Fall 2004 Physics 3
Tu-Th Section

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Web page:
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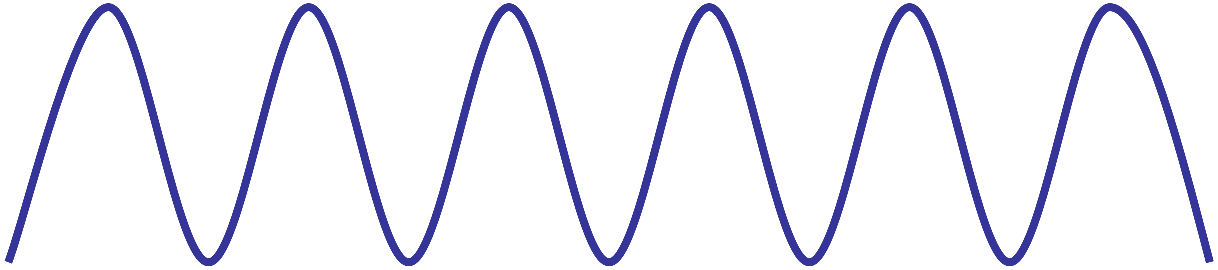
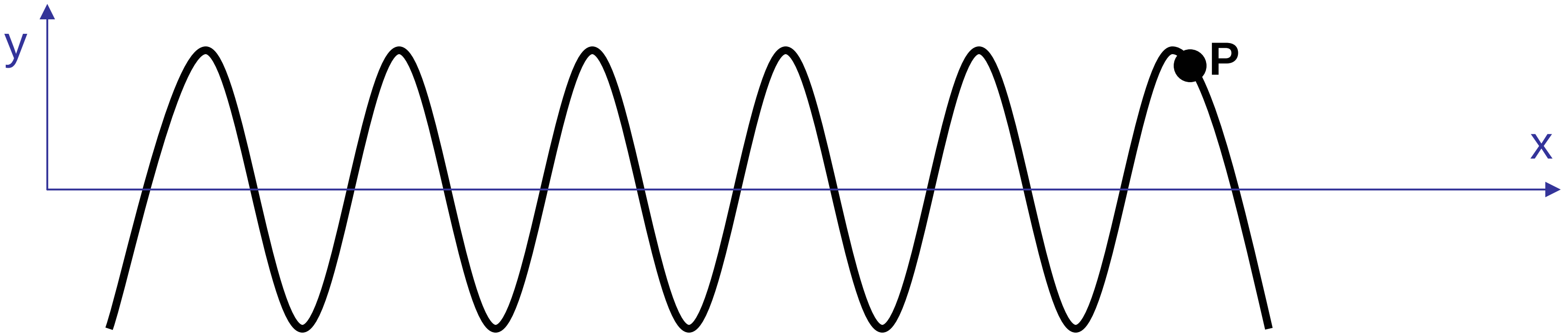
Last Time

- What is a wave?
  - A "disturbance" that moves through space.
  - Mechanical waves through a medium.
- Transverse vs. Longitudinal
  - e.g., string vs. sound
- Sinusoidal
  - each particle in the medium undergoes SHM.



