Fall 2004 Physics 3
Tu-Th Section

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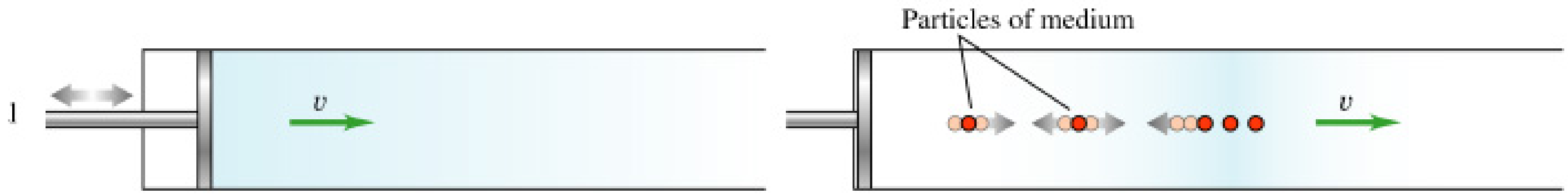
Web page:
http://hep.ucsb.edu/people/claudio/ph3-04/

Sound

- Sound = longitudinal wave in a medium.

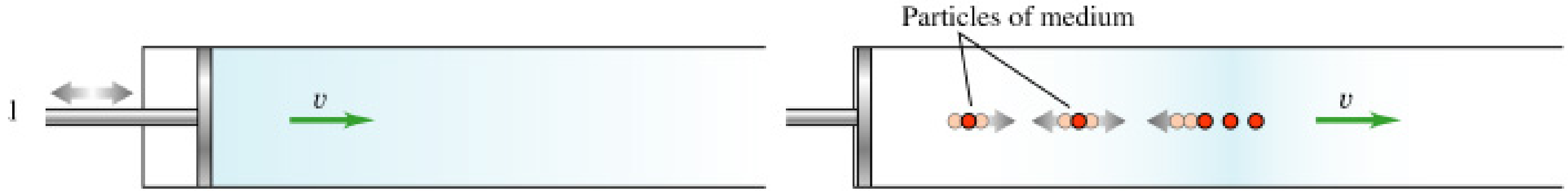


- The medium can be anything:
  - a gas, e.g., air
  - a liquid, e.g., water
  - even a solid, e.g., the walls
- The human hear is "sensitive" to frequency range ~ 20-20,000 Hz
  - "audible range"
  - higher frequency: "ultrasounds"
  - lower frequency: "infrasound"

How do we describe such a wave?

e.g., sinusoidal wave, traveling to the right

– idealized one dimensional transmission, like a sound wave within a pipe.

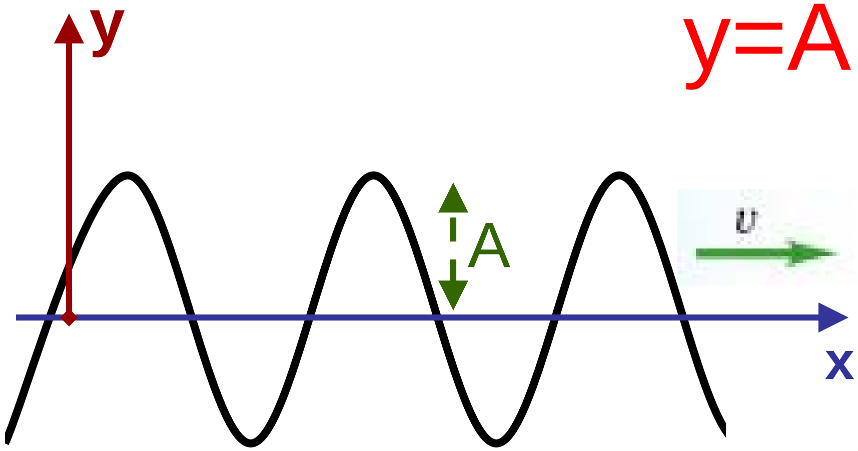


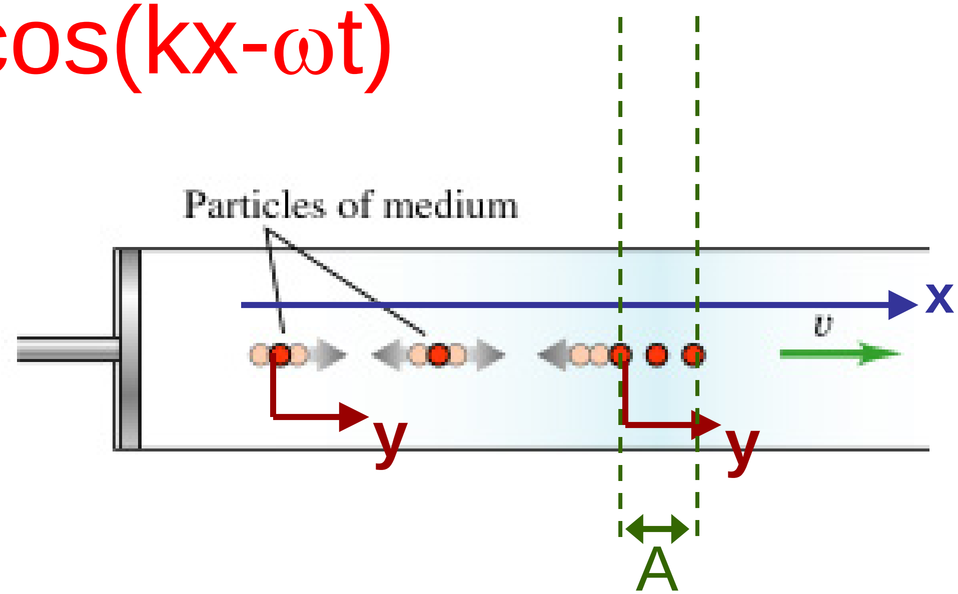
Mathematically, just like <u>transverse</u> wave on string:

$$y(x,t) = A\cos(kx-\omega t)$$

But the meaning of this equation is quite different for a wave on a string and a sound wave!!

