Fall 2004 Physics 3
Tu-Th Section

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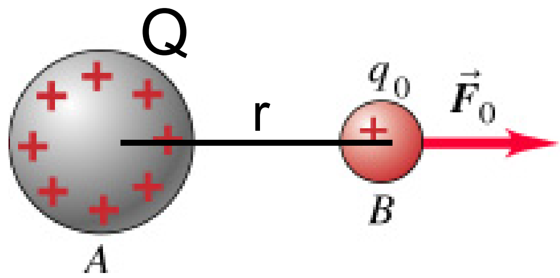
Lecture 7: 14 Oct. 2004

Web page:
http://hep.ucsb.edu/people/claudio/ph3-04/

Last time: Electric Field

- Different way of thinking about electric forces
- The presence of a charge Q modifies the properties of space → causes electric field
- Then another charge q_0 placed in space will feel a force $\vec{F}_0 = q_0 \vec{E}$
- Electric field $(\vec{E})$ is a vector
- Units of [F]/[charge] = N/C
- Electric fields from ≥ 1 charge add (vectorially)

Electric Field, last time (continued)



$$F_0 = k\frac{|Qq_0|}{r^2}$$

$$\vec{F}_0 = q_0\vec{E}$$

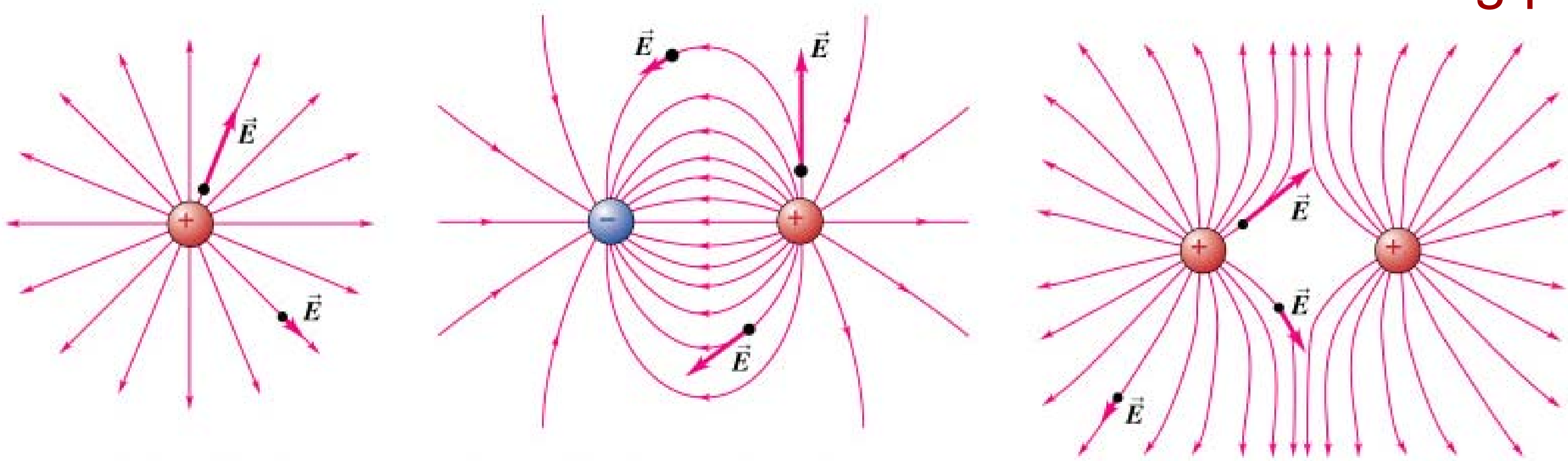
Definition of electric field due to charge Q at the point where charge q_0 is placed.

⇒ $$E = k\frac{Q}{r^2}$$

Magnitude of electric field due to Q at a distance r from Q.

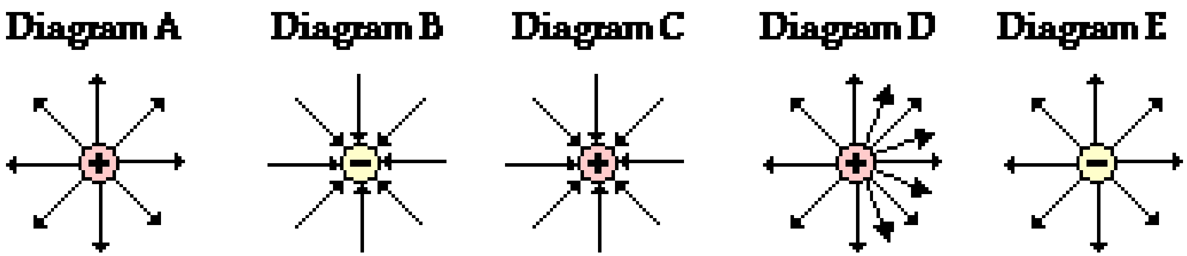
Electric Field Lines (last time)

- Visualization of the electric field
- Lines drawn <u>parallel</u> to the E-direction
  - ➢ With arrows pointing in the direction of E
  - ➢ Start on +ve charge, end on –ve charge
  - ➢ High density of lines ←→ strong field
  - ➢ Uniqueness of E-field → lines never cross
    - Otherwise would have two directions at crossing point

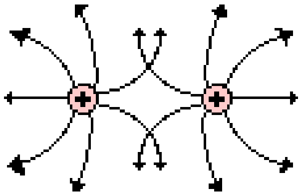


Are these field-lines pattern correct?

from http://www.glenbrook.k12.il.us/gbssci/phys/Class/estatics/u8l4c.html

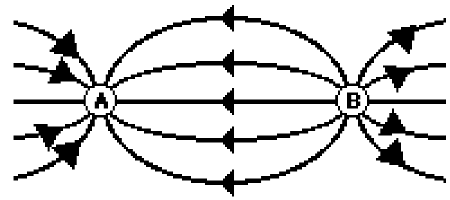


A. OK
B. OK
C. No
   - Direction of arrows is wrong
D. No
   - Density of lines suggests that field is stronger on one side of the charge
E. No
   - Direction of arrows is wrong



Are these OK field lines?

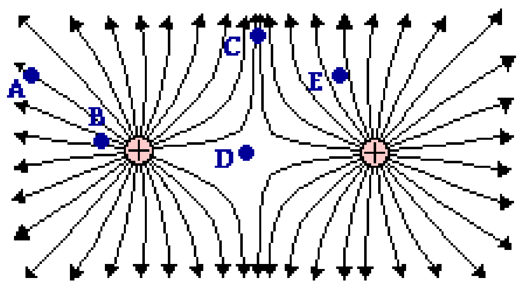
No. Lines cannot cross



What are the signs of charges A & B?