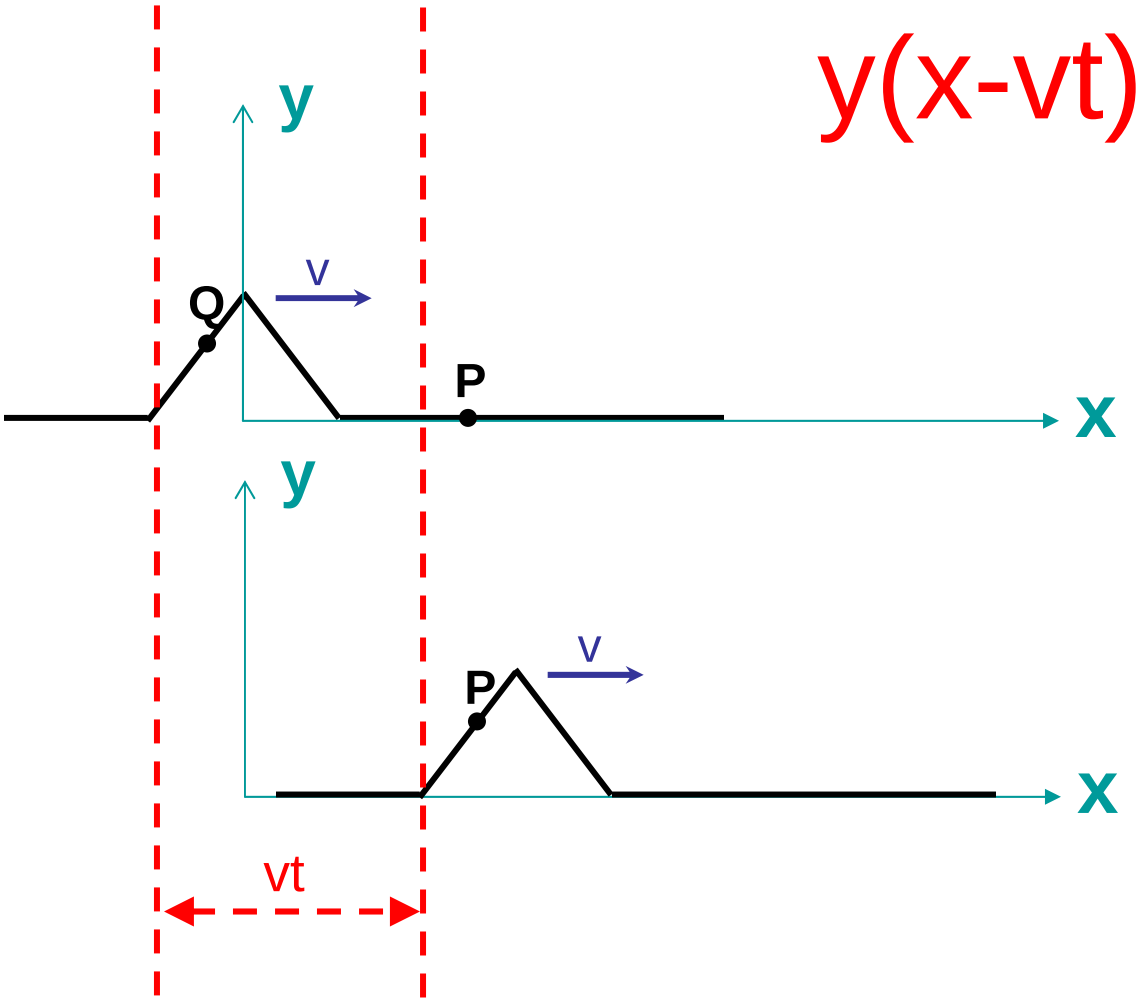
Last Time (cont.)

- Wave function y(x,t)
  - displacement (y) as a function of position along the direction of propagation (x) and time (t).



- Wave moves with velocity v in +ve x-direction
  - y(x-vt)
  - Do not confuse velocity of wave with velocity of particle in the medium!!

y(x-vt)



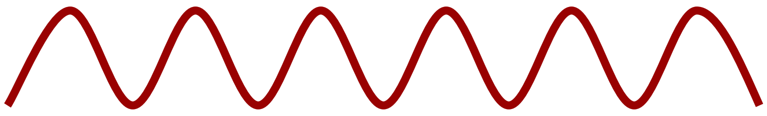
Pulse at t=0
y(x,t)=y(x,0)

Pulse at later time t

- At time t, an element of the string (P) at some x has the same y position as an element located at x-vt at t=0 (Q).
- y(x,t)=y(x-vt,0)=y(x-vt)

Last time (cont.)

Sinusoidal Wave:



$$y(x,t) = A\cos[\tfrac{2\pi}{\lambda}(x - vt)]$$
$$= A\cos[\omega(\tfrac{x}{v} - t)]$$
$$= A\cos(kx - wt)$$

Wave number: $k = \frac{2\pi}{\omega}$

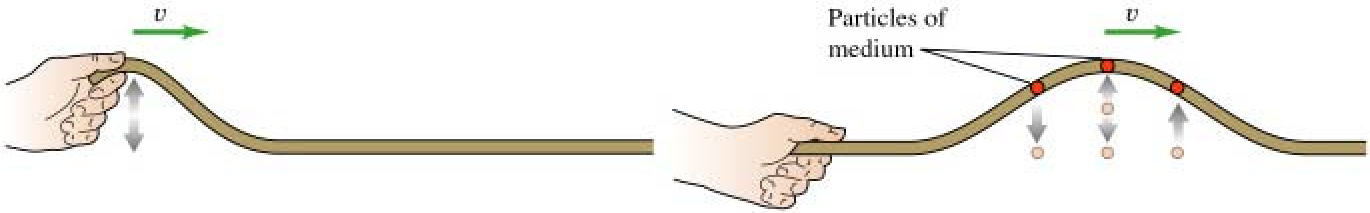
$$\lambda = \frac{2\pi}{k},\ f = \frac{\omega}{2\pi},\ v = \lambda f$$

Wave Equation:

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\ \frac{\partial^2 y(x,t)}{\partial t^2}$$