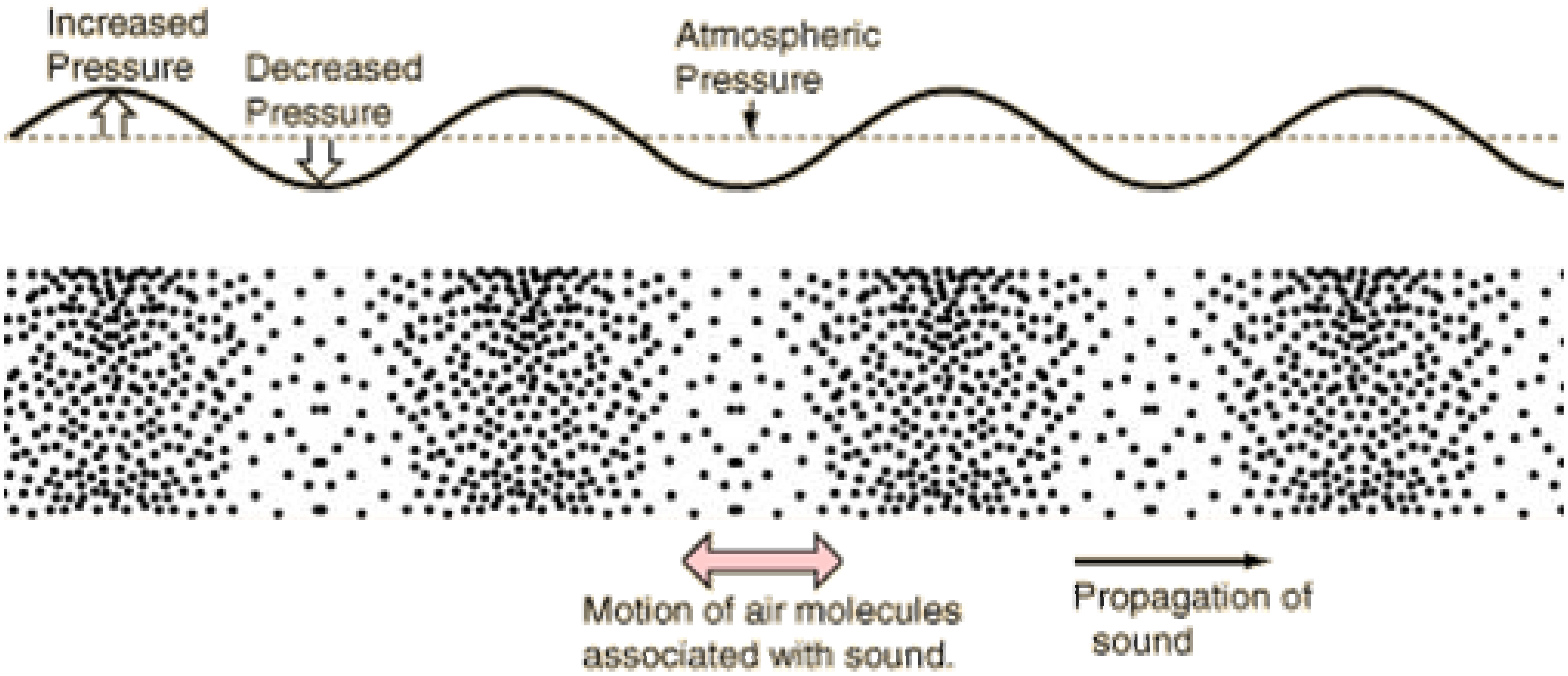
$y = A\cos(kx - \omega t)$



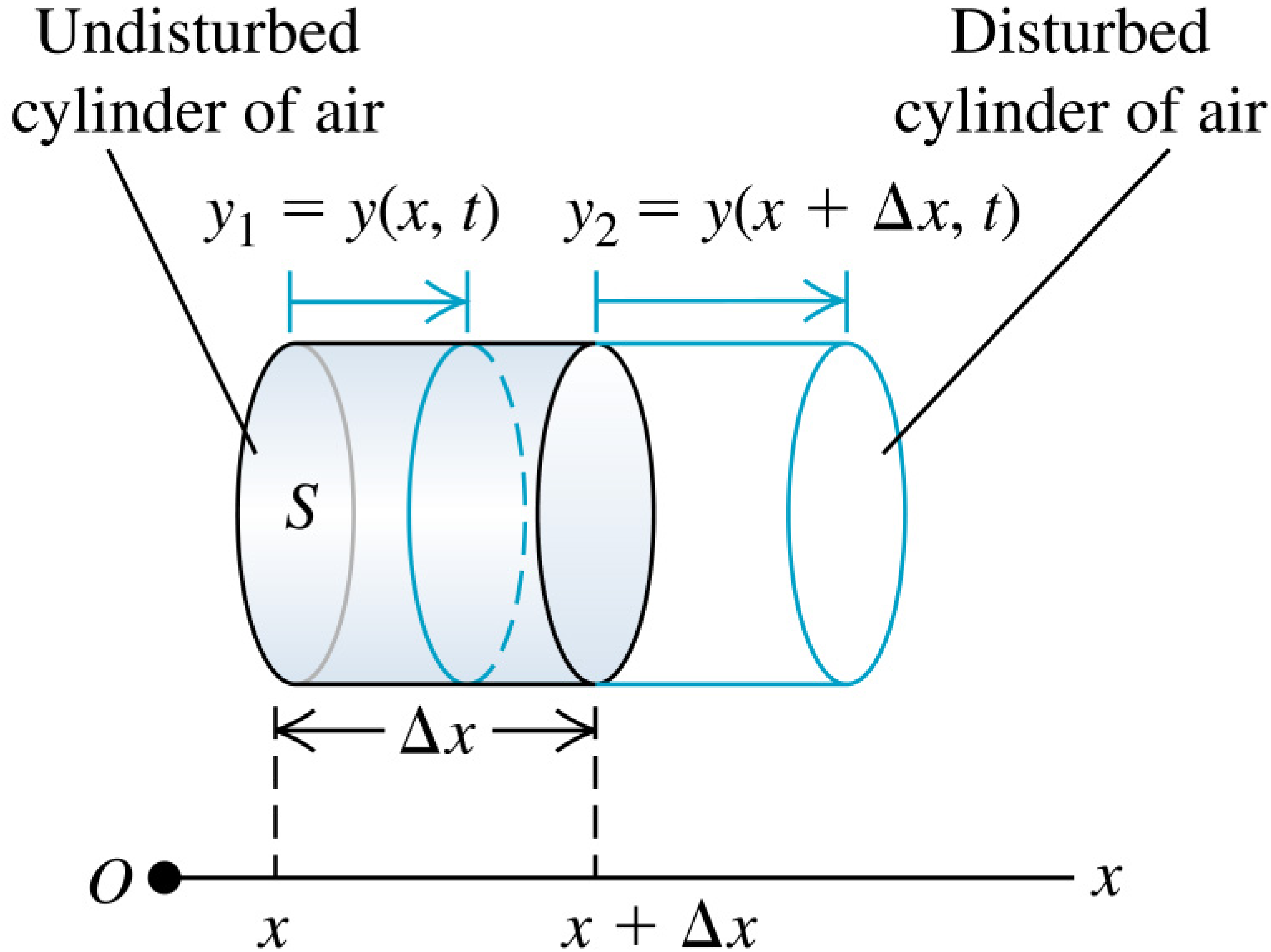


- "x" = the direction of propagation of the wave
- "y(x,t)" for string is the displacement at time t of the piece of string at coordinate x <u>perpendicular</u> to the direction of propagation.
- "y(x,t)" for sound is the displacement at time t of a piece of fluid at coordinate x <u>parallel</u> to the direction of propagation.
- "A" is the amplitude, i.e., the <u>maximum</u> value of displacement.

