Fall 2004 Physics 3
Tu-Th Section

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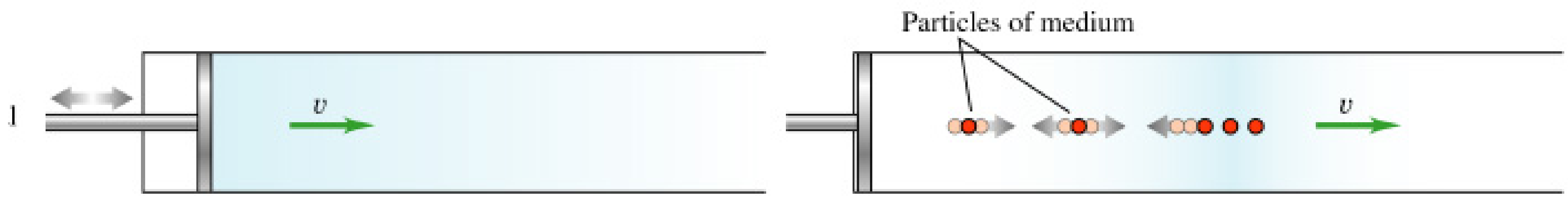
Lecture 4: 5 Oct. 2004

Web page:
http://hep.ucsb.edu/people/claudio/ph3-04/

Last time….

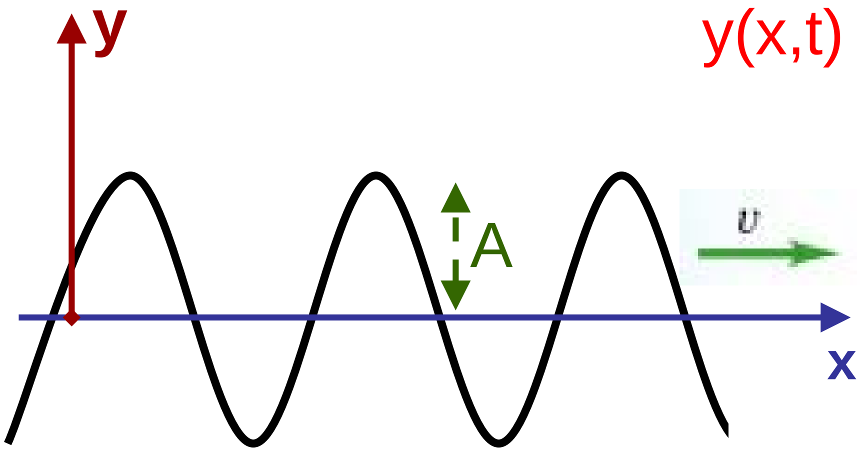
Sound:

- A <u>longitudinal</u> wave in a medium.

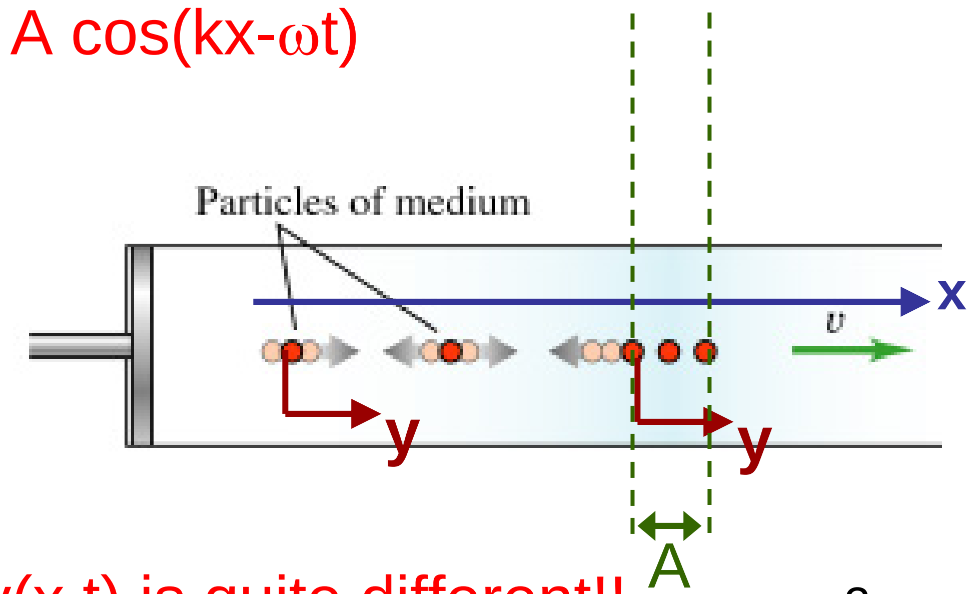


How do we describe such a wave?

- Mathematically, just like <u>transverse</u> wave on string:



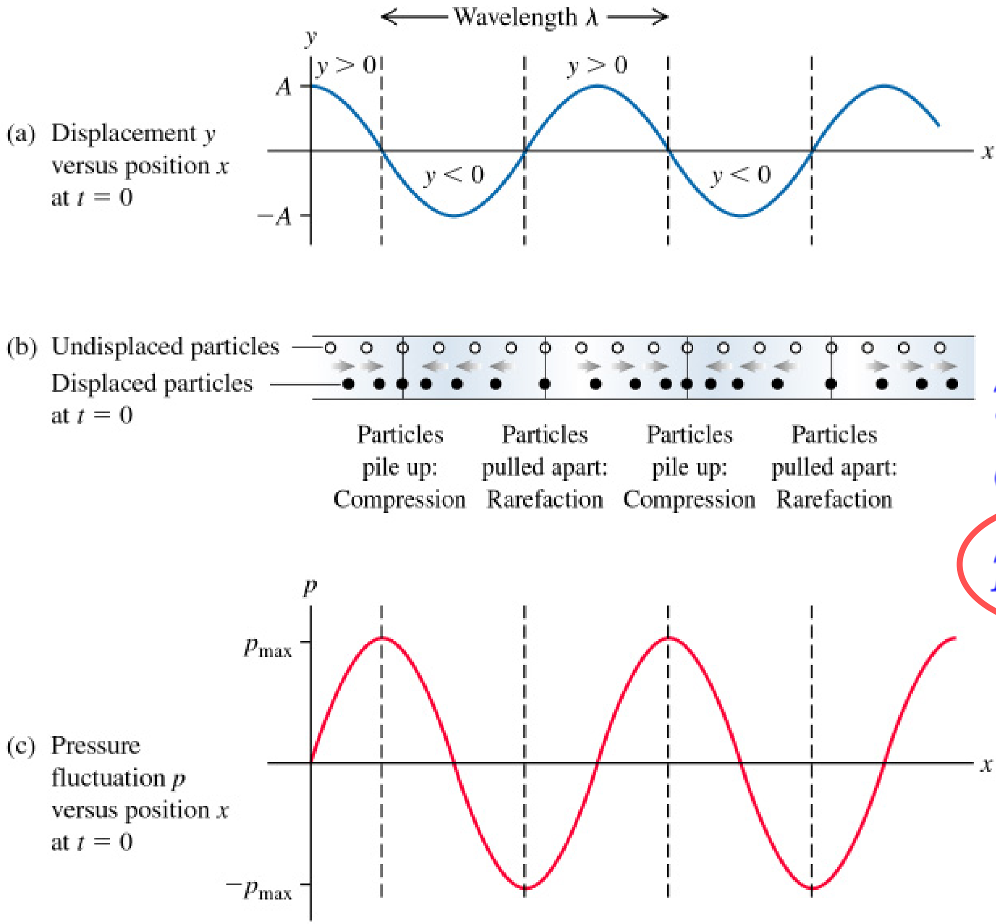
$$y(x,t) = A\cos(kx-\omega t)$$



Careful: the meaning of y(x,t) is quite different!!

Displacement vs. pressure (last time...)

- Can describe sound wave in terms of <u>displacement</u> or in terms of <u>pressure</u>



Displacement and pressure out of phase by 90^o.

$$y(x,t) = A\cos(kx - \omega t)$$

or

$$p(x,t) = (A \cdot B \cdot k)\sin(kx - \omega t)$$

Bulk Modulus

Pressure fluctuation

Bulk Modulus (last time...)