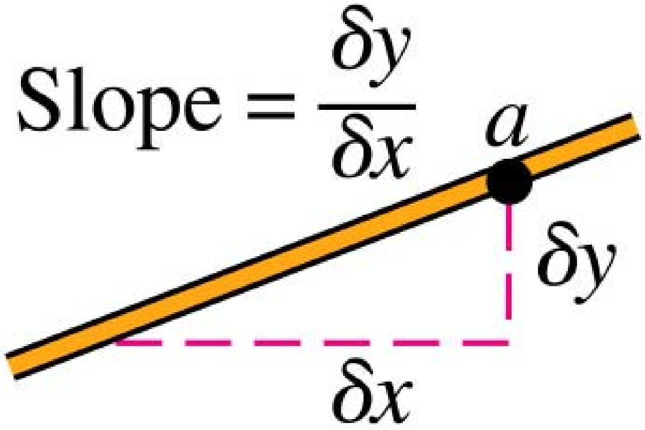
Energy Considerations

- Waves carry energy.
- Think of a pulse on a string.

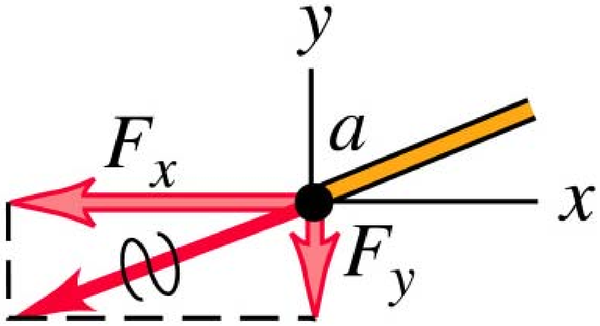


- Energy is <u>transferred</u> from hand to string.
- Kinetic energy <u>moves</u> down the string.

Consider an element
of a string in motion (left to right):



It moves (accelerates) because each piece of the medium exerts a force on its neighboring piece.
What is the force on "a" ?



$$F_y(x,t) = -F\frac{\partial y(x,t)}{\partial x}$$

What is the power?

$$P(x,t) = F_y(x,t)v_y(x,t) = -F\frac{\partial y(x,t)}{\partial x}\frac{\partial y(x,t)}{\partial t}$$

Power:

$$P(x,t) = F_y(x,t)v_y(x,t) = -F\frac{\partial y(x,t)}{\partial x}\frac{\partial y(x,t)}{\partial t}$$

This is the rate at which work is being done (P=W/t), and the rate at which energy travels down the string.

For sinusoidal wave $y(x,t) = A\cos(kx - \omega t)$

$$\frac{\partial y(x,t)}{\partial x} = -kA\sin(kx - \omega t)$$

$$\frac{\partial y(x,t)}{\partial t} = \omega A\sin(kx - \omega t)$$

$$P(x,t) = Fk\omega A^2\sin^2(kx - \omega t) > 0$$

$$P(x,t) = Fk\omega A^2 \sin^2(kx - \omega t)$$

Then $\omega = vk$ and $v^2 = \frac{F}{\mu}$

$\frac{\text{mass}}{\text{length}}$

$$P(x,t) = \sqrt{\mu F}\,\omega^2 A^2 \sin^2(kx - \omega t)$$

Property
of the string

Property
of the wave

<u>This is a general property of mechanical waves:</u>

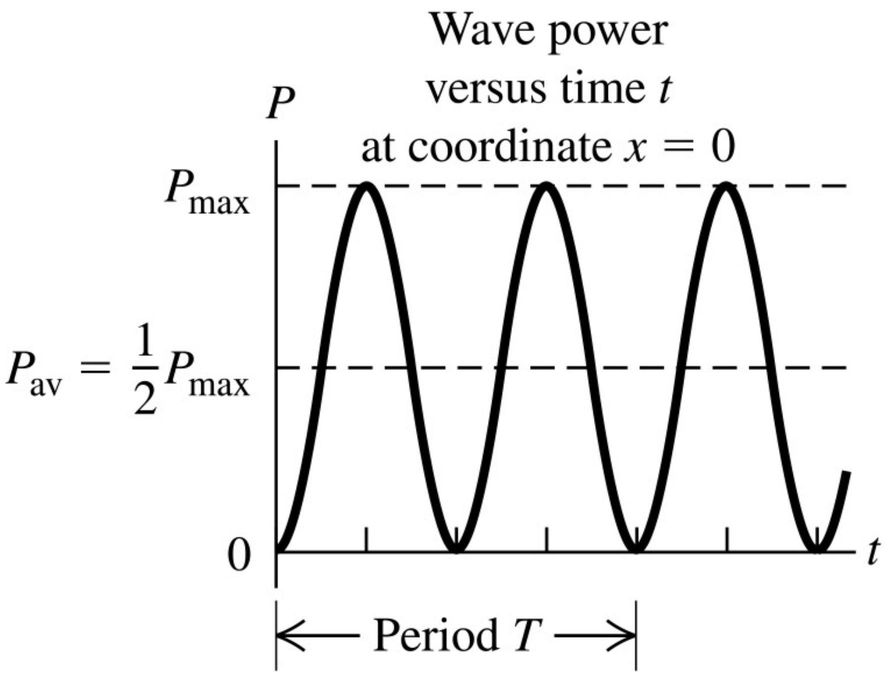
1. Power proportional to square of amplitiude
2. Power proportional to square of frequency