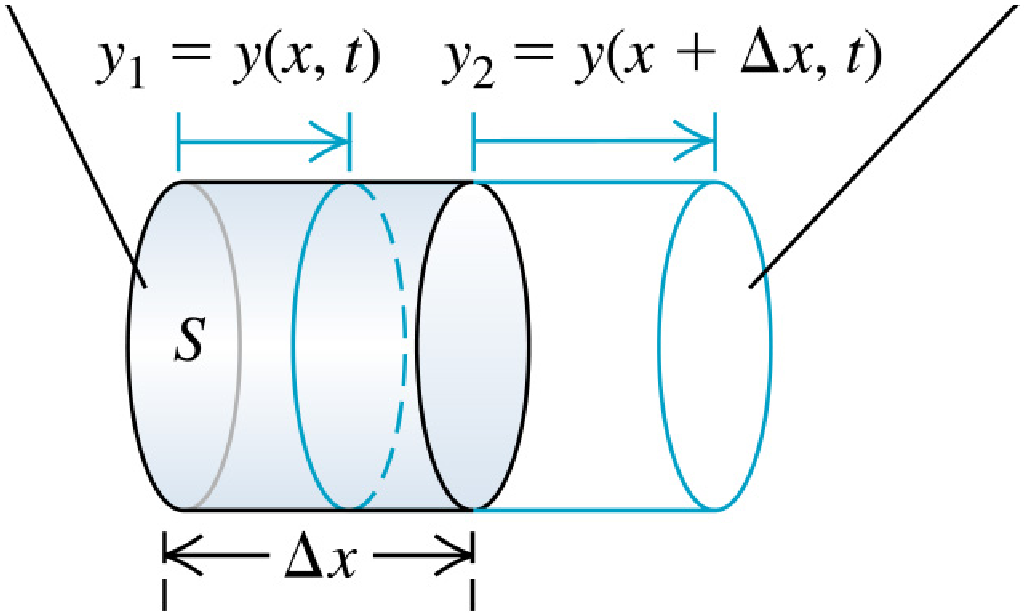
- It is more convenient to describe a sound wave not in term of the displacement of the particles in the fluid, but rather in terms of the pressure in the fluid.
- Displacement and pressure are clearly related:



- In a sound wave the pressure fluctuates around the equilibrium value .
  - For air, this would normally be the atmospheric pressure.
  - $p_{absolute}(x,t) = p_{atmospheric} + p(x,t)$
- Let's relate the fluctuating pressure p(x,t) to the displacement defined before.



Undisturbed cylinder of air
Disturbed cylinder of air
$y_1 = y(x, t)$
$y_2 = y(x + \Delta x, t)$
S
Δx
O
x
x
$x + \Delta x$



ΔV = change in volume of cylinder:

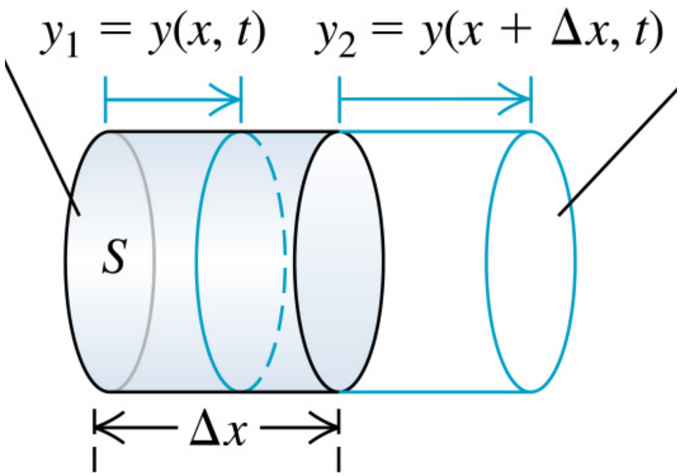
$\Delta V = S \cdot (y_2 - y_1) = S \cdot [\, y(x+\Delta x,t) - y(x,t)\,]$

Original volume:

$$V = S \cdot \Delta x$$

Fractional volume change:

$$\frac{\Delta V}{V} = \frac{S \cdot [y(x+\Delta x,t) - y(x,t)]}{S \cdot \Delta x}$$



$$\frac{\Delta V}{V} = \frac{S\cdot[y(x+\Delta x,t)-y(x,t)]}{S\cdot\Delta x}$$

Now make Δx infinitesimally small

- Take limit Δx → 0
- ΔV becomes dV

$$\frac{dV}{V} = \lim_{\Delta x \to 0} \frac{S\cdot[y(x+\Delta x,t)-y(x,t)]}{S\cdot\Delta x} = \frac{\partial y}{\partial x}$$

Bulk modulus: *B* (chapter 11) $B = \frac{-p}{dV/V}$

$$p(x,t) = -B\frac{\partial y(x,t)}{\partial x}$$

$$p(x,t) = -B\frac{\partial y(x,t)}{\partial x}$$

- This is the equation that relates the <u>displacement</u> of the particles in the fluid (<u>y</u>) to the <u>fluctuation in pressure</u> (<u>p</u>).
- They are related through the <u>bulk modulus</u> (<u>B</u>)

Definition of bulk modulus:

$$\frac{dV}{V} = -B \cdot p$$

Fractional volume change

Bulk modulus

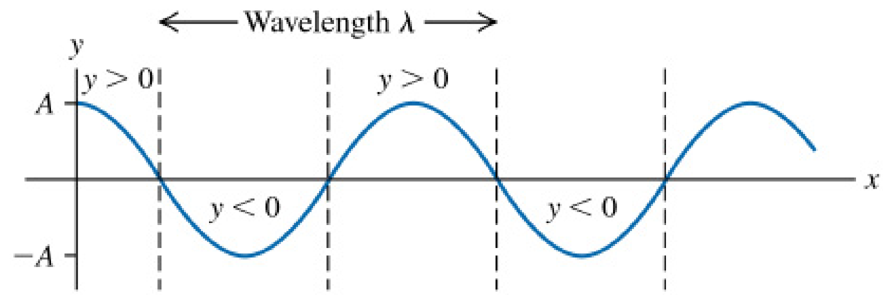
Applied pressure

Sinusoidal wave

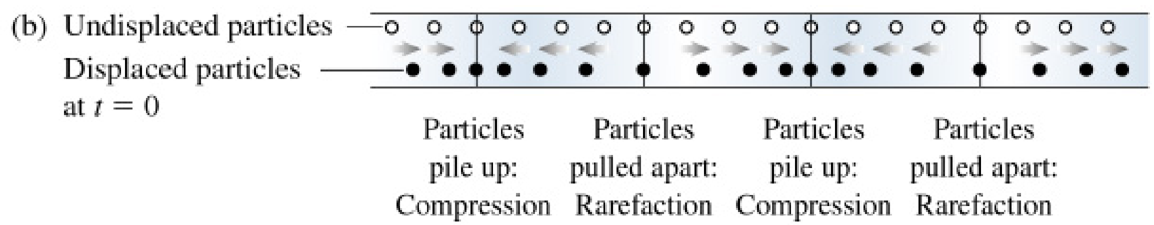
$$y(x,t) = A\cos(kx - \omega t)$$

and

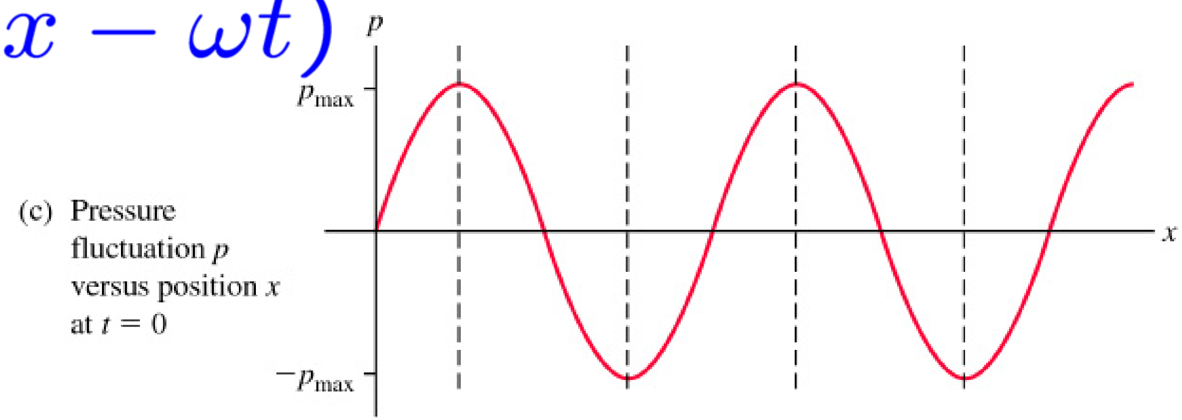
$$p(x,t) = -B\frac{\partial y(x,t)}{\partial x}$$