A measure of how easy it is to compress a fluid.

$$B = -V\frac{dP}{dV}$$

Ideal gas: $$B = \gamma P$$

$$\gamma = \frac{C_P}{C_V}$$

↑ Heat capacities at constant *P* or *V*

γ ~ 1.7 monoatomic molecules (He, Ar,..)
γ ~ 1.4 diatomic molecules (O_2, N_2,..)
γ ~ 1.3 polyatomic molecules (CO_2,..)

Speed of sound (last time...)

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma R T}{M}}$$

Molar mass

For air: v=344 m/sec at 20^{o} C

Intensity (last time...)

- The wave carries energy
- The intensity is the time average of the power carried by the wave crossing unit area.
- Intensity is measured in W/m2

$$I = \frac{<P>}{S} = \frac{<dE/dt>}{S}$$

Decibel (last time...)

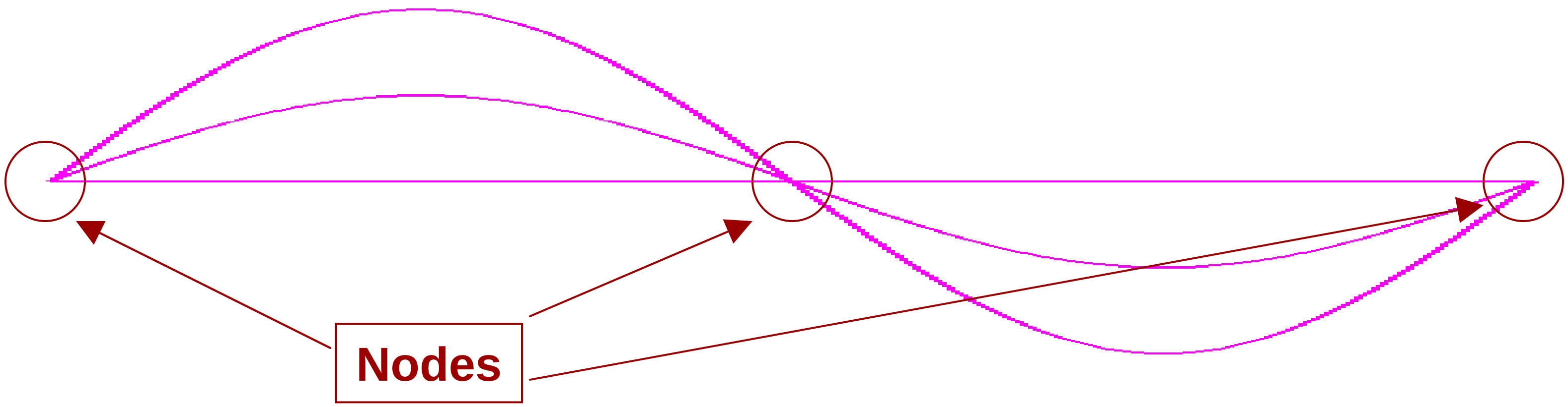
- A more convenient sound intensity scale
  - more convenient than W/m2.
- The <u>sound intensity level β</u> is defined as

$$\beta = (10\ \text{dB}) \log \frac{I}{I_0}$$

- Where $I_0 = 10^{-12}$ W/m^2
  - Approximate hearing threshold at 1 kHz
- It's a log scale
  - A change of 10 dB corresponds to a factor of 10

Standing sound waves

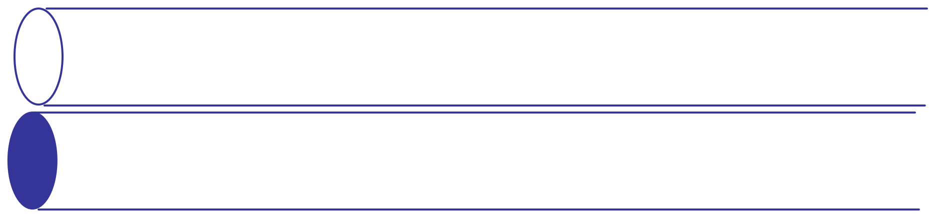
Recall standing waves on a string



- A standing wave on a string occurs when we have interference between wave and its reflection.
- The reflection occurs when the medium changes, e.g., at the string support.

- We can have sound standing waves too.
- For example, in a pipe.
- Two types of boundary conditions:

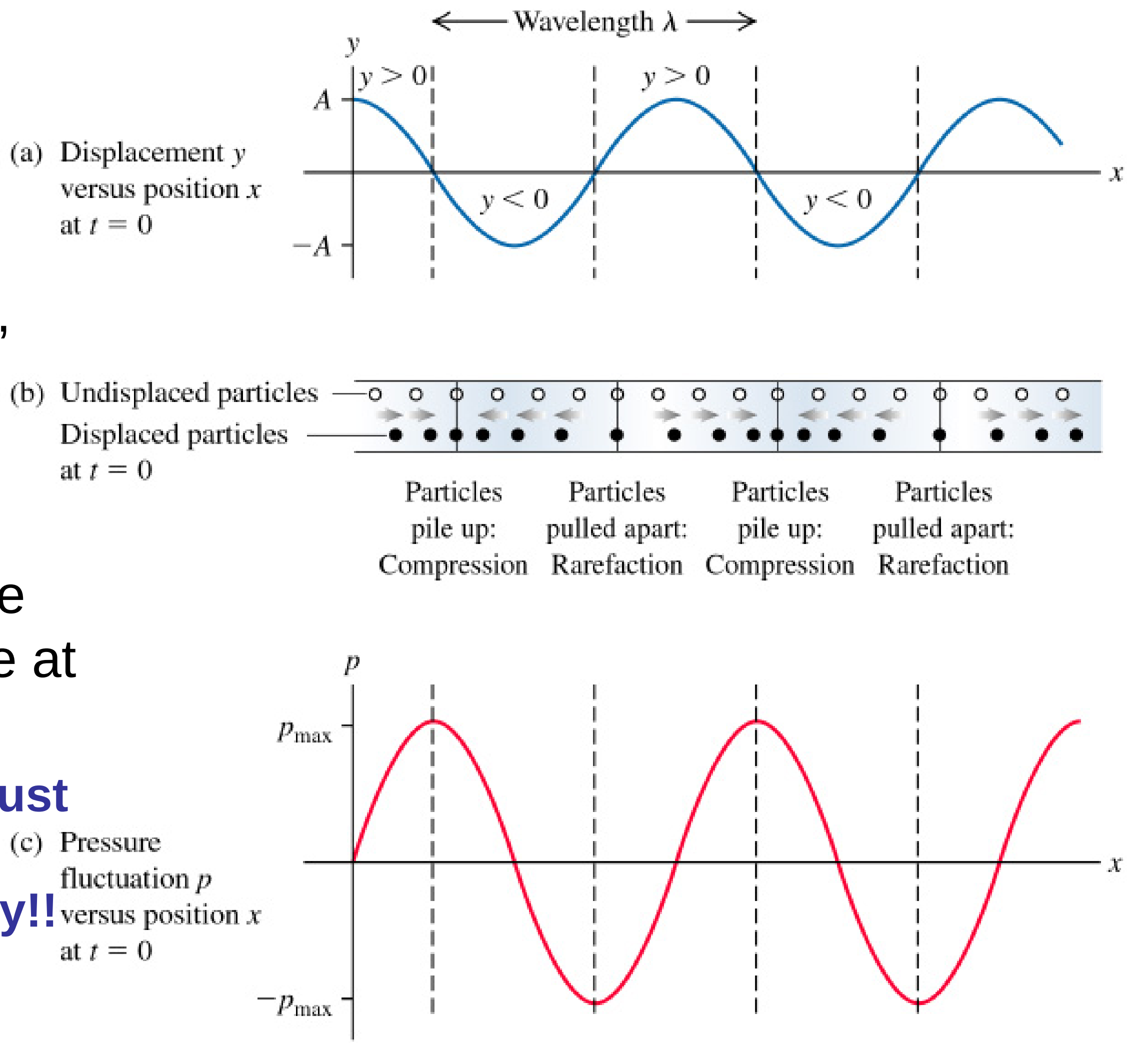
1. Open pipe
2. Closed pipe



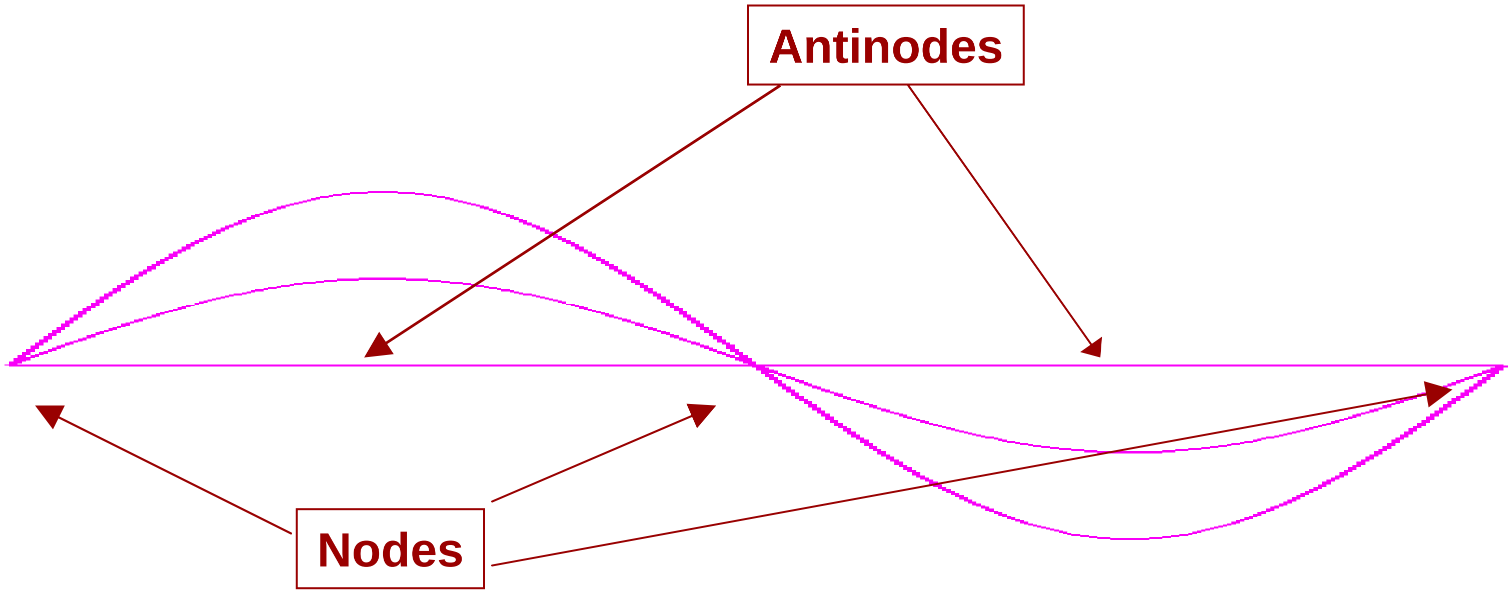
- In an closed pipe the boundary condition is that the displacement is zero at the end
  - ➢ Because the fluid is constrained by the wall, it can't move!
- In an open pipe the boundary condition is that the pressure fluctuation is zero at the end
  - ➢ Because the pressure is the same as outside the pipe (atmospheric)

Remember:

- Displacement and pressure are out of phase by 90º.
- When the displacement is 0, the pressure is $\pm p_{max}$.
- When the pressure is 0, the displacement is $\pm y_{max}$.
- So the nodes of the pressure and displacement waves are at different positions
  - **It is still the same wave, just two different ways to describe it mathematically!!**



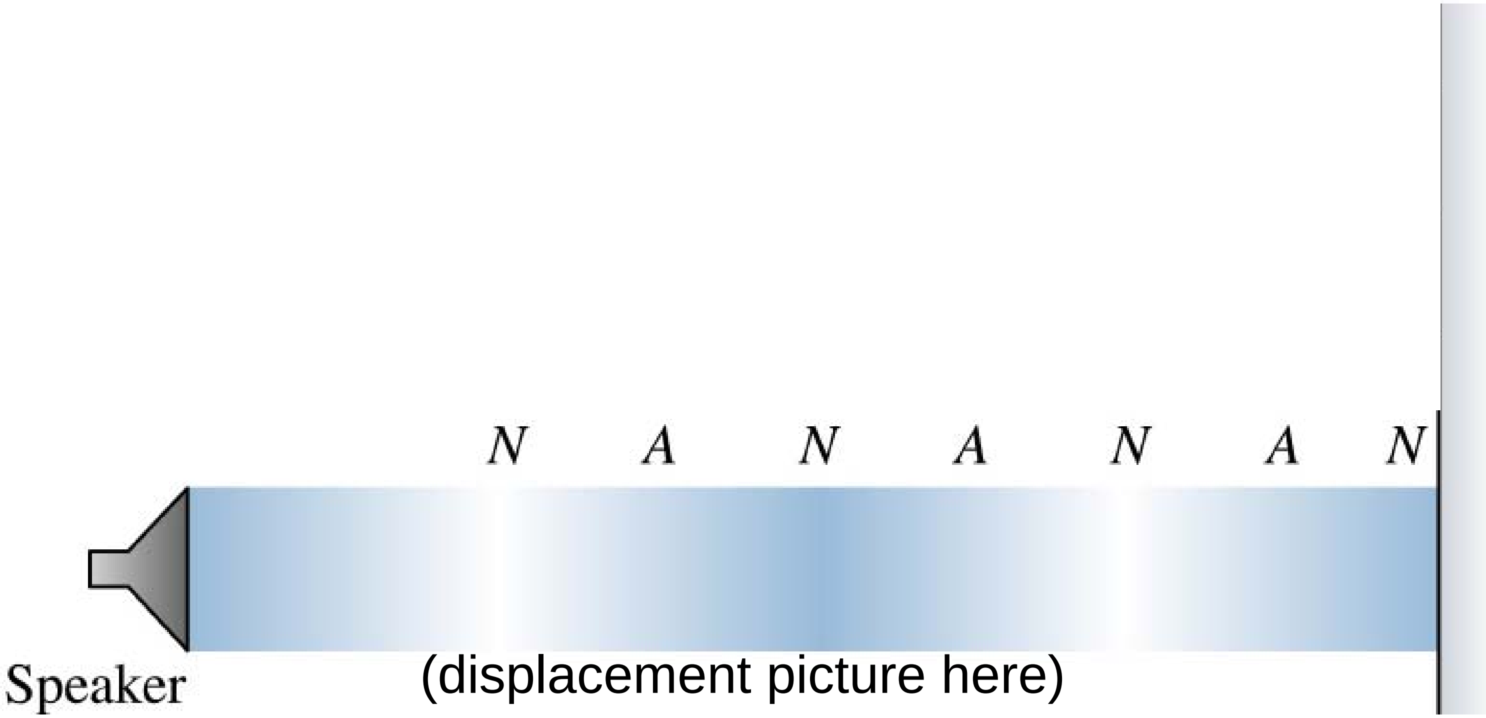
More jargon: nodes and antinodes



- In a sound wave the pressure nodes are the displacement antinodes and viceversa

Example

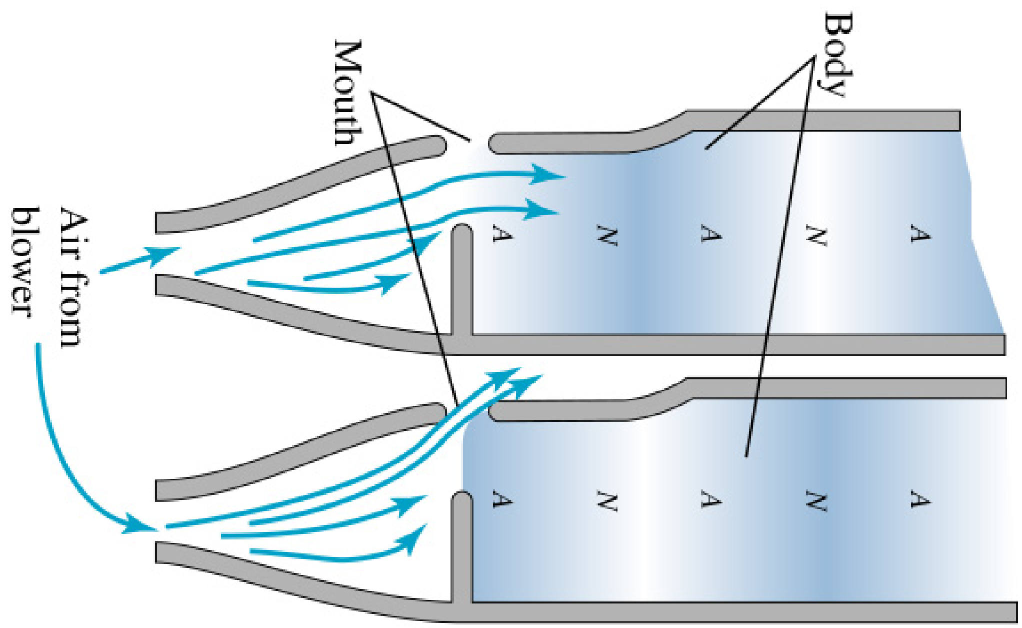
- A directional loudspeaker bounces a sinusoidal sound wave off the wall. At what distance from the wall can you stand and hear no sound at all?
- A key thing to realize is that the ear is sensitive to pressure fluctuations
- Want to be at pressure node
- The wall is a displacement node → pressure antinode