A: -ve   B: +ve

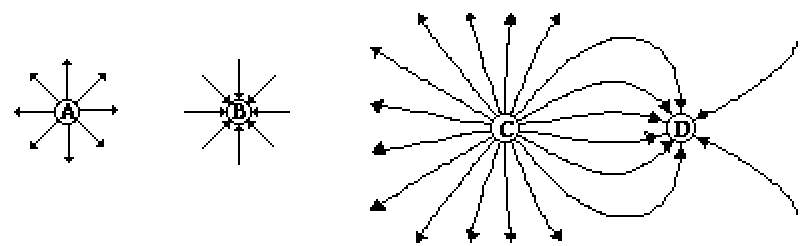
(lines start on +ve charge, end on –ve)

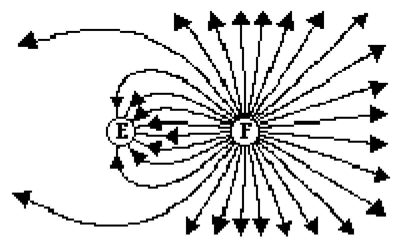


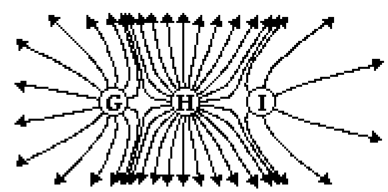
Several locations are labeled. Rank them in order of electric field strength, from smallest to largest

DAECB or perhaps DAEBC

high density of lines → high E

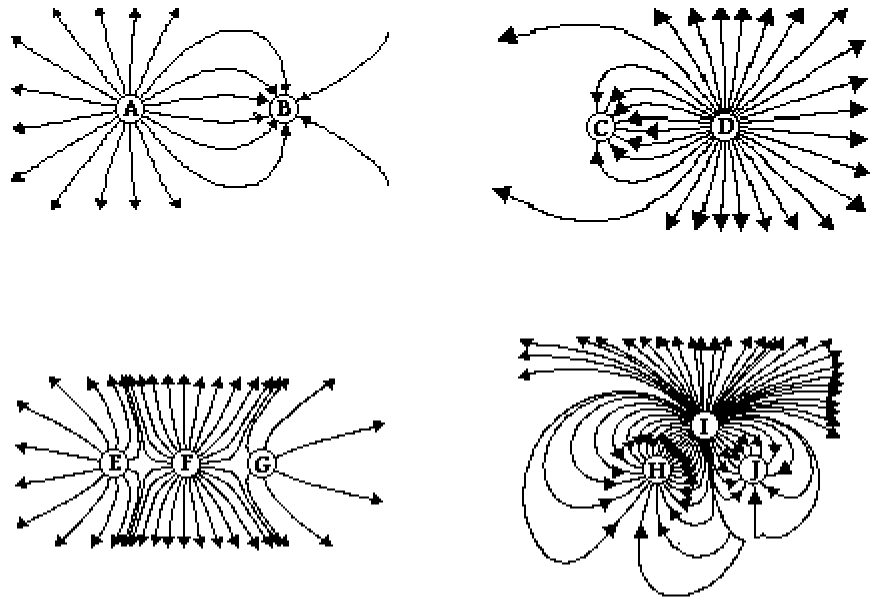






What are the signs of charges A through I?

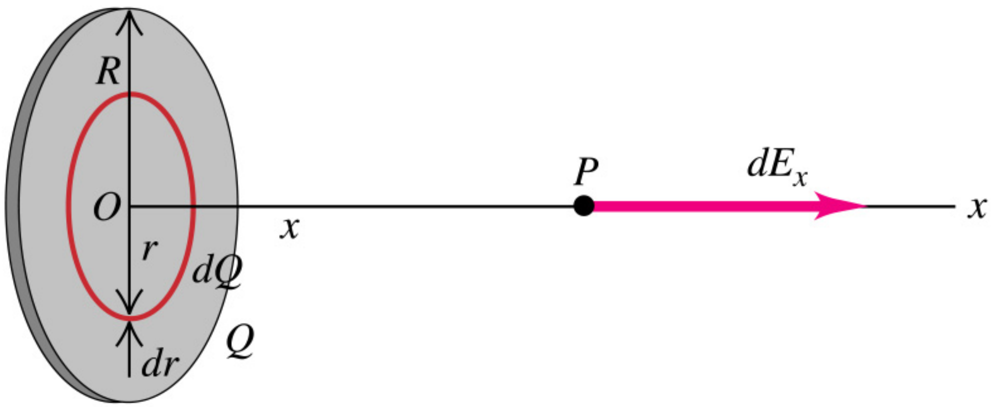
A. +	E. -
B. -	F. +
C. +	G. +
D. -	H. +
	I. +



Rank <u>magnitude</u> of charges in each sketch

- A > B
  - Density of lines to the left of A > density to right of B
- D > C
- F > E > G
- I > H > J

Another electric field example

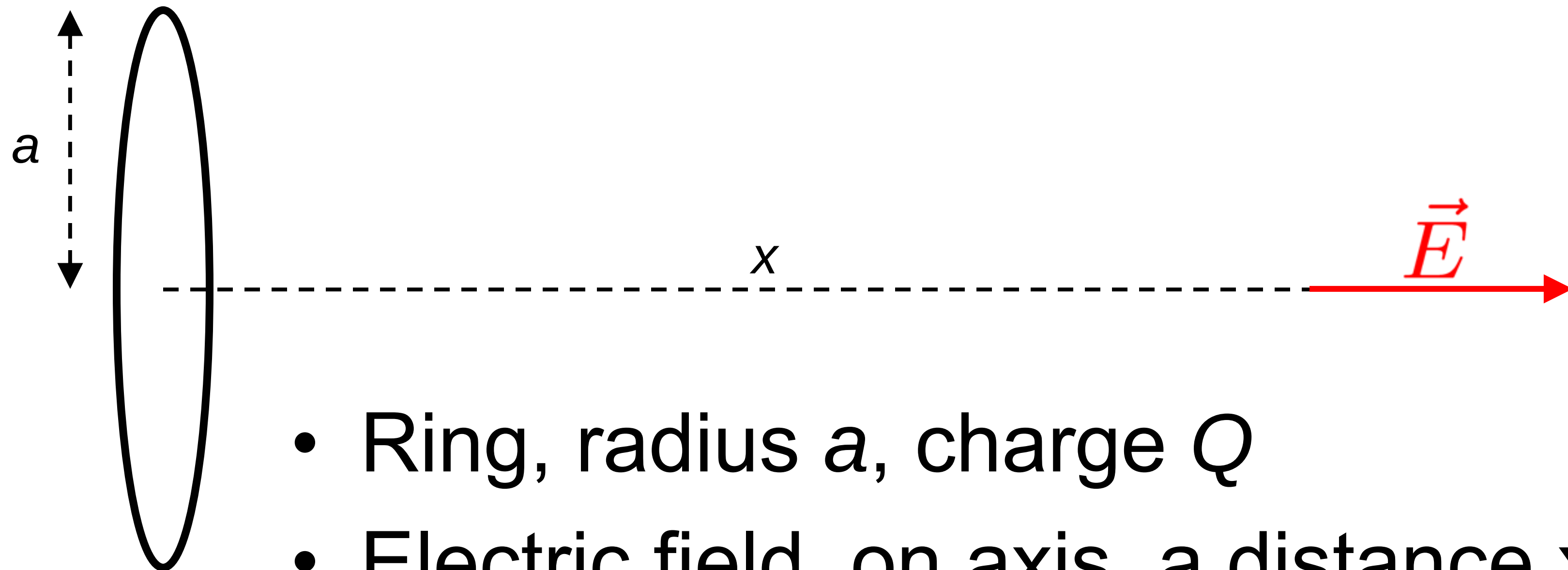


Electric field on axis of a uniformly charged disk?