This then gives



$$p(x,t) = (A \cdot B \cdot k)\sin(kx - \omega t)$$



Displacement (y) and pressure (p) oscillations are 90^o out of phase

Example

1 Pa = 1 N/m^2

- "Typical" sinusoidal sound wave, maximum pressure fluctuation 3×10^{-2} Pa.
  - Compare $p_{atmospheric} = 1 \times 10^5$ Pa
- Find maximum displacement at f=1 kHz given that B=1.4 10^5 Pa, v=344 m/sec

$$y(x,t) = A\cos(kx - \omega t)$$

$$p(x,t) = (A \cdot B \cdot k)\sin(kx - \omega t)$$

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$$p(x,t) = (A \cdot B \cdot k)\sin(kx - \omega t)$$

$$y_{max} = A \qquad \text{and}$$

$$p_{max} = A \cdot B \cdot k = y_{max} \cdot B \cdot k$$

$$\rightarrow \quad y_{max} = \frac{p_{max}}{B \cdot k}$$

We have p_{max} (= 3×10^{-2} Pa) and B (= 1.4×10^{5} Pa)

But what about k?

We have f and v → $\lambda = v/f$

And $k = 2\pi/\lambda$

→ $k = 2\pi/(v/f) = 2\pi\, f/v$

$$y_{max} = \frac{p_{max}}{B \cdot k} = \frac{p_{max} \cdot v}{2 \cdot \pi \cdot B \cdot f}$$

Now it is just a matter of plugging in numbers:

$$y_{max} = \frac{p_{max}}{B \cdot k} = \frac{p_{max} \cdot v}{2 \cdot \pi \cdot B \cdot f}$$

$$y_{max} = \frac{(3 \cdot 10^{-2}\ \mathrm{Pa})(344\ \mathrm{m/sec})}{2 \cdot \pi \cdot (1.4 \cdot 10^{5}\ \mathrm{Pa}) \cdot (10^{3}\ \mathrm{sec}^{-1})}$$

$$y_{max} = 1.2 \times 10^{-8}\ \mathrm{m} \quad \mathrm{TINY!!!}$$

Aside: Bulk Modulus, Ideal Gas

$$B = \frac{-p}{dV/V} = -V\frac{P}{dV}$$

Careful: *P* here is not the pressure, but its deviation from equilibrium, e.g., the additional pressure that is applied to a volume of gas originally at atmospheric pressure.

Better notation *P* → *dP*

$$B = -V\frac{dP}{dV}$$

For adiabatic process, ideal gas: $PV^{\gamma} = \text{const.}$

$$\gamma = \frac{C_P}{C_V}$$

$\gamma \sim 1.7$ monoatomic molecules (He, Ar,..)
$\gamma \sim 1.4$ diatomic molecules (O_2, N_2,..)
$\gamma \sim 1.3$ polyatomic molecules (CO_2,..)

Heat capacities at constant *P* or *V*

$$PV^{\gamma} = \text{const.}$$

Differentiate w.r.t. V:

$$V^{\gamma}\frac{dP}{dV} + \gamma P V^{\gamma - 1} = 0$$

$$\rightarrow V\frac{dP}{dV} = -\gamma P$$

But bulk modulus $B = -V\frac{dP}{dV}$

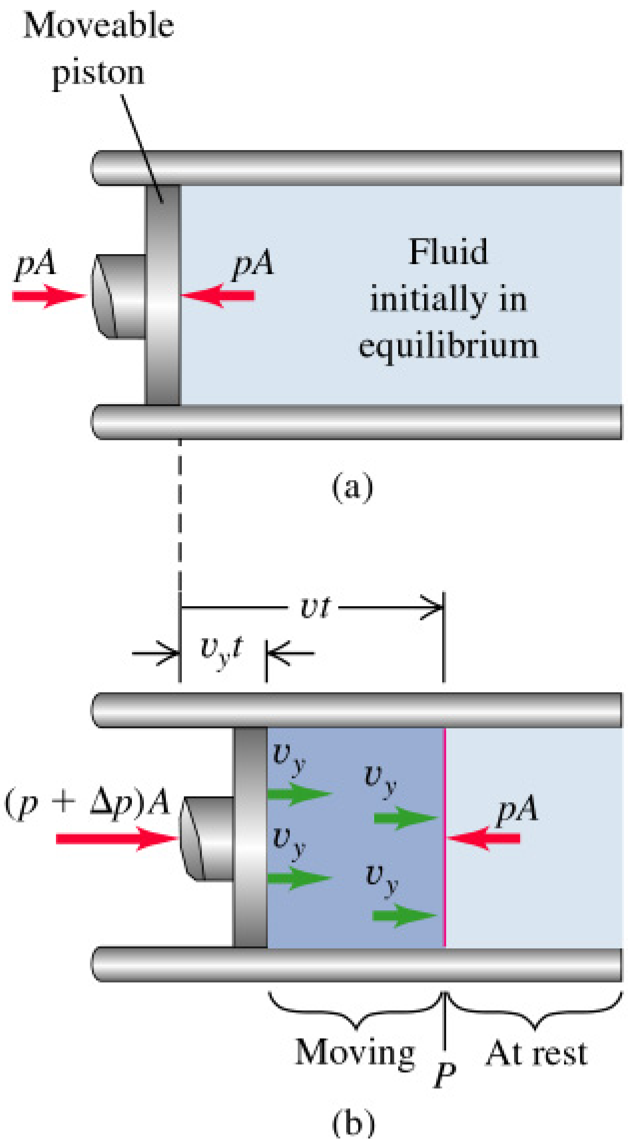
$$\rightarrow B = \gamma P$$

$$B = \gamma P \approx 1.4 \cdot P \text{ for air}$$

Note: the bulk modulus increases with pressure.

If we increase the pressure of a gas, it becomes harder to compress it further, i.e. the bulk modulus increases (makes intuitive sense)