(displacement picture here)

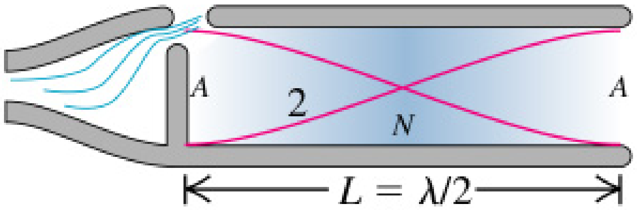
Organ pipes

- Sound from standing waves in the pipe
- Remember:
  - ➢Closed pipe:
    - Displacement node (no displacement possible)
      - → Pressure Antinode
  - ➢Open pipe:
    - Pressure node (pressure is atmospheric)
      - → Displacement Antinode

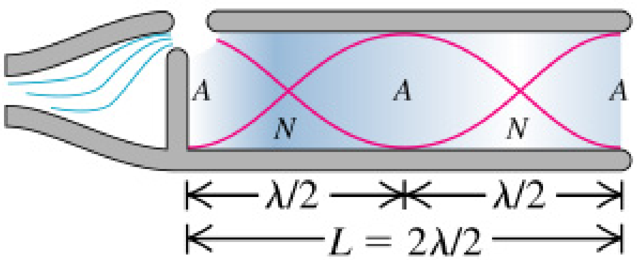


Open Pipe

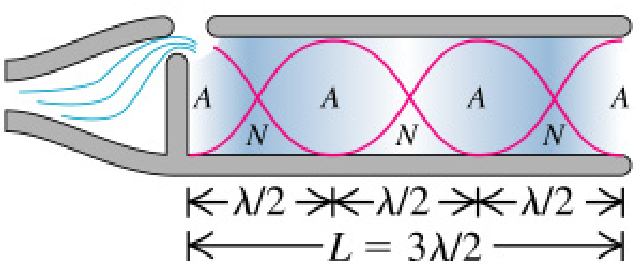
(displacement picture)



(a) $f_1 = \frac{v}{2L}$



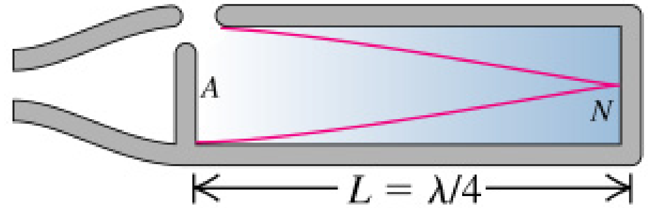
(b) $f_2 = 2\frac{v}{2L} = 2f_1$



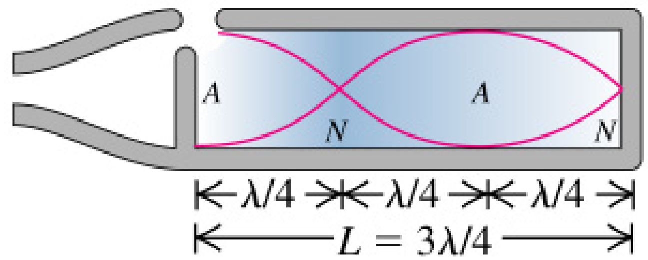
(c) $f_3 = 3\frac{v}{2L} = 3f_1$

Closed Pipe

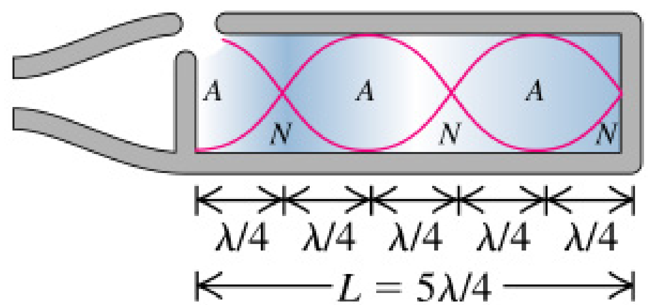
(displacement picture)



(a) $f_1 = \frac{v}{4L}$



(b) $f_3 = 3\frac{v}{4L} = 3f_1$



(c) $f_5 = 5\frac{v}{4L} = 5f_1$

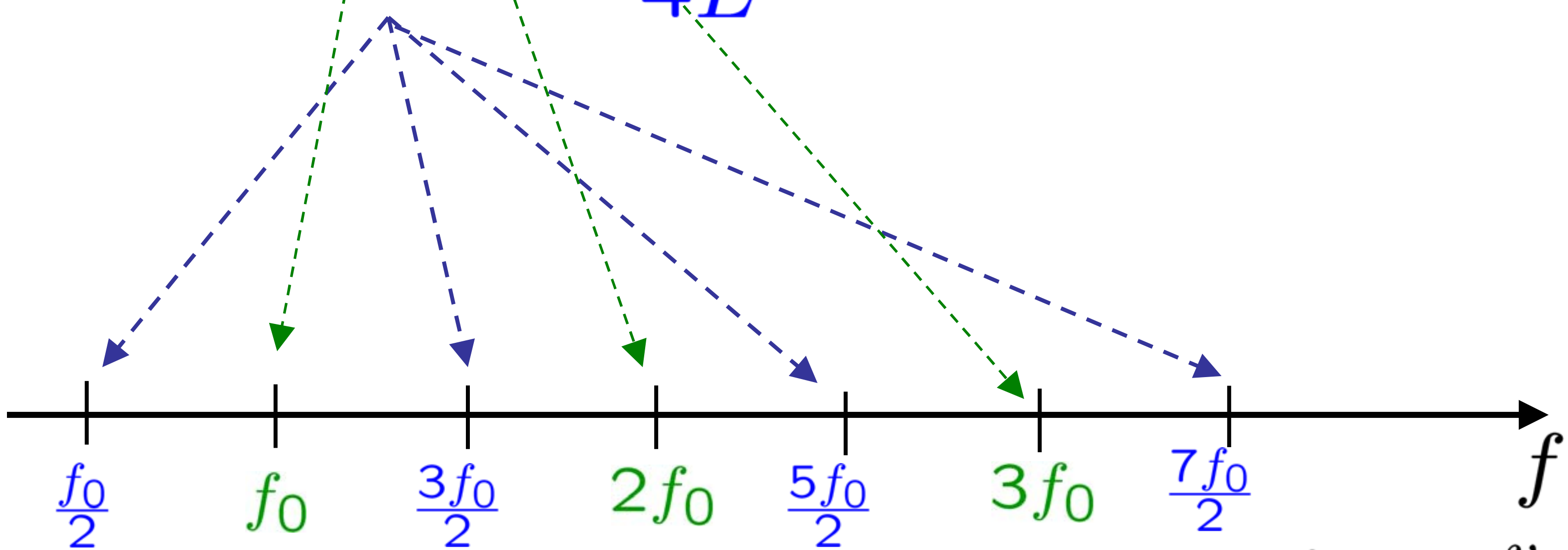
Organ pipe frequencies

- Open pipe:

$$f = n \cdot \frac{v}{2L} \qquad (n = 1, 2, 3..)$$

- Closed (stopped) pipe:

$$f = n \cdot \frac{v}{4L} \qquad (n = 1, 3, 5..)$$



$$f_0 = \frac{v}{2L}$$

Sample Problem

- A pipe is filled with air and produces a fundamental frequency of 300 Hz.
  - If the pipe is filled with He, what fundamental frequency does it produce?
  - Does the answer depend on whether the pipe is open or stopped?

