eqtn (3), $\boxed{u_2 = \frac{4}{3}V_1}$

Or, in terms of m, g, h

$$\boxed{u_1 = \frac{\sqrt{2gh}}{3}} \qquad \boxed{u_2 = \frac{4\sqrt{2gh}}{3}}$$

(c)



initial velocity $u_x = u$.
$u_y = 0$

~~Time~~ t = time taken to hit floor

$$y = -\tfrac{1}{2} g t^2$$

~~Also~~ $-H = -\frac{1}{2} g t^2 \implies t = \sqrt{\frac{2H}{g}}$

Distance travelled horizontally in $L = ut = u\sqrt{\frac{2H}{g}}$

$\implies$ For first marble

$$L_1 = u_1 t = \frac{\sqrt{2gh}}{3}\sqrt{\frac{2H}{g}} \qquad \boxed{L_1 = \frac{2}{3}\sqrt{hH}}$$

For second marble

$$L_2 = u_2 t = \frac{4\sqrt{2gh}}{3}\sqrt{\frac{2H}{g}} \qquad \boxed{L_2 = \frac{8}{3}\sqrt{hH}}$$