$$P(x,t) = \sqrt{\mu F}\omega^2 A^2 \sin^2(kx - \omega t)$$

Maximum value of power: $P_{max} = \sqrt{\mu F}\omega^2 A^2$

Average value of power $P_{av} = \frac{1}{2}\sqrt{\mu F}\omega^2 A^2$

Since average of $\sin^2$ is 1/2

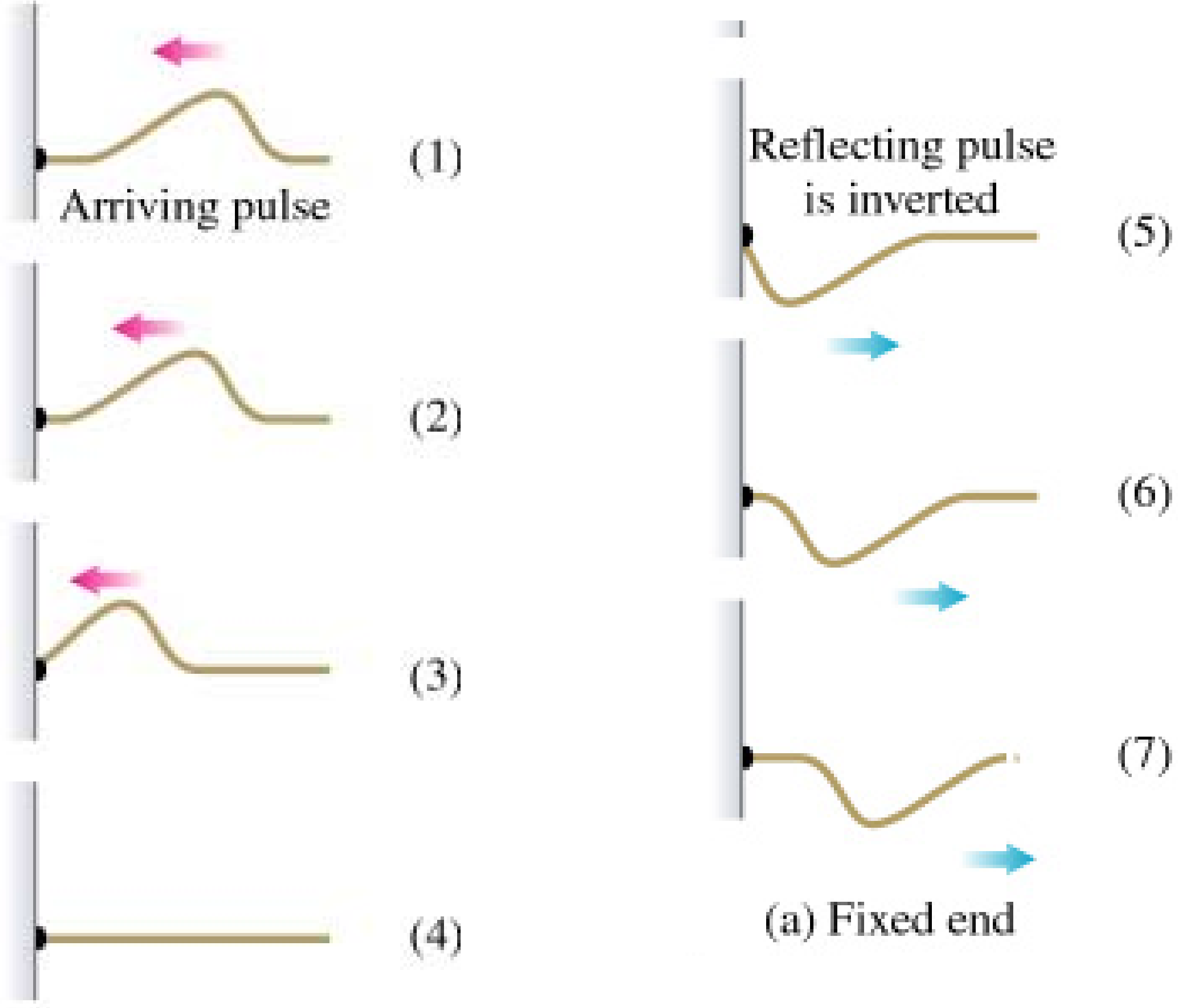


Wave reflection

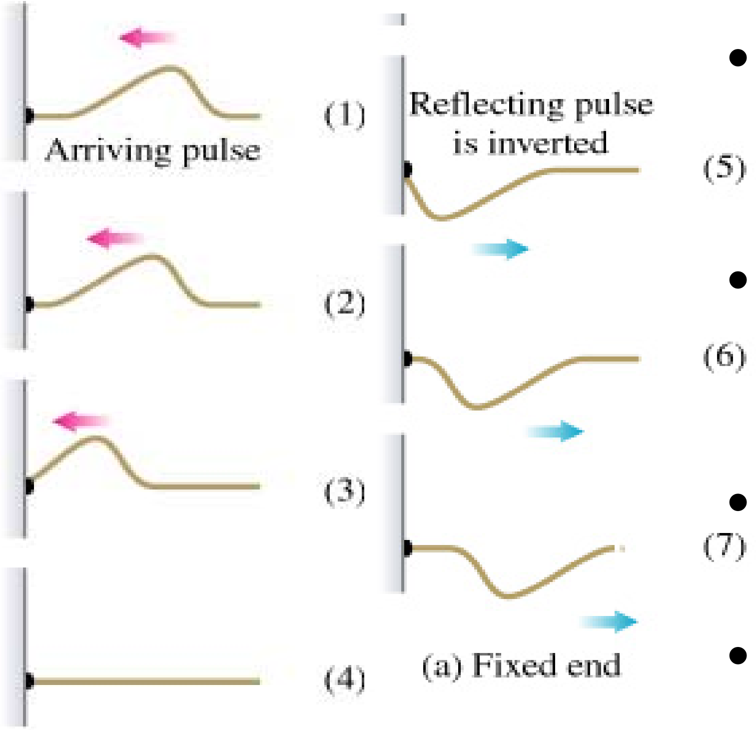
- When a wave encounters an "obstacle", .i.e., a "change in the medium" something happens.