Open pipe: $f = n \cdot \frac{v}{2L} \qquad (n = 1, 2, 3..)$

Closed (stopped) pipe: $f = n \cdot \frac{v}{4L} \qquad (n = 1, 3, 5..)$

→ Fundamental frequency v/2L (open) or v/4L (stopped)

What happens when we substitute He for air?

The velocity of sound changes!

From last week, speed of sound:

$$v = \sqrt{\frac{B}{\rho}} = \sqrt{\frac{\gamma RT}{M}}$$

Molar mass

$$\gamma = \frac{C_P}{C_V}$$

Heat capacities at constant *P* or *V*

γ ~ 1.7 monoatomic molecules (He, Ar,..)
γ ~ 1.4 diatomic molecules (O_2, N_2,..)
γ ~ 1.3 polyatomic molecules (CO_2,..)

$$\rightarrow \quad v_{air} = \sqrt{\frac{\gamma_{air}}{\gamma_{He}}}\sqrt{\frac{M_{He}}{M_{air}}}\ v_{He}$$

We had:

Open pipe: $f = n \cdot \frac{v}{2L} \qquad (n = 1, 2, 3..)$

Closed (stopped) pipe: $f = n \cdot \frac{v}{4L} \qquad (n = 1, 3, 5..)$

i.e., the fundamental frequency is proportional to velocity for <u>both</u> open and stopped pipes

$$f_{He} = \frac{v_{He}}{v_{air}} \quad f_{air}$$

But: $v_{air} = \sqrt{\frac{\gamma_{air}}{\gamma_{He}}}\sqrt{\frac{M_{He}}{M_{air}}} \; v_{He}$

So: $f_{He} = \sqrt{\frac{\gamma_{He}}{\gamma_{air}}}\sqrt{\frac{M_{air}}{M_{He}}} \; f_{air}$

$$f_{He} = \sqrt{\frac{1.7}{1.4}}\sqrt{\frac{29 \text{ g/mol}}{4 \text{ g/mol}}} \; (300\text{Hz}) = 890\text{Hz}$$

Resonance

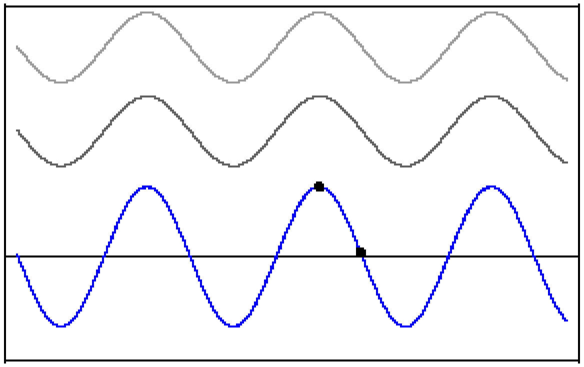
- Many mechanical systems have natural frequencies at which they oscillate.
  - a mass on a spring: $\omega^2 = k/m$
  - a pendulum: $\omega^2 = g/l$
  - a string fixed at both ends: $f=nv/(2L)$
- If they are driven by an external force with a frequency equal to the natural frequency, they go into resonance:
  - the amplitude of the oscillation grows
  - in the absence of friction, the amplitude would → infinity

Interference

- Occur when two (or more) waves overlap.
- The resulting displacement is the <u>sum of the displacements</u> of the two (or more) waves.
  - Principle of superposition.
  - We already applied this principle to standing waves:
    - Sum of a wave moving to the right and the reflected wave moving to the left.

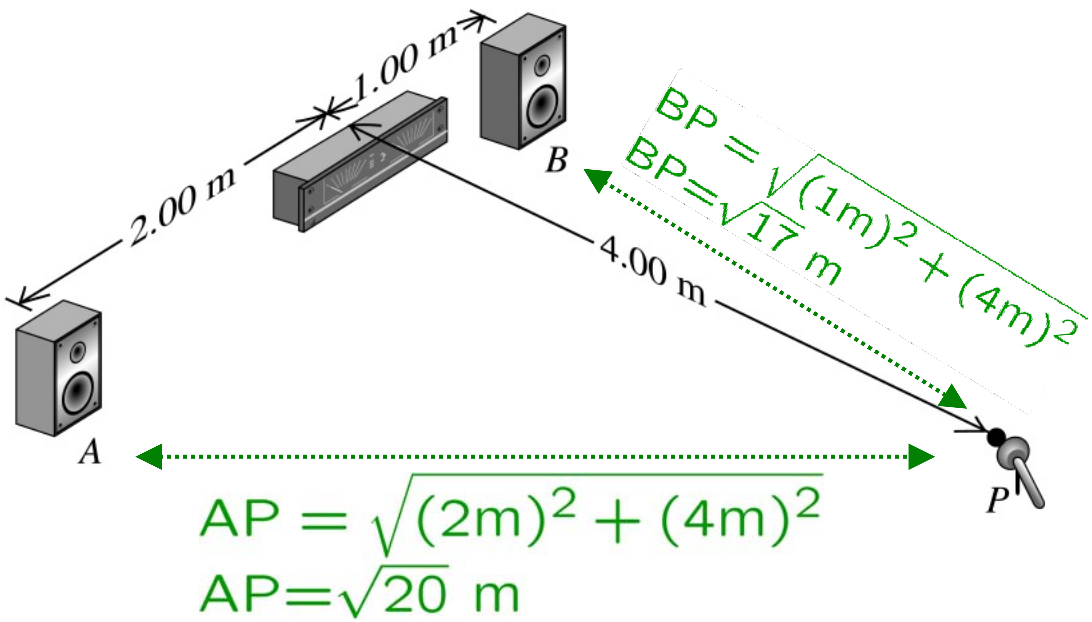
Interference (cont.)

- The displacements of the two waves can add to give
  - ➢ a bigger displacement.
    - Constructive Interference.
  - ➢ or they can even cancel out and give zero displacement.
    - Destructive interference.
    - Sometime, sound + sound = silence
    - Or, light + light = darkness



Interference Example

- Two loudspeakers are driven by the same amplifier and emit sinusoidal waves in phase. The speed of sound is v=350 m/sec. What are the frequencies for (maximal) constructive and destructive interference.



- Wave from speaker A at P (x_1=AP)

  $y_1(t) = A_1 \cos(\omega t - kx_1)$
- Wave from speaker B at P (x_2=BP)

  $y_2(t) = A_2 \cos(\omega t - kx_2)$
- Total amplitude

  $y(t) = y_1(t) + y_2(t)$

  $y(t) = A_1 \cos\omega t \cos kx_1 + A_1 \sin\omega t \sin kx_1$
  $\quad + A_2 \cos\omega t \cos kx_2 + A_2 \sin\omega t \sin kx_2$
- When $kx_1 = kx_2 + 2n\pi$ the amplitude of the resulting wave is largest.
  - In this case $\cos kx_1 = \cos kx_2$ and $\sin kx_1 = \sin kx_2$.
- Conversely, when $kx_1 = kx_2 + n\pi$ (with n odd), the amplitude of the resulting wave is the smallest.
  - Then $\cos kx_1 = -\cos kx_2$ and $\sin kx_1 = -\sin kx_2$.