- As we did last time:
  - Calculate the field due to a small piece of the disk
  - Add-up the contributions from all the small pieces
- Key question:
  - What is the most convenient way of breaking up the disk into small pieces?
  - Exploit result from last time: field due to a ring

→ Break up the disk into a bunch of concentric rings

Result from last time: electric field, on axis, due to a uniformly charged ring

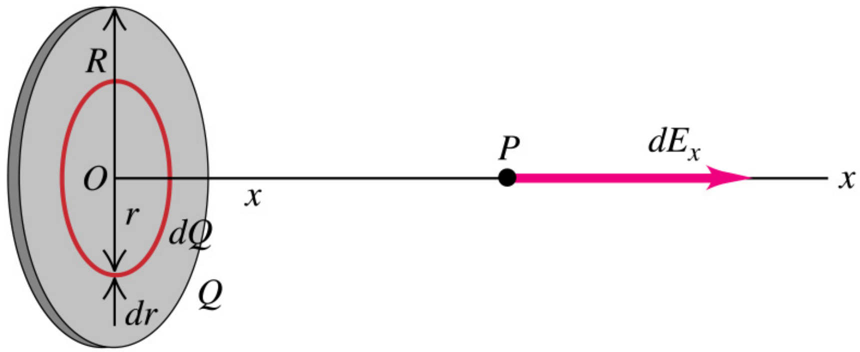


- Ring, radius *a*, charge *Q*
- Electric field, on axis, a distance x:

$$E = \frac{kxQ}{(x^2+a^2)^{\frac{3}{2}}}$$

- In the direction indicated

Back to the disk



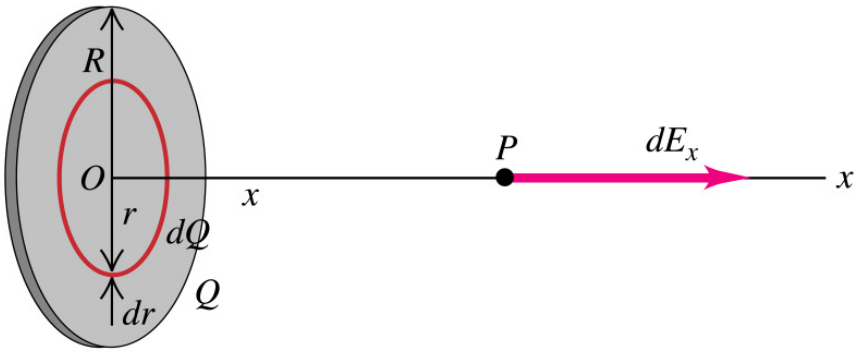
Strategy:

- Consider ring, at radius *r*, small width *dr*
- Compute electric field *dE* due to this ring
- Add up the electric fields of all the rings that make up the disk

Electric field due to ring, using prev. result:

$$dE_x = \frac{kx\, dQ}{(x^2+r^2)^{\frac{3}{2}}}$$

dQ = charge of ring



dQ = charge of ring

$$dE_x = \frac{kx\,dQ}{(x^2+r^2)^{\frac{3}{2}}}$$

Next question: what is *dQ*?

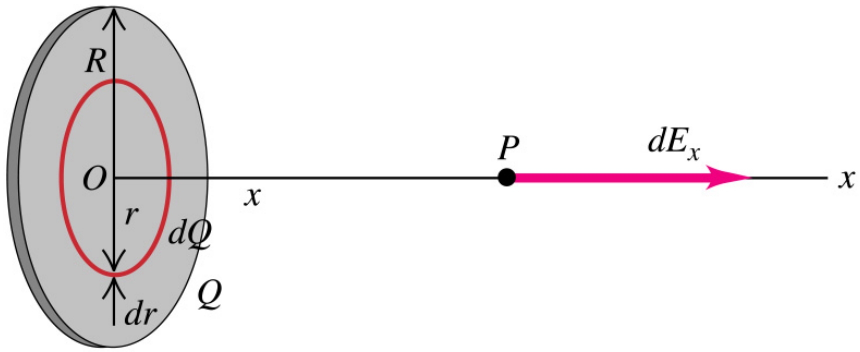
- Total charge of the disk = *Q*
- *dQ*/*Q* = [Area of the ring] / [Area of the disk]
- Area of the disk = πR^2
- Area of ring = $2\pi r\,dr$

width of the ring

circumference of the ring

- $dQ = Q(2r/R^2)dr$

$$dE_x = 2k\frac{Qxr\ \ dr}{R^2(x^2+r^2)^{\frac{3}{2}}}$$



$$dE_x = 2k\frac{Qxr\ dr}{R^2(x^2+r^2)^{\frac{3}{2}}}$$

Surface charge density:

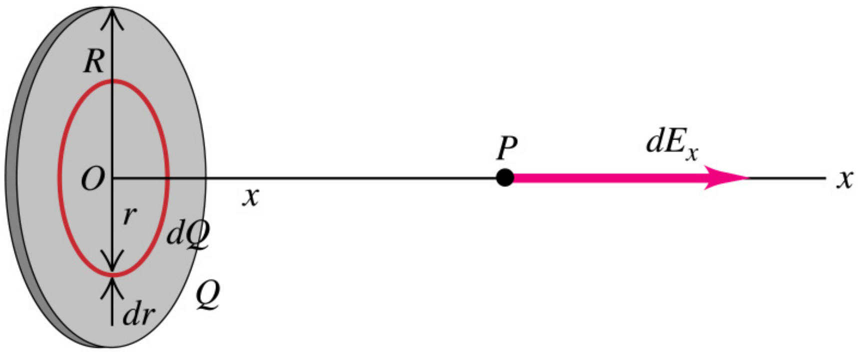
$\sigma = Q/\text{Area} = Q/(\pi R^2)$

$\rightarrow Q = \pi\sigma R^2$

$$dE_x = 2\pi k\frac{\sigma xr\ dr}{(x^2+r^2)^{\frac{3}{2}}}$$

Now use $k=1/(4\pi\varepsilon_0)$

$$dE_x = \frac{\sigma x}{2\epsilon_0}\frac{rdr}{(x^2+r^2)^{\frac{3}{2}}}$$



$$dE_x = \frac{\sigma x}{2\epsilon_0} \frac{r\,dr}{(x^2+r^2)^{\frac{3}{2}}}$$

Now we sum over the rings → we integrate:

$$E_x = \int dE_x$$

What are we integrating over?