- For example:
  - a sound wave hitting a wall is "reflected"
  - a light wave originally traveling in air when it reaches the surface of a lake is partially "reflected" and partially "transmitted".

Wave reflection, string

Imagine that one end of the string is held fixed:

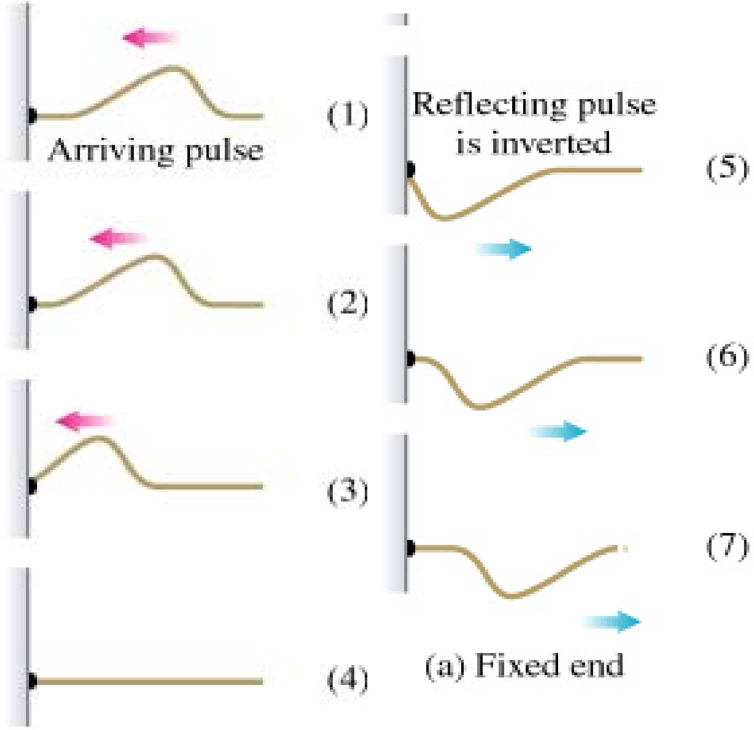


Why is the reflected pulse inverted?