Constructive Interference

$$kx_1 = kx_2 + 2n\pi$$

$$\frac{2\pi}{\lambda}x_1 = \frac{2\pi}{\lambda}x_2 + 2n\pi$$

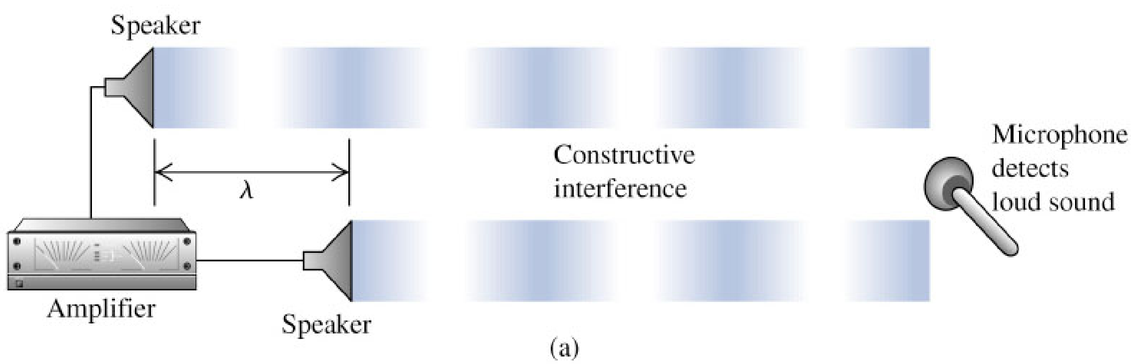
$$x_1 - x_2 = n\lambda$$

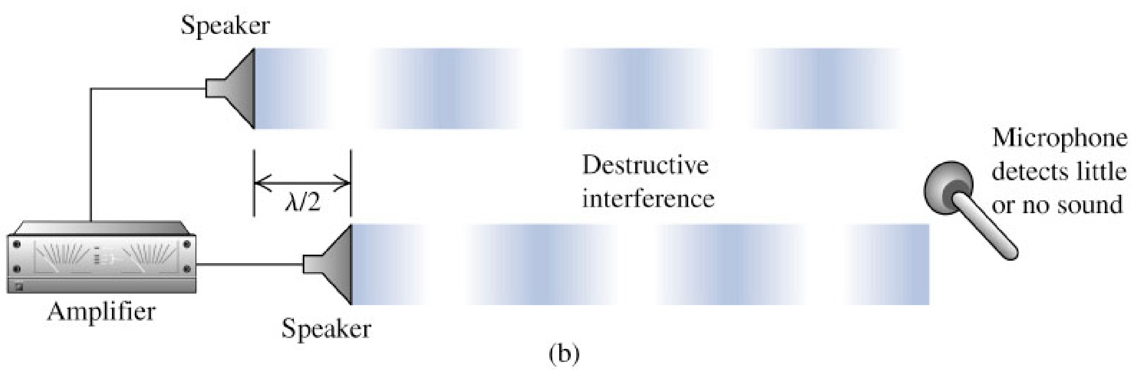
Destructive Interference

$$kx_1 = kx_2 + (2n+1)\pi$$

$$\frac{2\pi}{\lambda}x_1 = \frac{2\pi}{\lambda}x_2 + (2n+1)\pi$$

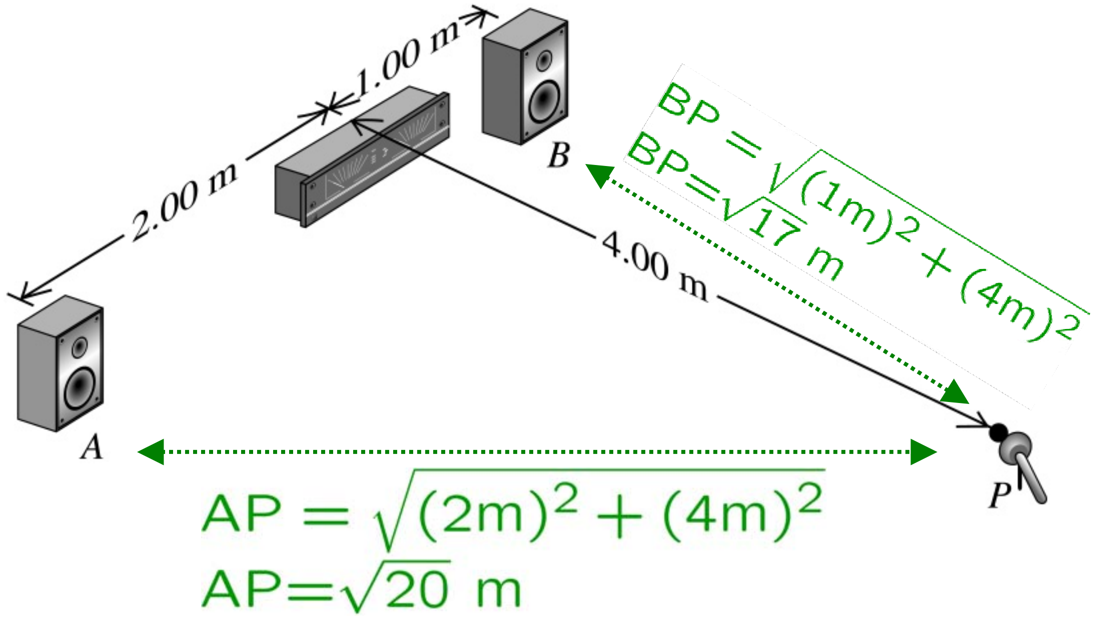
$$x_1 - x_2 = \frac{2n+1}{2}\lambda$$





- Constructive interference occurs when the difference in path length between the two waves is <u>equal to an integer number of wavelengths</u>.
- Destructive interference when the difference in path length is <u>equal to a half-integer number of wavelengths</u>.
- CAREFUL: this applies if
  - The two waves have the same wavelength.
  - The two waves are emitted in phase.
- What would happen if they were emitted (say) 180^o out of phase?

Back to our original problem:



The waves are generated in phase; v=350 m/sec

Constructive interference:

AP-BP=nλ

λ = v/f

→ f = nv / (AB-BP)

f = n 350/0.35 Hz

f = 1,2,3,... kHz

Destructive interference:

AP-BP=nλ/2 (n odd)

λ = v/f

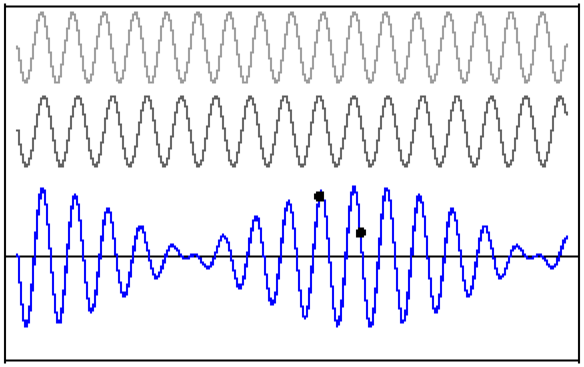
→ f = nv / [2(AB-BP)]

f = n 350/0.70 Hz

f = 0.5, 1.5, 2,5 kHz

Beats

- Consider interference between two sinusoidal waves with similar, but not identical, frequencies:



- The resulting wave looks like a single sinusoidal wave with a varying amplitude between some maximum and zero.
- The intensity variations are called <u>beats</u>, and the frequency with which these beats occur is called the <u>beat frequency</u>.