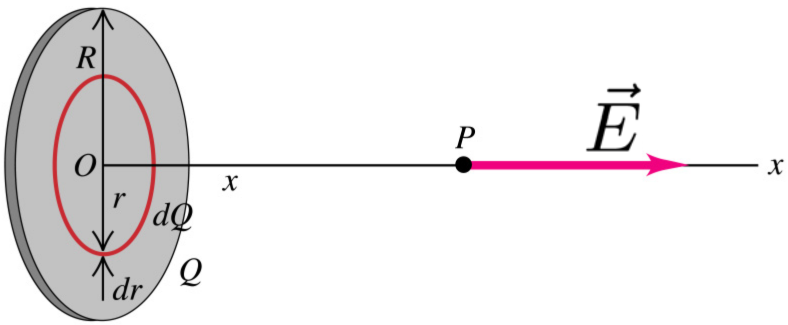
Integrate over *r*, from *r=0* to *r=R*

$$E_x = \frac{\sigma x}{2\epsilon_0} \int_{r=0}^{r=R} \frac{r\,dr}{(x^2+r^2)^{\frac{3}{2}}}$$

$$E_x = \frac{\sigma x}{2\epsilon_0} \left. \frac{-1}{\sqrt{x^2+r^2}} \right|_0^R$$

$$E_x = \frac{\sigma x}{2\epsilon_0} \left(\frac{1}{x} - \frac{1}{\sqrt{x^2+R^2}}\right)$$

A curious result (counterintuitive ?)

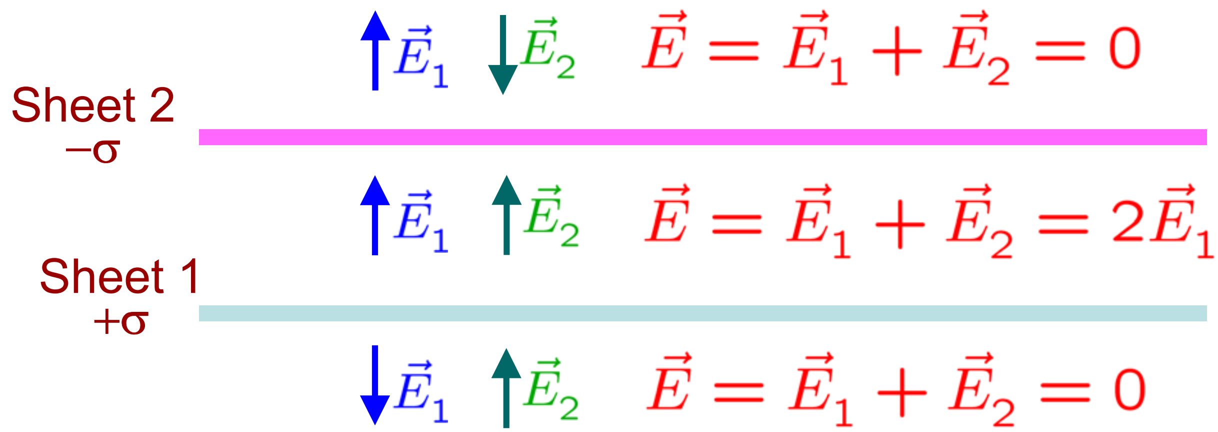


$$E = \frac{\sigma x}{2\epsilon_0}\left(\frac{1}{x} - \frac{1}{\sqrt{x^2+R^2}}\right)$$

- Consider infinitely large disk, i.e., $R \rightarrow \infty$
  - $x^2+R^2 \rightarrow \infty$, the 2nd term in parenthesis $\rightarrow 0$
  - $E \rightarrow \sigma/(2\varepsilon_0)$
  - Constant, independent of x !!
- The electric field of an infinitely large, uniformly charged plane is perpendicular to the plane, and constant in magnitude $E=\sigma/(2\varepsilon_0)$
- In the limit, this holds also for a finite plane provided the distance from the plane is small compared to the size of the plane

Yet Another E-field Example

Two oppositely charge infinite sheets



Principle of superposition: $\vec{E} = \vec{E}_1 + \vec{E}_2$

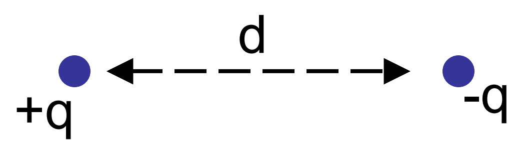
- $\vec{E}$: Total Field
- $\vec{E}_1$: Field due to sheet 1
- $\vec{E}_2$: Field due to sheet 2

In magnitude: $E_1 = E_2 = \sigma/2\varepsilon_0$
In direction: see Figure

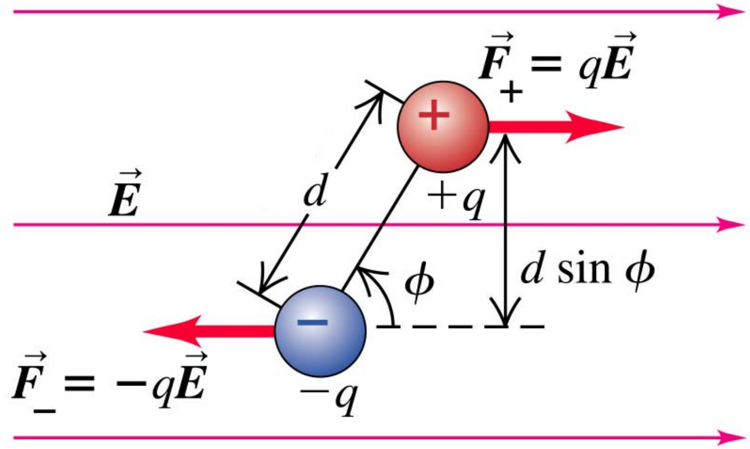
No field outside the plates

(Electric) Dipole

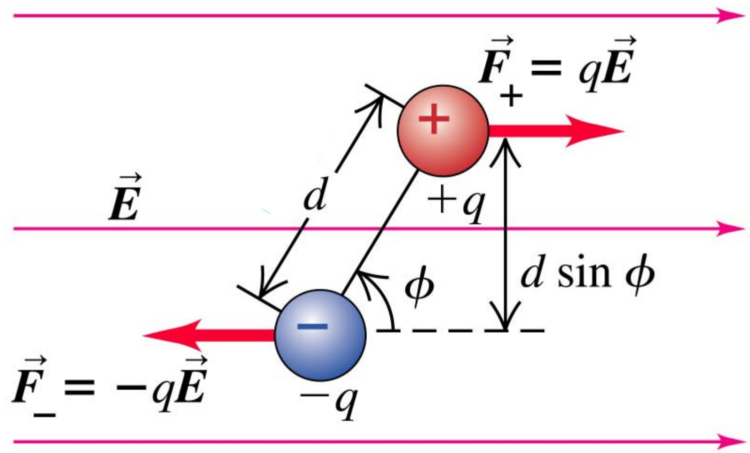
- We saw this in the previous lecture
- Two equal and opposite charges