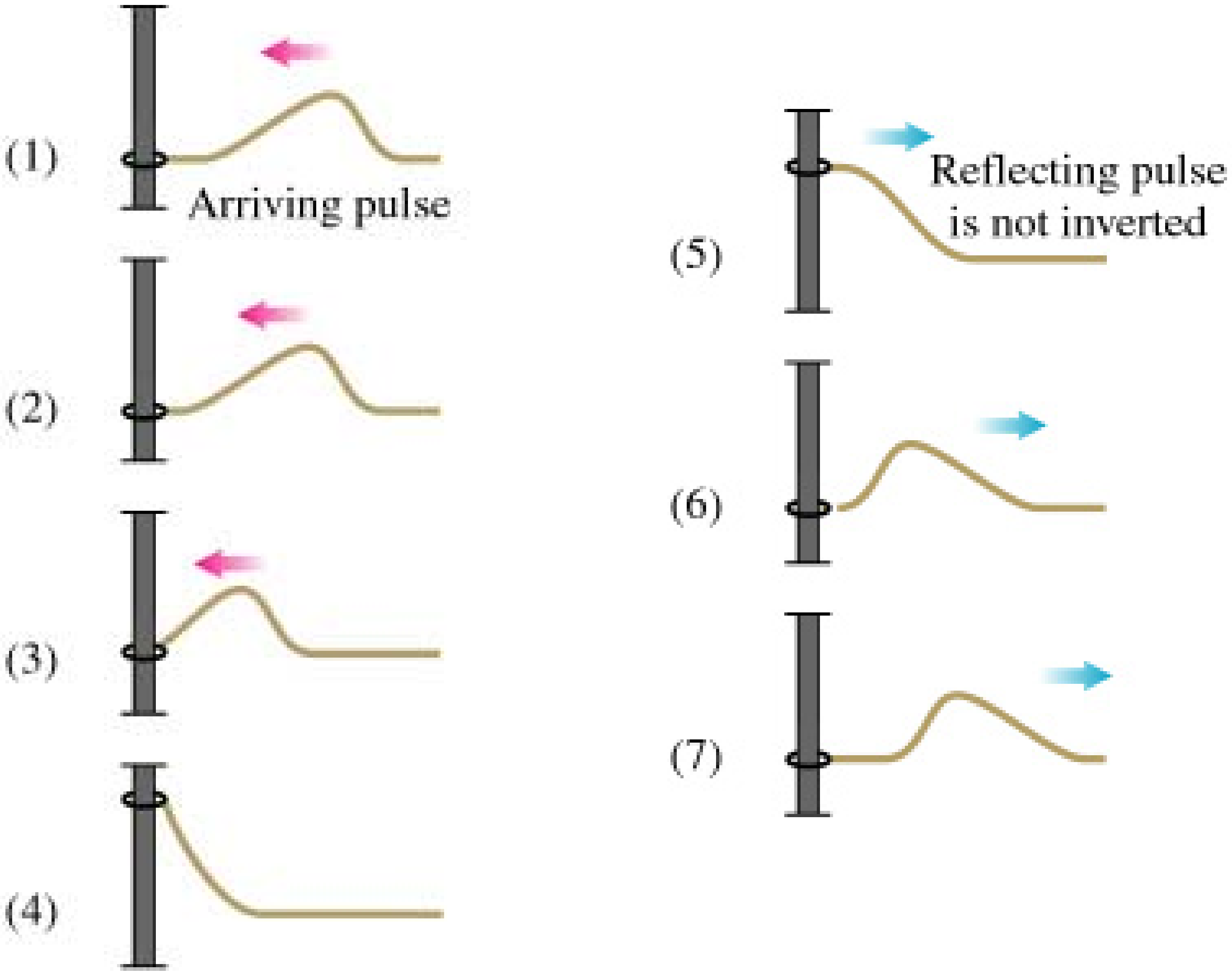
- Pulse was initially created with <u>upward</u> and then <u>downward</u> force on end of string.
- When pulse arrives at fixed end, string exerts <u>upward</u> force on support.
- The end of the string does not move (it is fixed!).
- By Newton's 3rd law, support exerts <u>downward</u> force on string.
- When the top of the pulse arrives, the string exerts a <u>downward</u> force on support.
- Newton's 3rd law → support exerts <u>upward</u> force on string.
- Support-to-string force: <u>downward</u> then <u>upward</u>.
  - Opposite order as pulse creation (was upward then downward).