Beats, mathematical representation

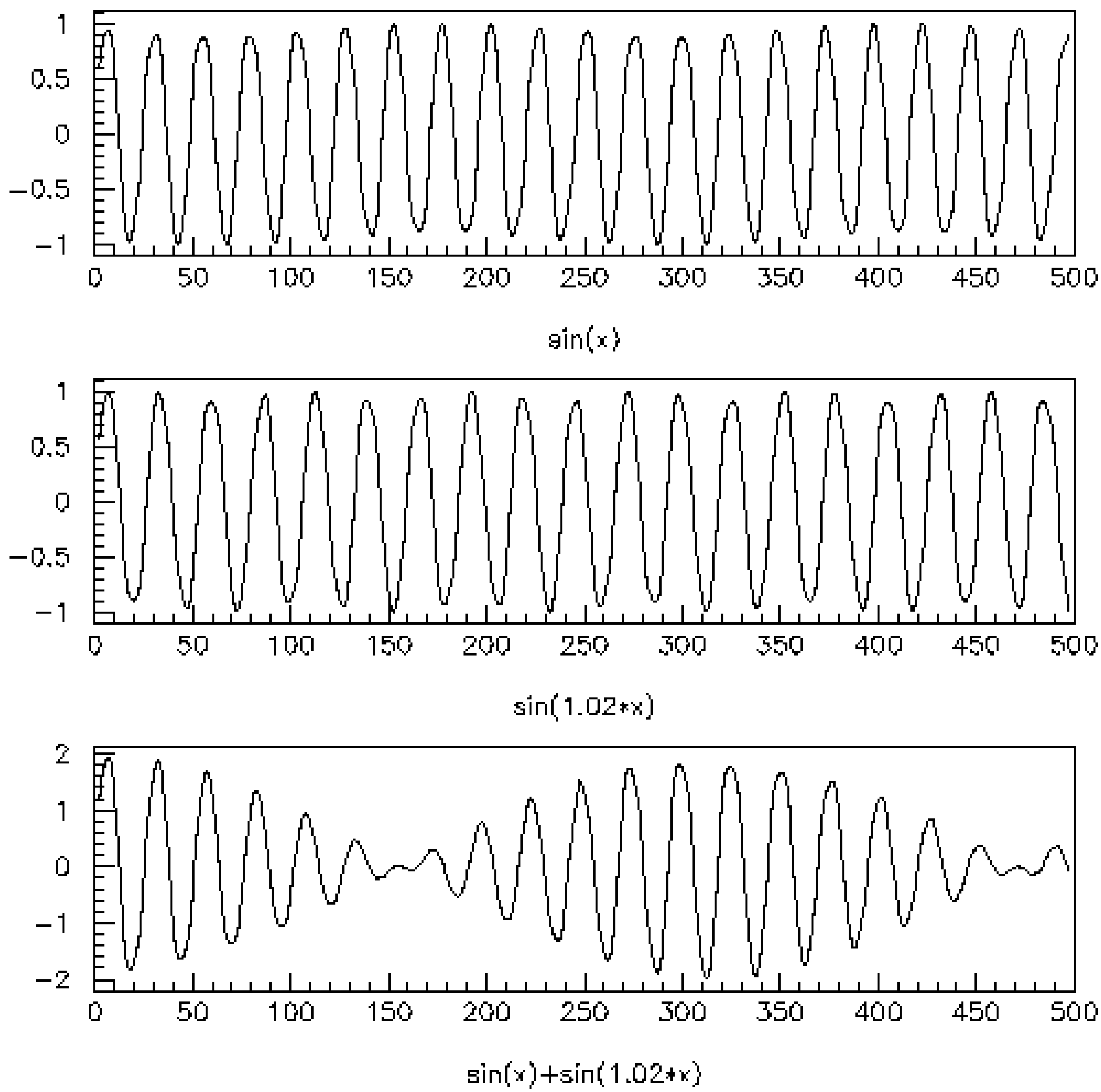
- Consider two waves, equal amplitudes, different frequencies:
  - $y_1(x,t) = A\cos(2\pi f_1 t - k_1 x)$
  - $y_2(x,t) = A\cos(2\pi f_2 t - k_2 x)$
- Look at the total displacement at some point, say x=0.
  - $y(0,t) = y_1(0,t) + y_2(0,t) = A\cos(2\pi f_1 t) + A\cos(2\pi f_2 t)$
- Trig identity:
  - $\cos A + \cos B = 2\cos[(A-B)/2]\cos[(A+B)/2]$
- This gives
  - $y(0,t) = 2A\cos[\tfrac{1}{2}(2\pi)(f_1 - f_2)t]\cos[\tfrac{1}{2}(2\pi)(f_1 + f_2)t]$

$$y(0,t) = 2A\cos[\tfrac{1}{2}(2\pi)(f_1-f_2)t]\ \cos[\tfrac{1}{2}(2\pi)(f_1+f_2)t]$$

An amplitude term which oscillates with frequency $\frac{1}{2}(f_1-f_2)$. If $f_1 \approx f_2$ then f_1-f_2 is small and the amplitude varies slowly.

A sinusoidal wave term with frequency $f = \frac{1}{2}(f_1 + f_2)$.

Beat frequency is $\frac{1}{2}|(f_1 - f_2)|$


1
0.5
0
−0.5
−1
0 50 100 150 200 250 300 350 400 450 500
sin(x)
1
0.5
0
−0.5
−1
0 50 100 150 200 250 300 350 400 450 500
sin(1.02*x)
2
1
0
−1
−2
0 50 100 150 200 250 300 350 400 450 500
sin(x)+sin(1.02*x)

Example problem

- While attempting to tune the note C at 523 Hz, a piano tuner hears 2 beats/sec.
  - (a) What are the possible frequencies of the string?
  - (b) When she tightens the string a little, she hears 3 beats/sec. What is the frequency of the string now?
  - (c) By what percentage should the tuner now change the tension in the string to "fix" it?

(a) $f_{beat} = ½ |f_C – f_{piano}|$

$f_{beat} = 2$ Hz and f_C=523 Hz

→ f_{piano} = 527 or 519 Hz

(b) f_{beat} = 3 Hz

→ f_{piano} = 529 or 517 Hz

To decide which of the two, use the fact that the tension increased

For string fixed at both end, we had f=nv/2L, i.e., f proportional to v.

But $v^2=F/\mu$ → higher F → higher v → higher f → f_{piano} = 529 Hz

(c) The frequency is f_{piano} = 529 Hz, we want f_C = 523 Hz.

frequency is proportional to v (f=nv/2L)

velocity is proportional to square root of tension ($v^2 = F/\mu$)

→ frequency is proportional square root of tension

$$\frac{f_{\text{piano}}}{f_C} = \sqrt{\frac{F_{\text{piano}}}{F_C}}$$

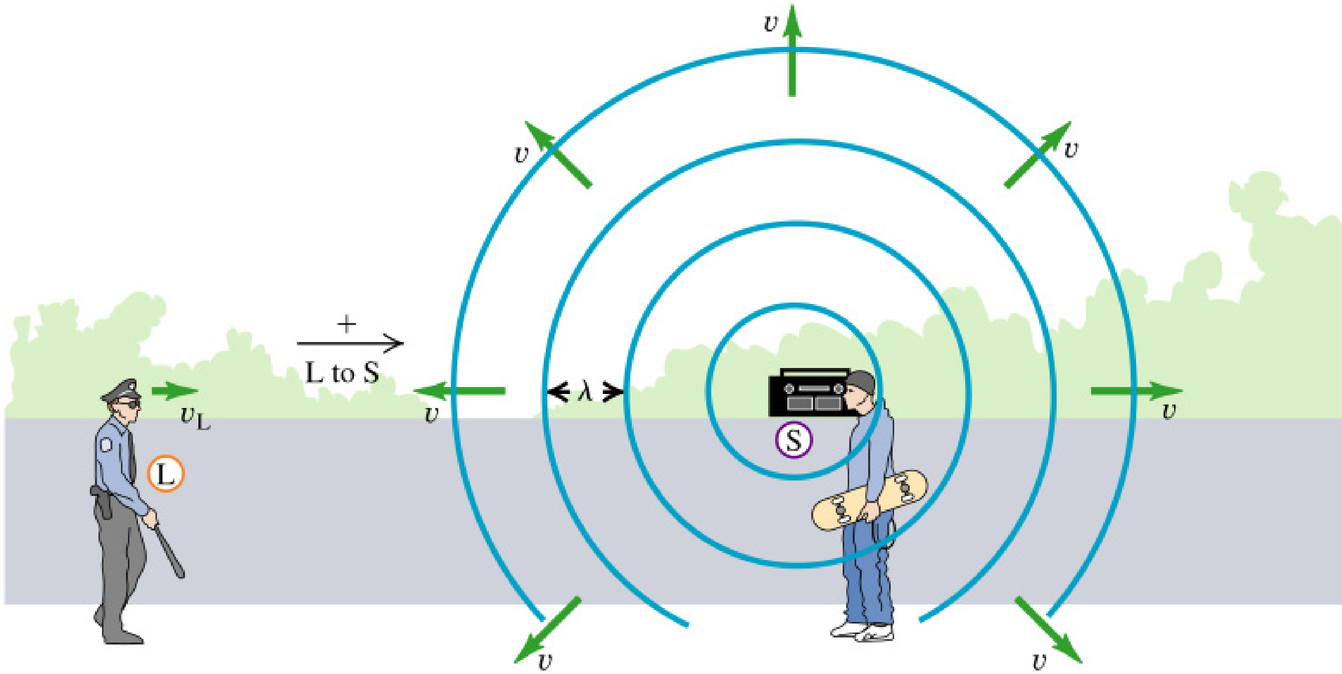
$$\frac{F_{\text{piano}}}{F_C} = \frac{f^2_{\text{piano}}}{f^2_C} = \frac{529^2}{523^2} = 1.023$$

<u>The tension must be changed (loosened) by 2.3%</u>

Doppler Effect

- When a car goes past you, the pitch of the engine sound that you hear changes.
- Why is that?
- This must have something to do with the velocity of the cars with respect to you (towards you vs. away from you).
  - Unless it is because the driver is doing something "funny" like accelerating to try to run you over ☺