- Consider what happens in a uniform E-field



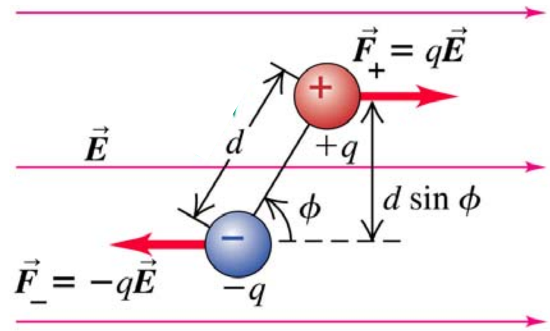
- No net force because $\vec{F}_+ = -\vec{F}_-$



No net force

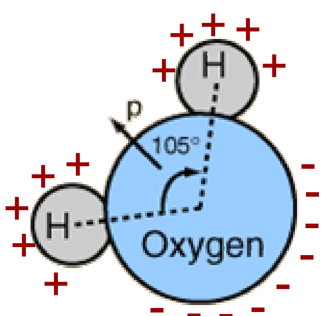
- But there is a net torque
  - Because the two forces act along different lines
- Torque (magnitude) $\tau = Fd \sin\phi = qEd \sin\phi$
  - Note: no torque if $\sin\phi=0$, i.e., if E field is parallel to line joining the two charges (makes sense)
  - Wants to rotate dipole to be parallel with E-field
- Definition of electric dipole moment
  - $p = qd$
    - Magnitude. We'll see in a minute that this is a vector

Aside:



- It makes sense to talk about the torque or force on the dipole (on the two charges) if the two charges are "bound" to each other somehow,
  - e.g. connected by a rod, like a dumbbell
  - or, a more physical situation, chemically bound atoms in a molecule

H_20 molecule:

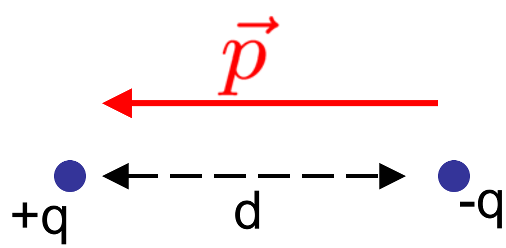


(Almost) all neutral objects where the charge distribution is not spherically symmetric can be thought of as dipoles

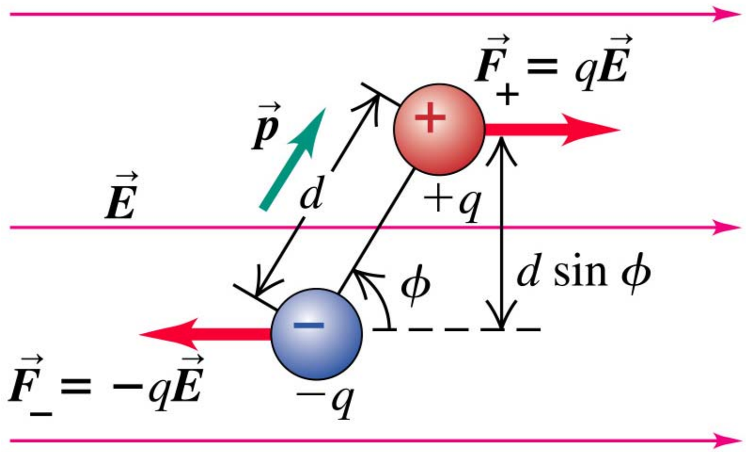
The dipole moment is a vector quantity

Convention:

- Vector along the line that joins the two charges
- Direction from negative to positive charge



$$p = qd$$



$p = qd$

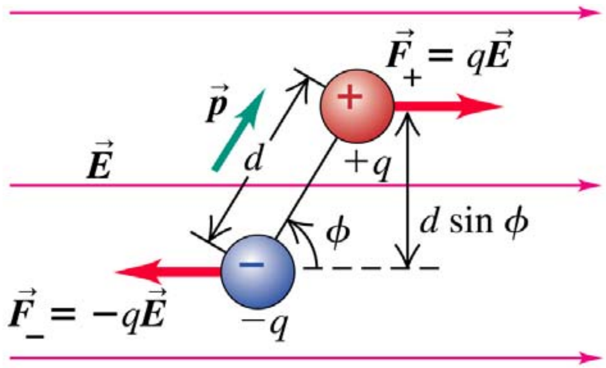
Torque (magnitude) $\tau = Fd \sin\phi = qEd \sin\phi = pE \sin\phi$

But remember, torque is also a vector.

- Perpendicular to the plane of induced rotation
  - In this case, rotate in the plane of the screen
  - → torque vector along line in-and-out of screen
- Direction set by the right-hand-rule
- Then, with convention for direction of dipole moment:

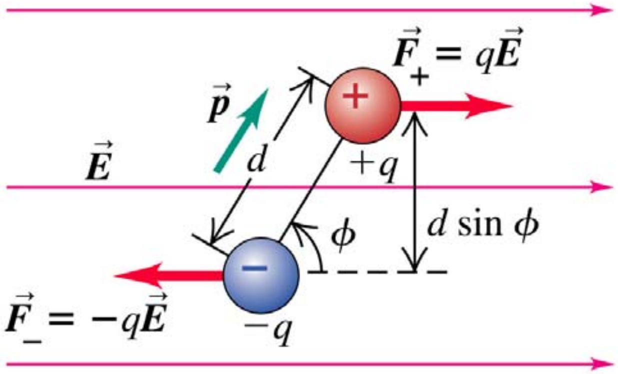
$$\vec{\tau} = \vec{p} \times \vec{E}$$

Energy Considerations



- Dipole rotated by torque → work done on dipole
- dW = work done on dipole as it rotates by $d\phi$
- $|dW| = |\tau\, d\phi| = |pE \sin\phi\, d\phi|$
- Now let's figure out the signs!
  - Work done on the dipole is positive
  - Rotation caused by torque is clockwise
    → ϕ gets smaller, $d\phi < 0$

$$dW = -pE \sin\phi\, d\phi$$



Work done on dipole as it rotates by dϕ

$$dW = -pE \sin\phi \, d\phi$$

Work done on dipole as it rotates $\phi_1 \rightarrow \phi_2$:

$$W = \int_{\phi_1}^{\phi_2} (-pE \sin\phi) d\phi$$

$$W = pE \cos\phi_2 - pE \cos\phi_1$$

Definition of potential energy:

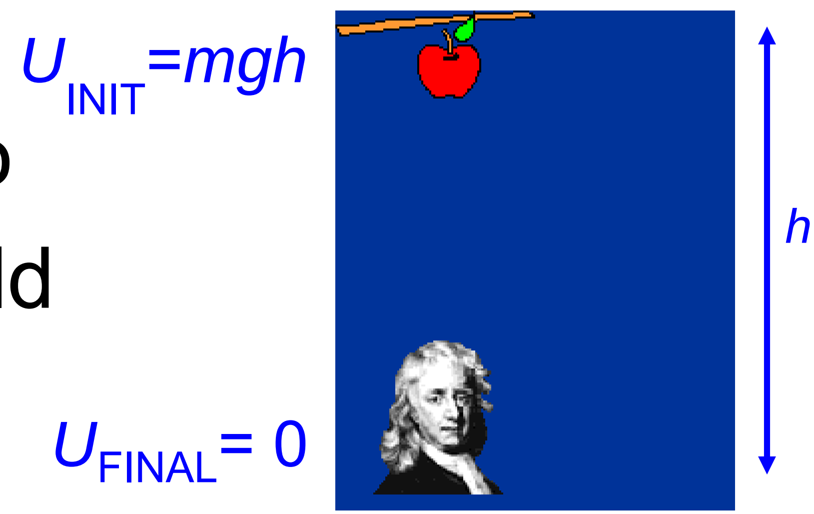
$$W = -\Delta U = -(U_2 - U_1)$$

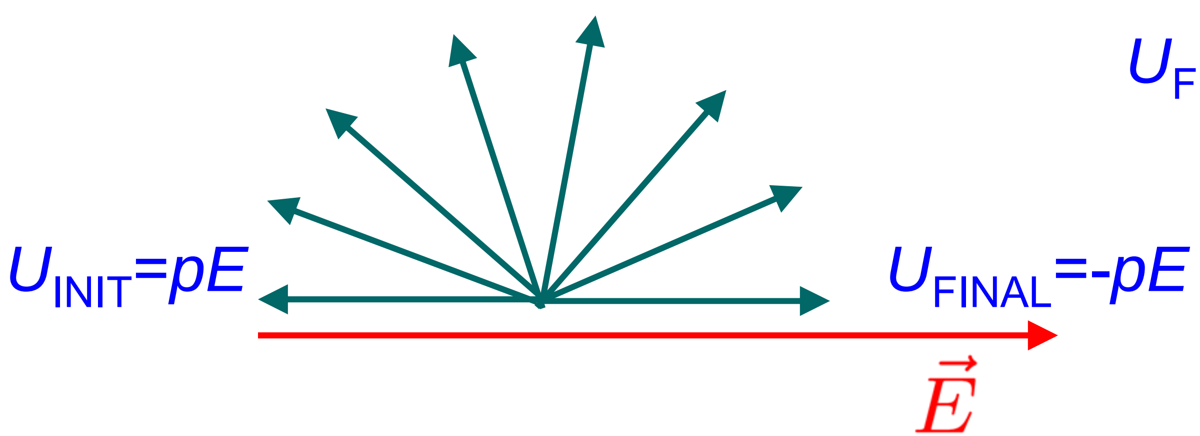
Potential energy of dipole in electric field:

$$U(\phi) = -pE \cos\phi = -\vec{p} \cdot \vec{E}$$

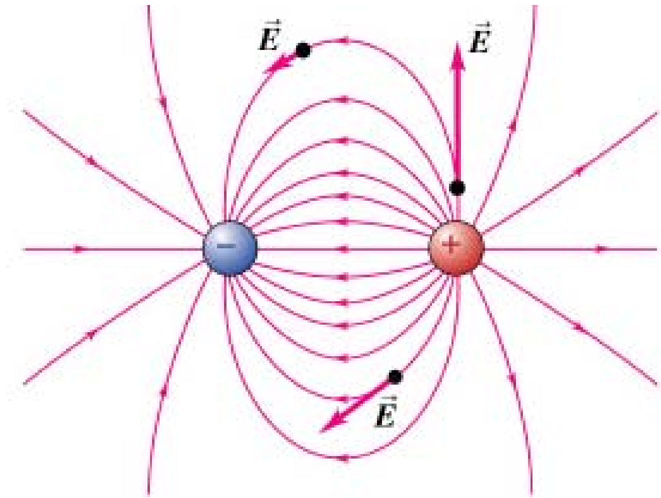
Analogy with gravitational potential energy

- When left to "do its thing" a system tries to <u>minimize</u> its potential energy
- An object in the earth's gravitational field "falls down"
- A dipole rotates so as to align itself with the E field





More on field of electric dipole



Quite complicated!

But we know in principle how to calculate it at any point in space:

1) Calculate E_1 and E_2 from the two charges

2) Add them vectorially

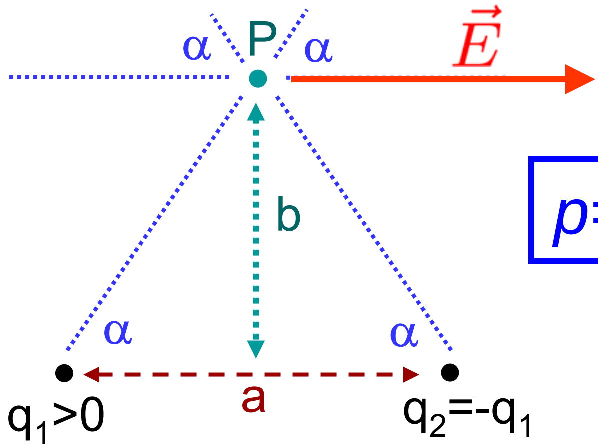
Tedious, but very doable. We did it in the last lecture for three points!

Interesting results for dipole

- Far away from the dipole, the field amplitude drops off as $1/r^3$ everywhere
  - r = distance from the dipole

- Far away from the dipole, the field amplitude is proportional to the dipole moment

Illustration of 1/r^3 behavior



$p=aq_1$

Last time:

$$E = 8k\frac{q_1 a}{(a^2+4b^2)^{\frac{3}{2}}}$$

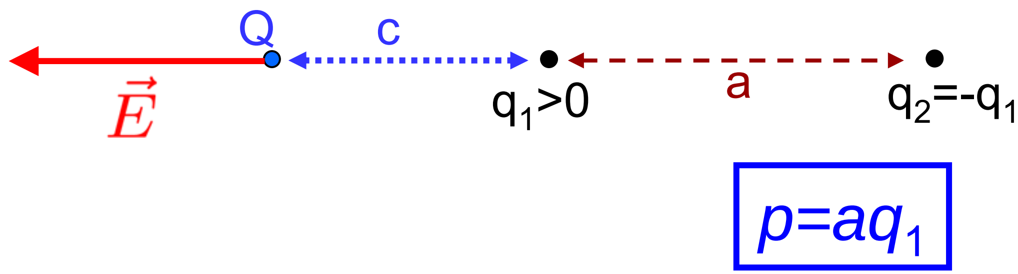
Very far away: *b* >> *a*

→ $a^2 + 4b^2 \sim 4b^2$

$$E \approx 8k\frac{q_1 a}{(4b^2)^{\frac{3}{2}}} = k\,\frac{aq_1}{b^3} = k\frac{p}{q^3}$$

1/b^3, as advertised!