→ Reflected pulse is inverted

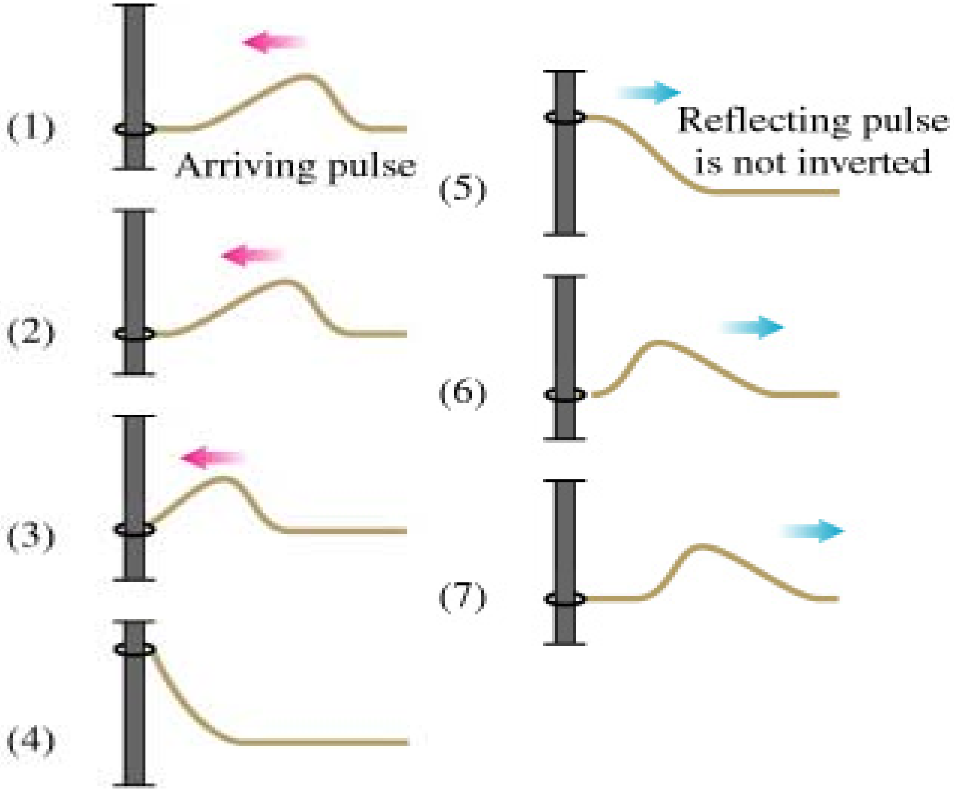


- In this (idealized) situation the reflected wave has the same amplitude (magnitude) and velocity (magnitude) as the incoming wave.
- No energy is lost in the reflection.

Now imagine that one end of the string is free:



Why is the reflected pulse not inverted?



- Pulse was initially created with upward and then downward force on the far end of the string.
- When pulse first arrives at free end, there is an upward force on the end of the string.
- When the top of the pulse arrives, the direction of the force becomes downward.

→ upward and then downward force on the free end of the string

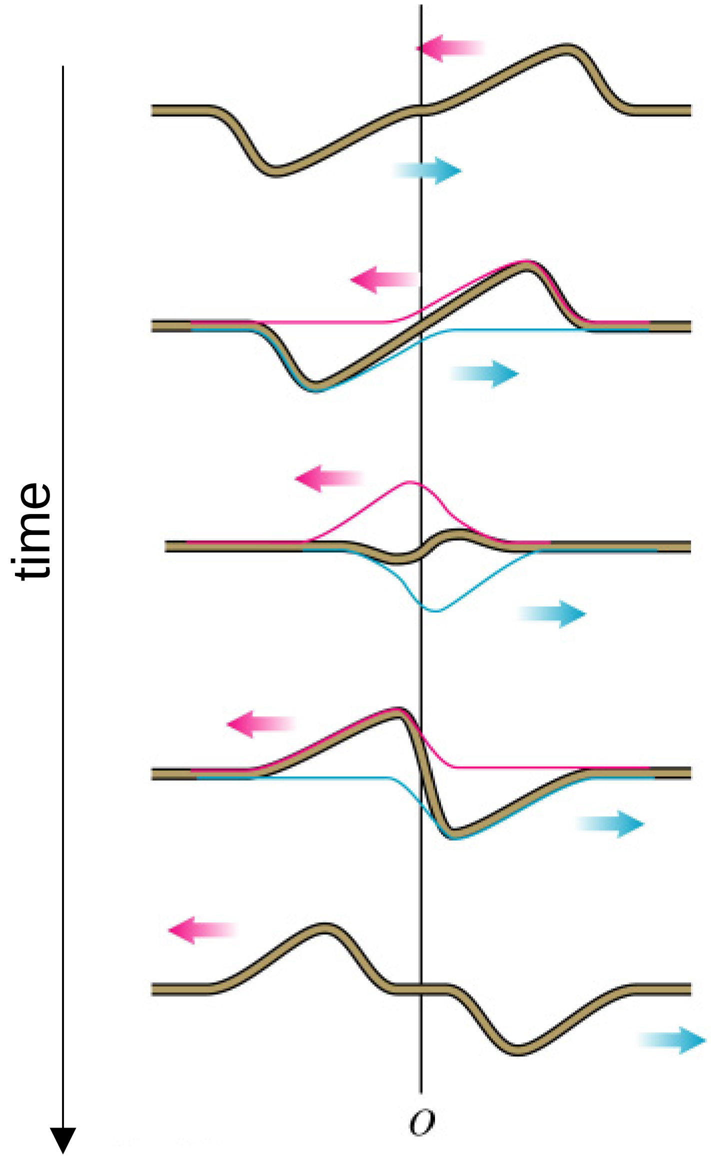
- Forces on free end like at far end where the pulse was 1st generated.