Consider listener moving towards sound sorce:

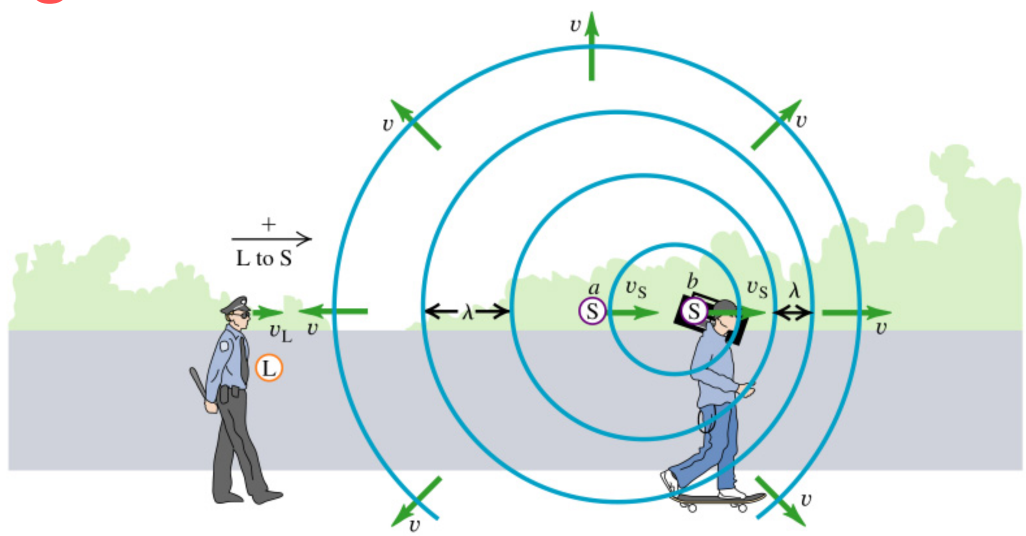


- Sound from source: velocity v, frequency f_s, wavelength λ, and v=λ f_s.
- The listener sees the wave crests approaching with velocity $v+v_L$.
- Therefore the wave crests arrive at the listener with frequency:

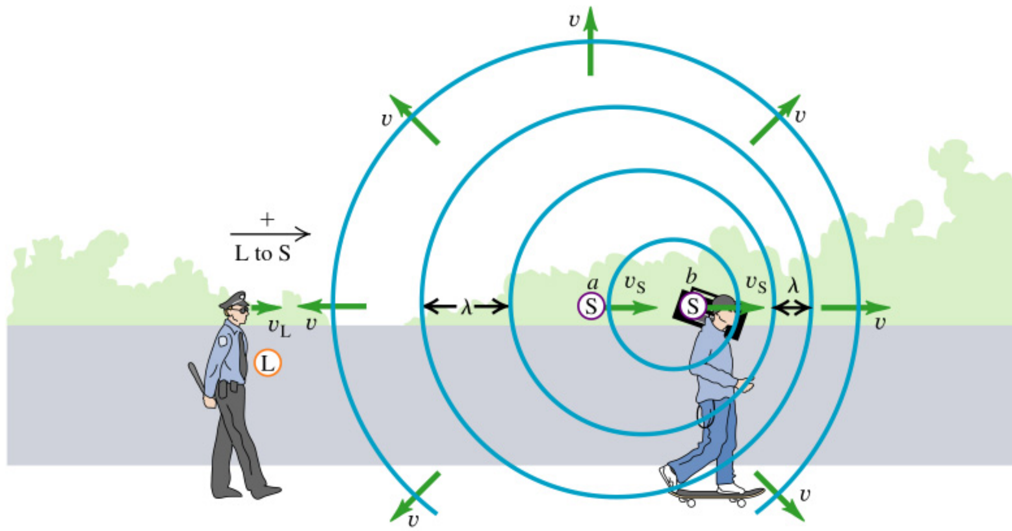
$$f_L = \frac{v+v_L}{v} f_S = (1 + \frac{v_L}{v}) f_S$$

→ The listener "perceives" a different frequency (Dopple shift)

Now imagine that the source is also moving:



- The wave speed relative to the air is still the same (v).
- The time between emissions of subsequent crests is the period $T=1/f_s$.
- Consider the crests in the direction of motion of the source (to the right)
  - ➢ A crest emitted at time t=0 will have travelled a distance vT at t=T
  - ➢ In the same time, the source has travelled a distance v_sT.
  - ➢ At t=T the subsequent crest is emitted, and this crest is at the source.
  - ➢ So the distance between crests is $vT-v_sT=(v-v_s)T$.
  - ➢ But the distance between crests is the wavelength
    - ❖ $\lambda = (v-v_s)T$
  - ➢ But $T=1/f_s$ → $\lambda = (v-v_s)/f_s$ (in front of the source)



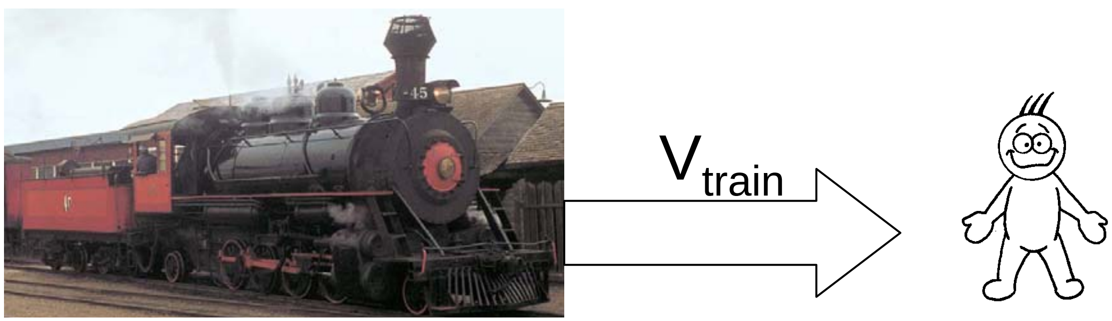
- $\lambda = (v - v_s)/f_s$ (in front of the source)
- Clearly, behind the source $\lambda = (v + v_s)/f_s$
- For the listener, $f_L = (v + v_L)/\lambda$
    - Since he sees crests arriving with velocity $v + v_L$

$$\rightarrow \quad f_L = \frac{v + v_L}{v + v_S} f_S$$

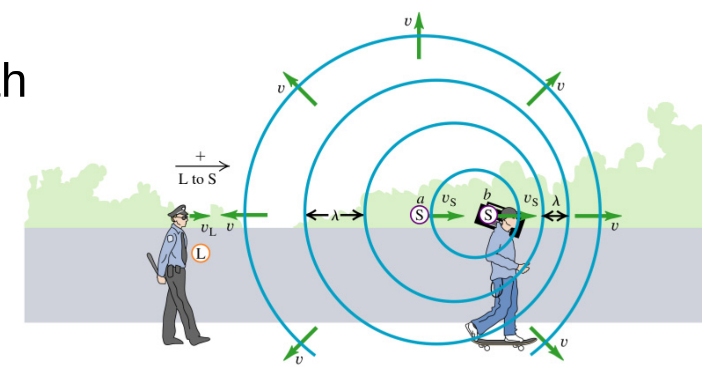
Sample problem

- A train passes a station at a speed of 40 m/sec. The train horn sounds with f=320 Hz. The speed of sound is v=340 m/sec.
  - What is the change in frequency detected by a person on the platform as the train goes by.

Approaching train:



Compare with



$$f_L = \frac{v + v_L}{v + v_S} f_S$$

In our case v_L=0 (the listener is at rest) and the source (train) is mowing <u>towards</u> rather than <u>away from</u> the listener.

→ I must switch the sign of v_S

$$f_L = \frac{v+v_L}{v+v_S} f_S \quad \text{becomes} \quad f_{L1} = \frac{v}{v-v_{\text{train}}} f$$

When the train moves away:

Clearly I need to switch the sign of v_{train}: $$f_{L2} = \frac{v}{v+v_{\text{train}}} f$$

$$\Delta f = f_{L1} - f_{L2} = \text{.. (algebra) ..} = -2\frac{v v_{\text{train}}}{v^2 - v_{\text{train}}^2} f = 76 \text{ Hz}$$