Speed of sound



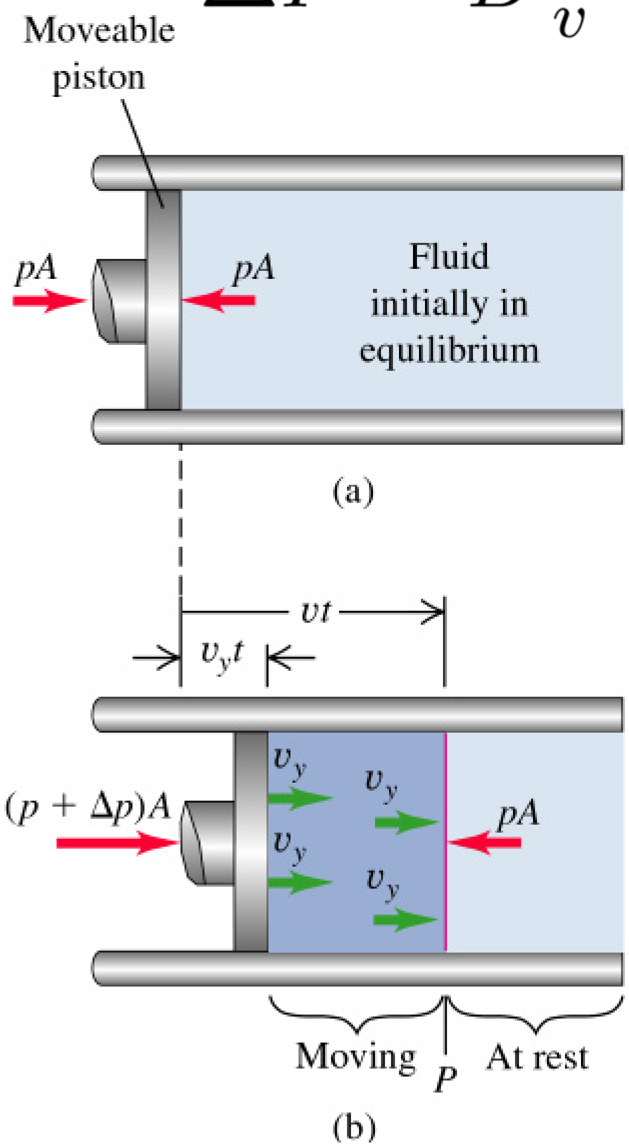
- At t=0 push the piston in with constant v_y
  This triggers wave motion in fluid
- At time t, piston has moved distance $v_y t$
- If v is the speed of propagation of the wave, i.e. the speed of sound, fluid particles up to distance vt are in motion
- Mass of fluid in motion $M=\rho Avt$
- Speed of fluid in motion is v_y
- Momentum of fluid in motion is $Mv_y=\rho Avtv_y$
- ΔP is the change in pressure in the region where fluid is moving

$$B = -V\frac{\Delta P}{\Delta V}$$

- $V = Avt$ and $\Delta V = -Av_y t$

$$\rightarrow \Delta P = B\frac{v_y}{v}$$

$$\Delta P = B\frac{v_y}{v}$$



Net force on fluid is
F = (p+Δ p)·A – p·A = Δp·A

This force has been applied for time t:

$$\text{Impulse} = F \cdot t = \Delta p \cdot A \cdot t$$

$$\text{Impulse} = B \cdot \frac{v_y}{v} \cdot A \cdot t$$

Impulse = change in momentum

Initial momentum = 0
Final momentum = $\rho A v t v_y$

$$\rho \cdot A \cdot v \cdot t \cdot v_y = B \cdot \frac{v_y}{v} \cdot A \cdot t$$

$$\rightarrow v = \sqrt{\frac{B}{\rho}}$$

Speed of sound: $v = \sqrt{\frac{B}{\rho}}$

Speed of waves on a string: $v = \sqrt{\frac{F}{\mu}}$

B and *F* quantify the restoring "force" to equilibrium.

ρ and μ are a measure of the "inertia" of the system.

Speed of sound: $v = \sqrt{\frac{B}{\rho}}$

Bulk modulus: $B = \gamma P$

R = gas constant
R ~ 8.3 J/mol K

One mole of ideal gas: $PV = RT$

$$\rightarrow B = \gamma \frac{RT}{V} \quad \rightarrow \quad v = \sqrt{\frac{\gamma RT}{V\rho}}$$

Since ρV = mass per mole = M:

$$v = \sqrt{\frac{\gamma RT}{M}}$$

$$v = \sqrt{\frac{\gamma RT}{M}}$$

A function of the gas and the temperature

For air:

- $\gamma = 1.4$
- M = 28.95 g/mol

$$\rightarrow v = 20 \text{ m/sec} \sqrt{T}$$

At T = 20^{o}C = 293^{o}K:

$$v = 343 \text{ m/sec} = 769 \text{ mph}$$

He: 1000 m/sec H_2: 1330 m/sec

Intensity

- The wave carries energy
- The intensity is the <u>time average</u> of the power carried by the wave crossing unit area.
- Intensity is measured in W/m^2

$$I = \frac{<P>}{S} = \frac{<dE/dt>}{S}$$

Intensity (cont.)

Sinusoidal sound wave:

$$y(x,t) = A\cos(kx - \omega t)$$

$$p(x,t) = B \cdot k \cdot A\sin(kx - \omega t)$$

Power = Force × velocity

Particle velocity
NOT
wave velocity

Intensity = <Power>/Area

= <Force × velocity>/Area

= < Pressure × velocity>

$$I = < p(x,t) \cdot v_y(x,t) > = < p(x,t) \cdot \frac{\partial y}{\partial t} >$$

$$I = < [B \cdot k \cdot A\sin(kx - \omega t)][\omega A\sin(kx - \omega t)] >$$

$$I = B \cdot \omega \cdot k \cdot A^2 < \sin^2(kx - \omega t) >$$

Intensity (cont.)

½

$$I = B \cdot \omega \cdot k \cdot A^2 \; < \sin^2(kx - \omega t) >$$

$$I = \frac{1}{2} \cdot B \cdot \omega \cdot k \cdot A^2$$

Now use ω = vk and v^2=B/ρ:

$$I = \frac{1}{2}\sqrt{\rho B}\,\omega^2 A^2$$

Or in terms of p_{max} = BkA:

$$I = \frac{\omega p_{max}^2}{2Bk} = \frac{v p_{max}^2}{2B}$$

And using v^2=B/ρ:

$$I = \frac{p_{max}^2}{2v\rho} = \frac{p_{max}^2}{2\sqrt{\rho B}}$$

Decibel

- A more convenient sound intensity scale
  - more convenient than W/m^2.
- The sound intensity level β is defined as

$$\beta = (10\ \text{dB}) \log \frac{I}{I_0}$$

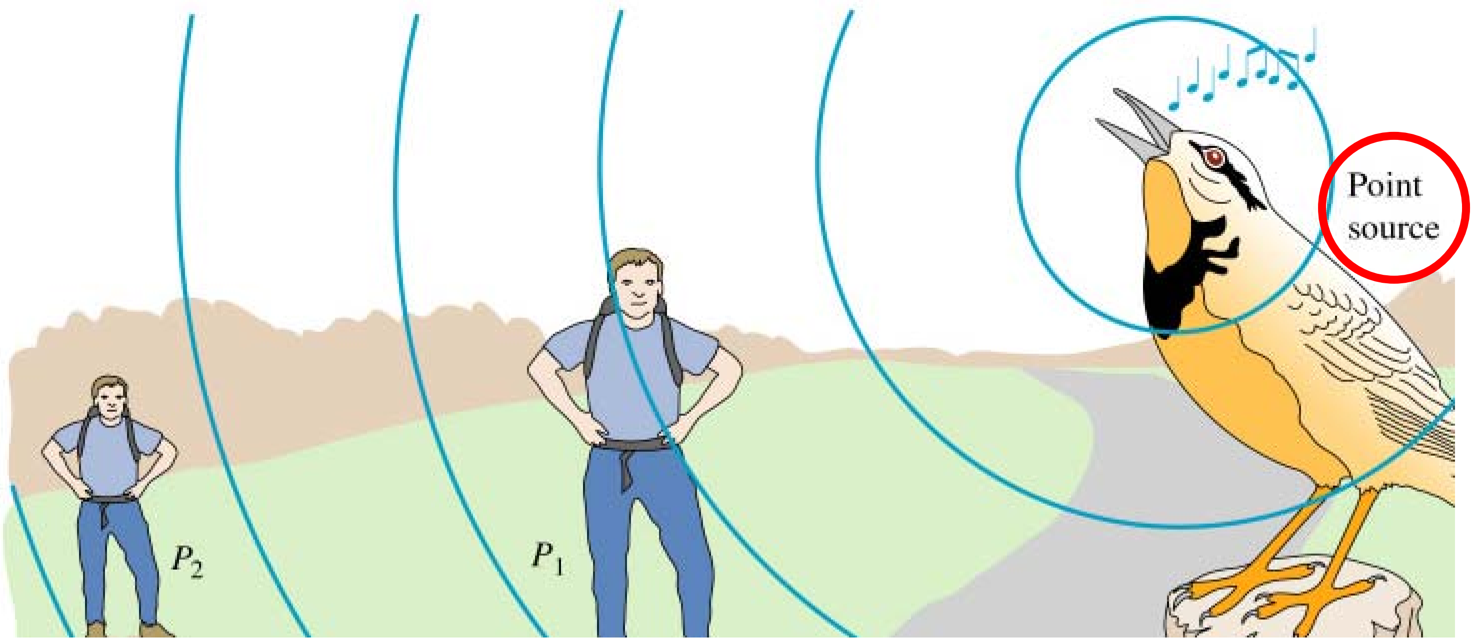
- Where $I_0 = 10^{-12}\ W/m^2$
  - Approximate hearing threshold at 1 kHz
- It's a log scale
  - A change of 10 dB corresponds to a factor of 10