Another illustration of $1/r^3$ behavior



Last time:

$$E = kq_1\left(\frac{1}{c^2} - \frac{1}{(a+c)^2}\right)$$

Very far away: *c* >> *a*

$$E = kq_1\left(\frac{1}{c^2} - \frac{1}{c^2(1+\frac{a}{c})^2}\right) = \frac{kq_1}{c^2}\left(1 - \frac{1}{(1+\frac{a}{c})^2}\right)$$

A very useful trick for making approximations, use binomial expansion:

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots.$$

Remember: k! = k(k-1)(k-2)...1

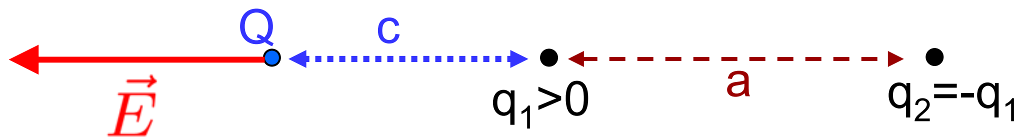
Why is the binomial expansion so useful?

$$(1+x)^n = 1+nx+\frac{n(n-1)}{2!}x^2+\frac{n(n-1)(n-2)}{3!}x^3+\ldots.$$

- Suppose x<1
- Then each additional term in the series is smaller than the preceeding one, since $x^{r+1} < x^r$
- So, approximate by keeping only a few terms
- You decide on a case-by-case basis how many terms to keep. More terms → more accurate
- Example: take x=0.02 (a small number)

$$(1+x)^2 \approx 1+2x$$
$$(1+0.02)^2 = 1.0404$$
$$1+2\cdot 0.02 = 1.0400$$

$$(1+x)^{-2} \approx 1-2x$$
$$(1+0.02)^{-2} = 0.9612$$
$$1-2\cdot 0.02 = 0.9600$$



Very far away: *c* >> *a*

$p=aq_1$

$$E = \frac{kq_1}{c^2}\left(1 - \frac{1}{(1+\frac{a}{c})^2}\right)$$

If *c* >> *a*, (*a*/*c*)<<1

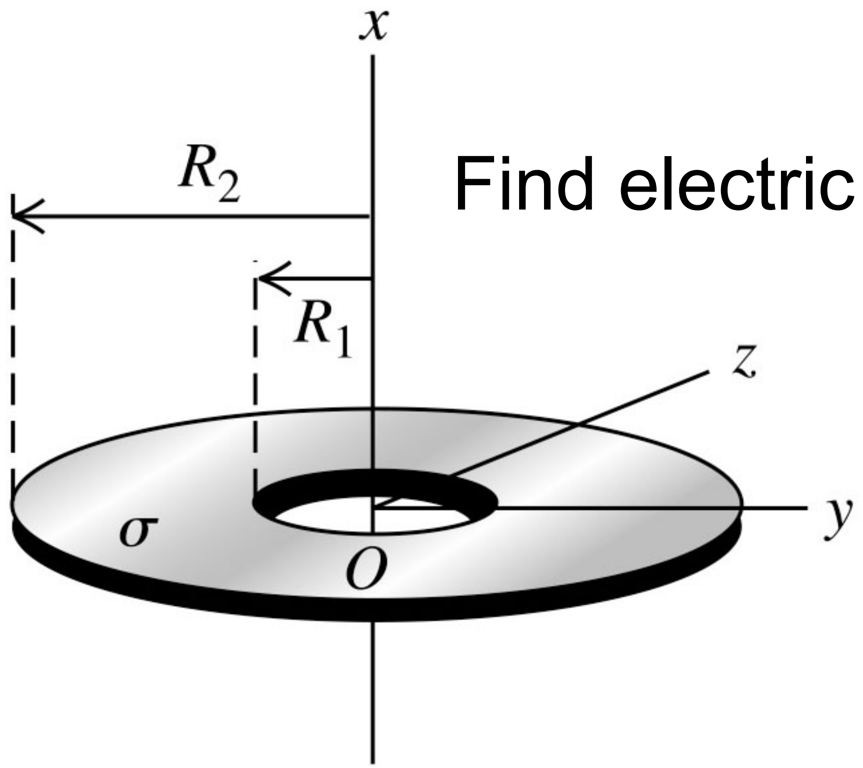
→ expand the denominator, keep only 1st term

$$E \approx \frac{kq_1}{c^2}[1 - (1 - 2\frac{a}{c})]$$

$$E \approx \frac{2kaq_1}{c^3} = \frac{2kp}{c^3}$$

$1/c^3$, as advertised!

Another example (Problem 21.102)



Find electric field at any point on x-axis

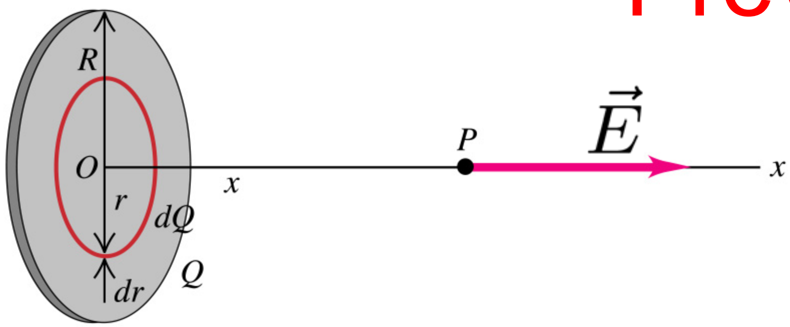
Brute force approach:

- find the field due to a small piece of the disk
- sum over all of the pieces (integrate)

There is a better way:

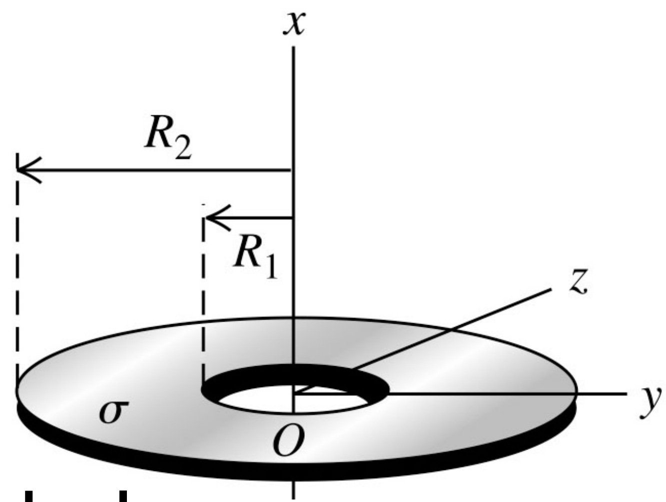
- use previous results + a trick !