Table 16.2 Sound Intensity Levels from Various Sources (Representative Values)

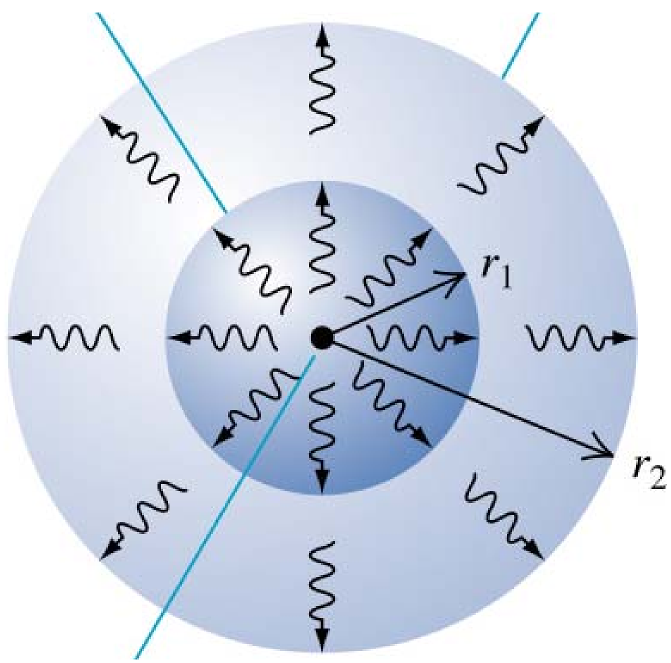
Source or Description of Sound	Sound Intensity Level, β (dB)	Intensity, I (W/m^2)
Military jet aircraft 30 m away	140	10^2
Threshold of pain	120	1
Riveter	95	3.2×10^{-3}
Elevated train	90	10^{-3}
Busy street traffic	70	10^{-5}
Ordinary conversation	65	3.2×10^{-6}
Quiet automobile	50	10^{-7}
Quiet radio in home	40	10^{-8}
Average whisper	20	10^{-10}
Rustle of leaves	10	10^{-11}
Threshold of hearing at 1000 Hz	0	10^{-12}

Example

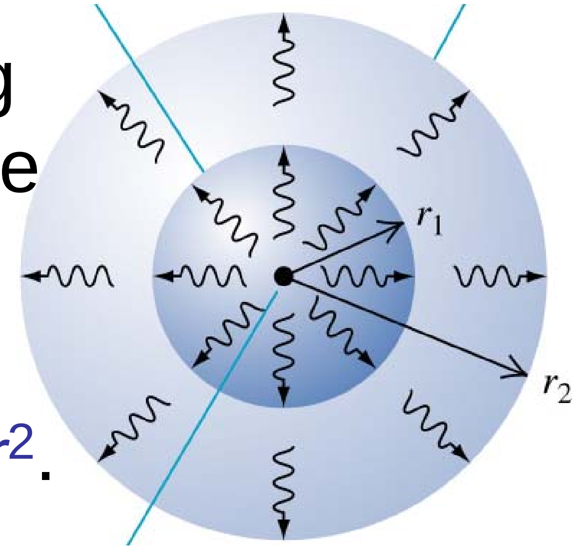
- Consider a sound source.
- Consider two listeners, one of which twice as far away as the other one.
- What is the difference (in decibel) in the sound intensity perceived by the two listeners?



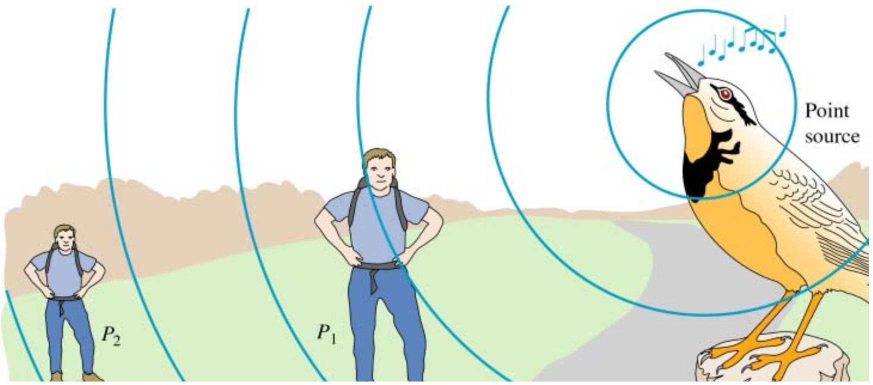
- To answer the question we need to know something about the directionality of the emitted sound.
- Assume that the sound is emitted uniformly in all directions.



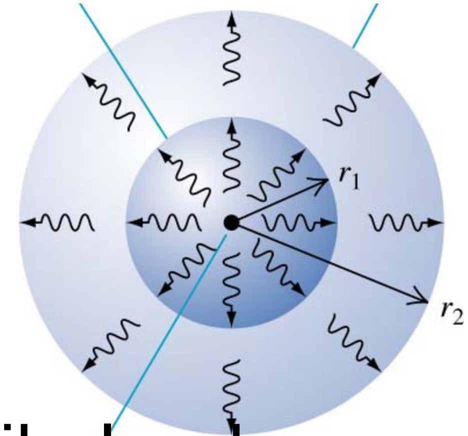
- How does the intensity change with r (distance from the source)?
- The key principle to apply is <u>conservation of energy</u>.
- The <u>total</u> energy per unit time crossing a spherical surface at r_1 must equal the total energy crossing a spherical surface at r_2
- Surface area of sphere of radius r: $4\pi r^2$.
- Intensity = Energy/unit time/unit area.
- $4\pi r_1^2\, I_1 = 4\pi r_2^2\, I_2$.



$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$



$$\frac{I_1}{I_2} = \frac{r_2^2}{r_1^2}$$



The original question was: change in decibels when second "listener" is twice as far away as first one.

$r_2 = 2r_1$

$I_1/I_2 = 4$

Definition of decibel : $\beta = (10\text{ dB})\log\frac{I}{I_0}$

$$\beta_2 - \beta_1 = (10\text{dB})(\log\frac{I_2}{I_0} - \log\frac{I_1}{I_0})$$

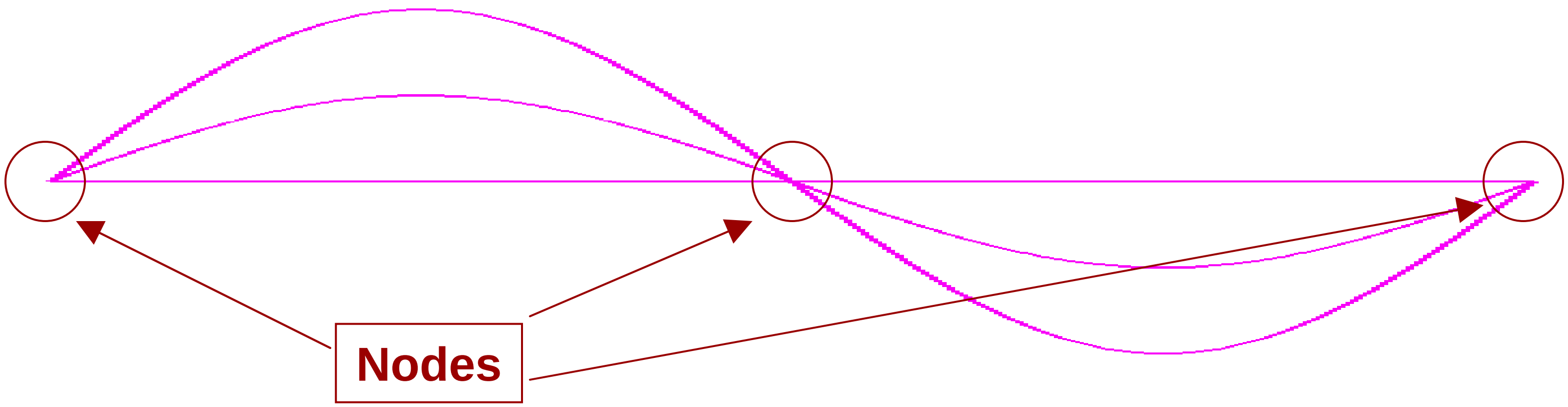
$$\beta_2 - \beta_1 = (10\text{dB})(\log I_2 - \log I_0 - \log I_1 + \log I_0)$$

$$\beta_2 - \beta_1 = (10\text{dB})(\log I_2 - \log I_1) = (10\text{dB})\log\frac{I_2}{I_1}$$

$$\beta_2 - \beta_1 = (10\text{dB})\log\frac{1}{4} = -6.0\text{ dB}$$

Standing sound waves

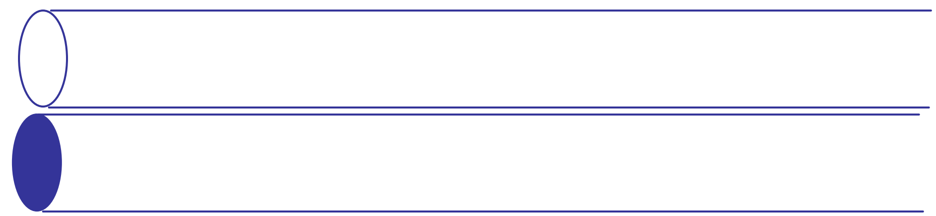
Recall standing waves on a string



- A standing wave on a string occurs when we have interference between wave and its reflection.
- The reflection occurs when the medium changes, e.g., at the string support.

- We can have sound standing waves too.
- For example, in a pipe.
- Two types of boundary conditions:

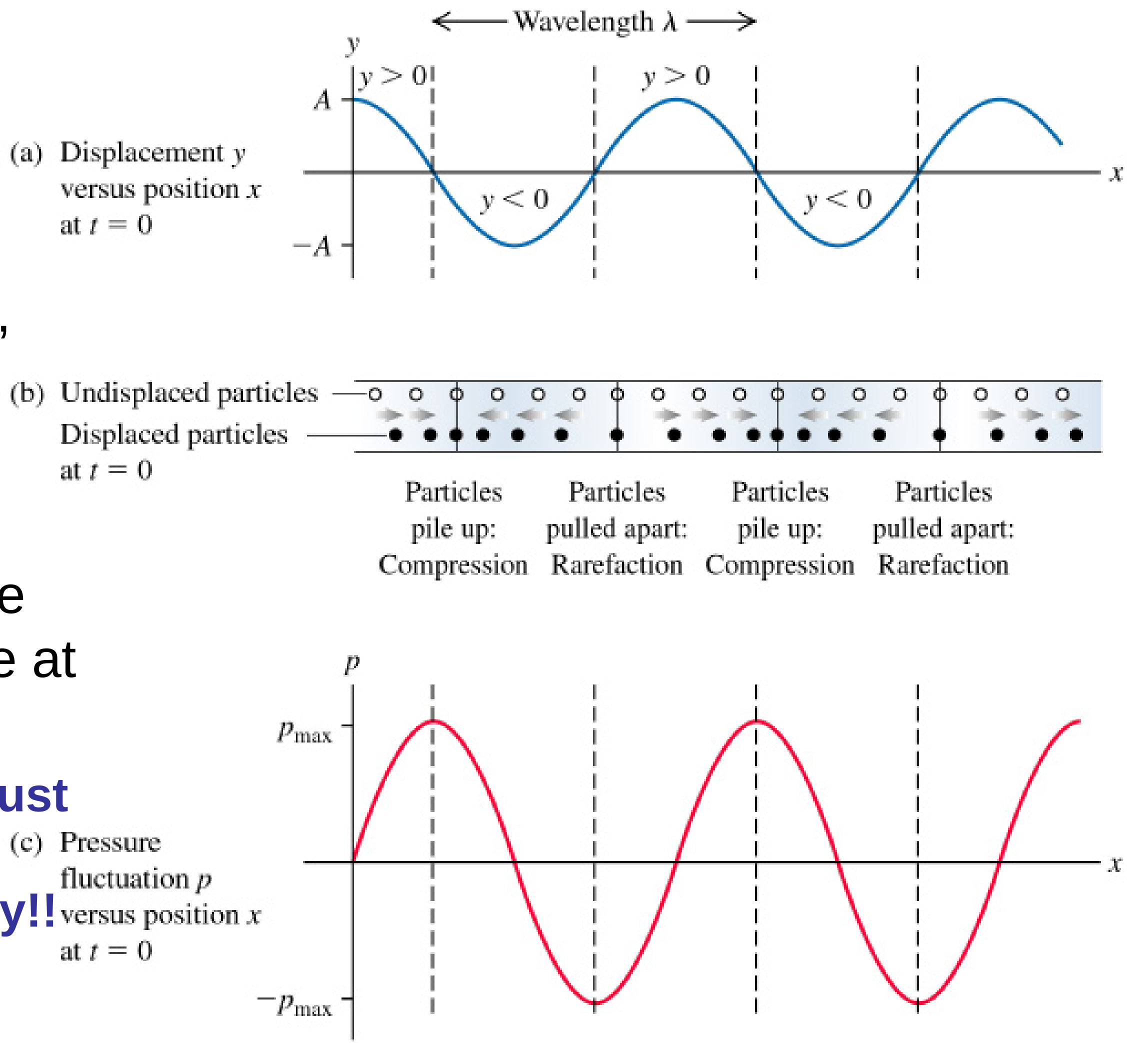
1. Open pipe
2. Closed pipe



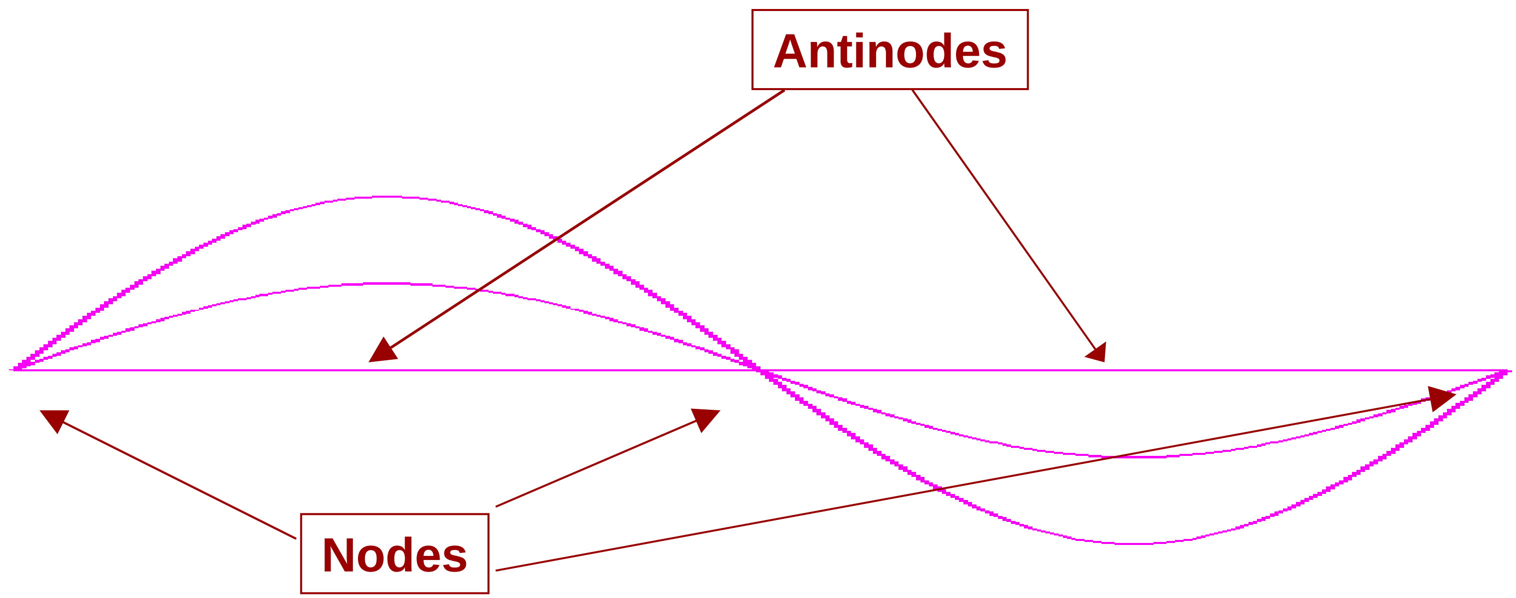
- In an closed pipe the boundary condition is that the displacement is zero at the end
    - Because the fluid is constrained by the wall, it can't move!
- In an open pipe the boundary condition is that the pressure fluctuation is zero at the end
    - Because the pressure is the same as outside the pipe (atmospheric)

Remember:

- Displacement and pressure are out of phase by 90º.
- When the displacement is 0, the pressure is ± p_{max}.
- When the pressure is 0, the displacement is ± y_{max}.
- So the nodes of the pressure and displacement waves are at different positions
  - **It is still the same wave, just two different ways to describe it mathematically!!**



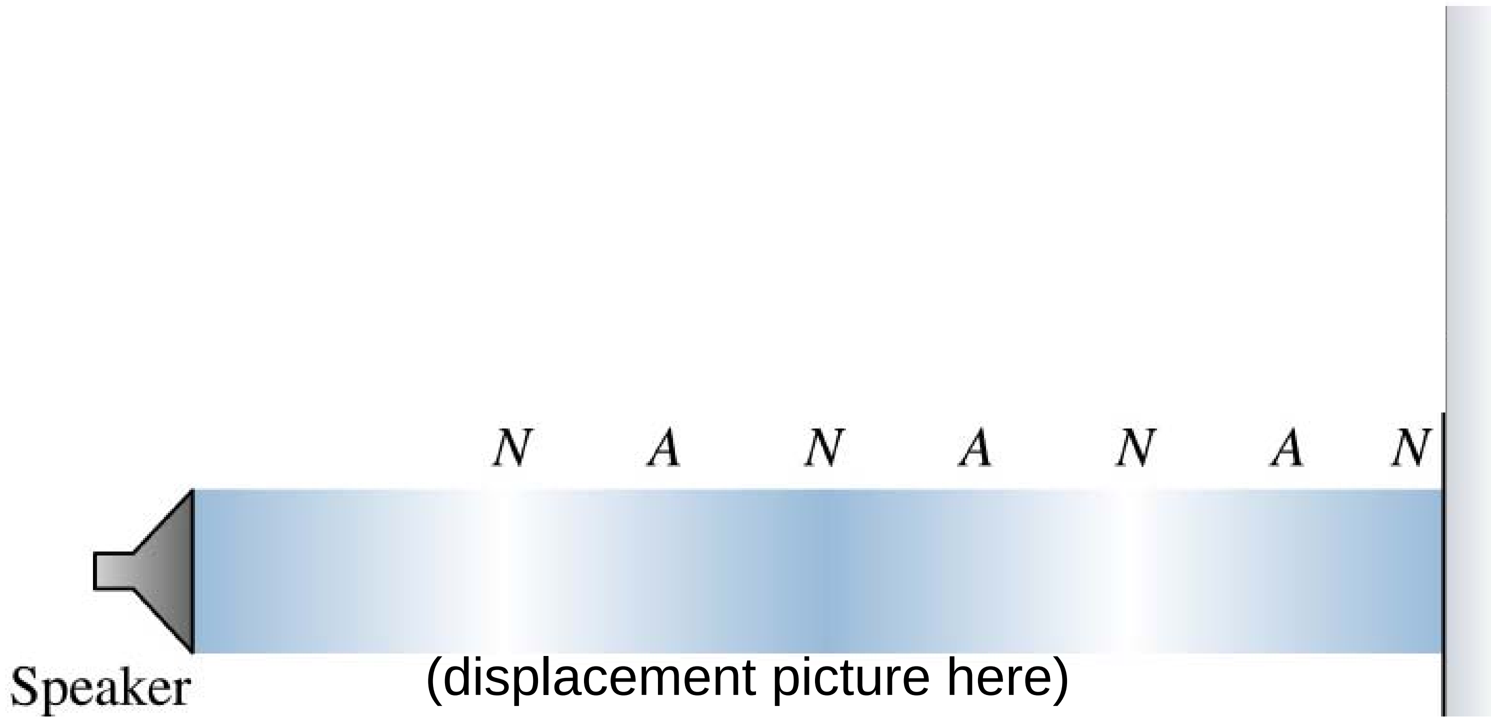
More jargon: nodes and antinodes



- In a sound wave the pressure nodes are the displacement antinodes and viceversa

Example

- A directional loudspeaker bounces a sinusoidal sound wave of the wall. At what distance from the wall can you stand and hear no sound at all?
- A key thing to realize is that the ear is sensitive to pressure fluctuations
- Want to be at pressure node
- The wall is a displacement node → pressure antinode



(displacement picture here)