Previous result:

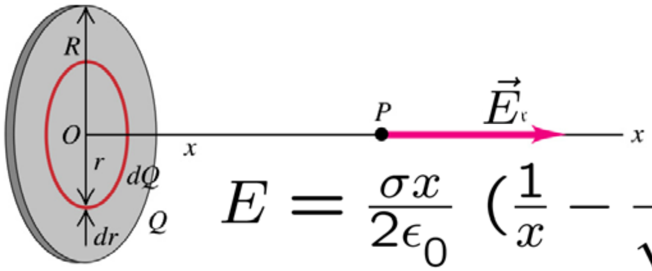


$$E = \frac{\sigma x}{2\epsilon_0}\left(\frac{1}{x} - \frac{1}{\sqrt{x^2+R^2}}\right)$$

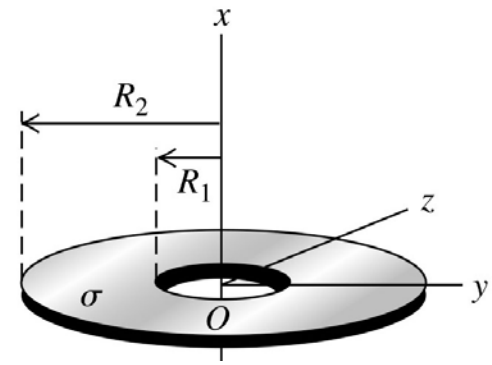
How can we apply it to:



- Imagine two rings with no holes
  - Radius R_2, charge density $+\sigma$
  - Radius R_1, charge density $-\sigma$
- The sum of the fields due to these two rings will be the same as the field of the ring with the hole!



$$E = \frac{\sigma x}{2\epsilon_0}\left(\frac{1}{x} - \frac{1}{\sqrt{x^2+R^2}}\right)$$



Field due to ring of radius R_2, surface charge density $+\sigma$:

$$E = \frac{\sigma x}{2\epsilon_0}\left(\frac{1}{x} - \frac{1}{\sqrt{x^2+R_2^2}}\right)$$

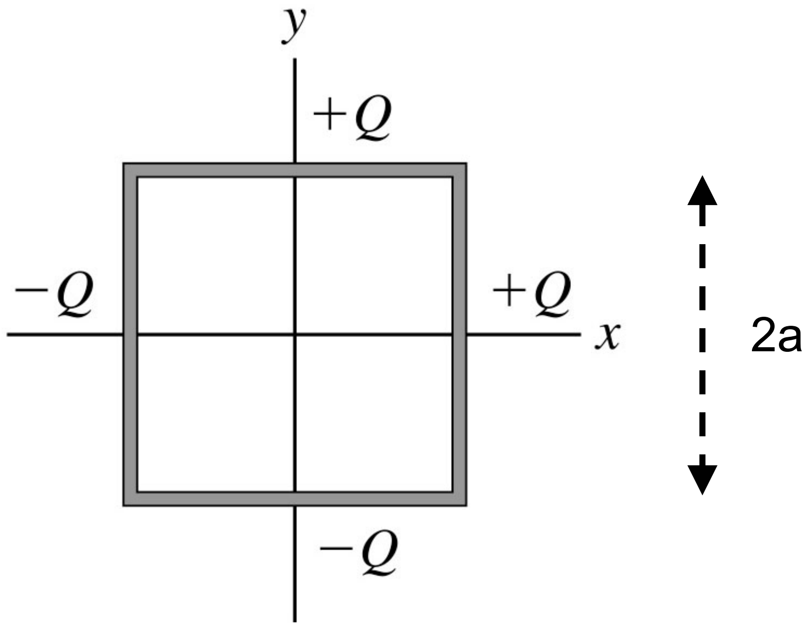
Field due to ring of radius R_1, surface charge density $-\sigma$:

$$E = -\frac{\sigma x}{2\epsilon_0}\left(\frac{1}{x} - \frac{1}{\sqrt{x^2+R_1^2}}\right)$$

Total field is the sum of the two:

$$E = -\frac{\sigma x}{2\epsilon_0}\left(\frac{1}{\sqrt{x^2+R_1^2}} - \frac{1}{\sqrt{x^2+R_2^2}}\right)$$

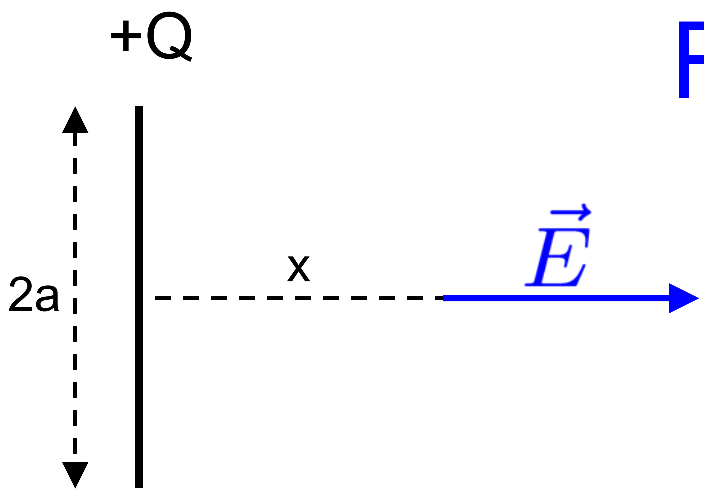
Another example (Problem 21.98)



Find electric field at the center of this square

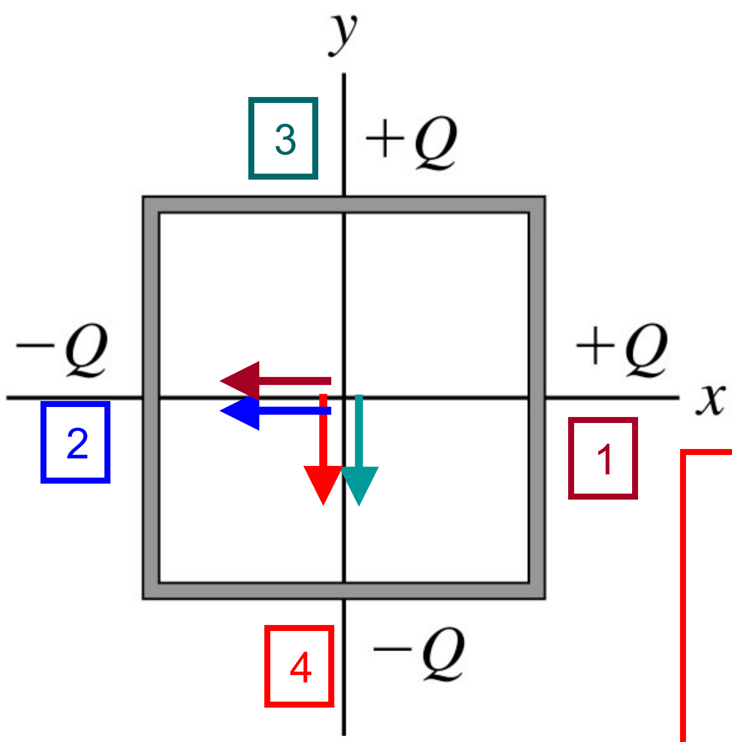
Use result for electric field due to wire

Find the four electric fields, then add them



Result from last time:

$$E = k\frac{Q}{x\sqrt{x^2+a^2}}$$



1. Label the sides
2. Sketch the 4 fields
3. In magnitude, all fields are the same
4. Distance from each wire = a

$$E = k\frac{Q}{a\sqrt{a^2+a^2}} = \frac{k}{\sqrt{2}}\frac{Q}{a^2}$$

$$E_x = -2E = -\sqrt{2}k\frac{Q}{a^2}$$

$$E_y = -2E = -\sqrt{2}k\frac{Q}{a^2}$$