→ No inversion on reflection

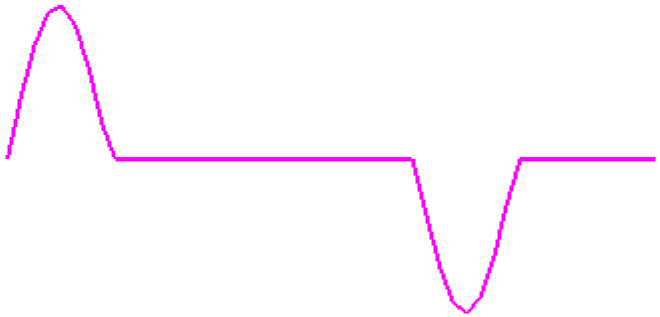
Boundary Conditions

- The properties ("conditions") at the end of the string (or more generally where the medium changes) are called "boundary conditions".
- This is jargon, but it is used in many places in physics, so try to remember what it means.

Interference



- Imagine that the incoming pulse is long.
- Near the boundary at some point we will have a "meeting" of the incoming pulse and the reflected pulse.
- The deflection of the string will be the sum of the two pulses. <u>(principle of superposition)</u>



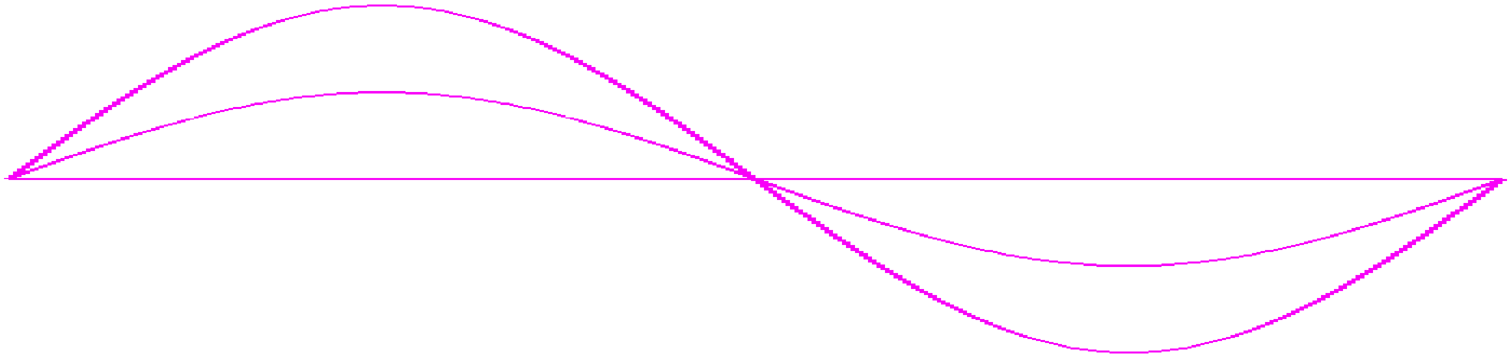
Principle of Superposition

- When two (or more) waves overlap, the actual displacement at any point is the sum of the individual displacements.

$$\underset{\underset{\text{Total displacement}}{\uparrow}}{y(x,t)} = \underset{\underset{\text{First wave}}{\uparrow}}{y_1(x,t)} + \underset{\underset{\text{Second wave}}{\uparrow}}{y_2(x,t)}$$

- Consequence of the fact that wave equation is linear in the derivatives.

Standing Waves



Consider a sinusoidal wave traveling to the left:

$$y_1(x,t) = -A\cos(kx+\omega t)$$

String held fixed at x=0 → reflected wave:

$$y_2(x,t) = +A\cos(kx-\omega t+\delta)$$

- kx+ωt → kx-ωt because reflected wave travels to the right.
- what about δ?
  - Must choose it to match the boundary conditions!

- Boundary condition: string is fixed at x=0
- Mathematically y(x=0,t) = 0 at all times t

$$y(x,t) = y_1(x,t) + y_2(x,t)$$
$$y(x,t) = -A\cos(kx+\omega t) + A\cos(kx-\omega t+\delta)$$
$$y(x=0,t) = -A\cos(\omega t) + A\cos(-\omega t+\delta)$$
$$y(x=0,t) = -A\cos(\omega t) + A\cos(\omega t-\delta)$$

- But y(x=0,t)=0:

$$\rightarrow 0 = -A\cos(\omega t) + A\cos(\omega t-\delta)$$
$$\rightarrow \delta = 0$$

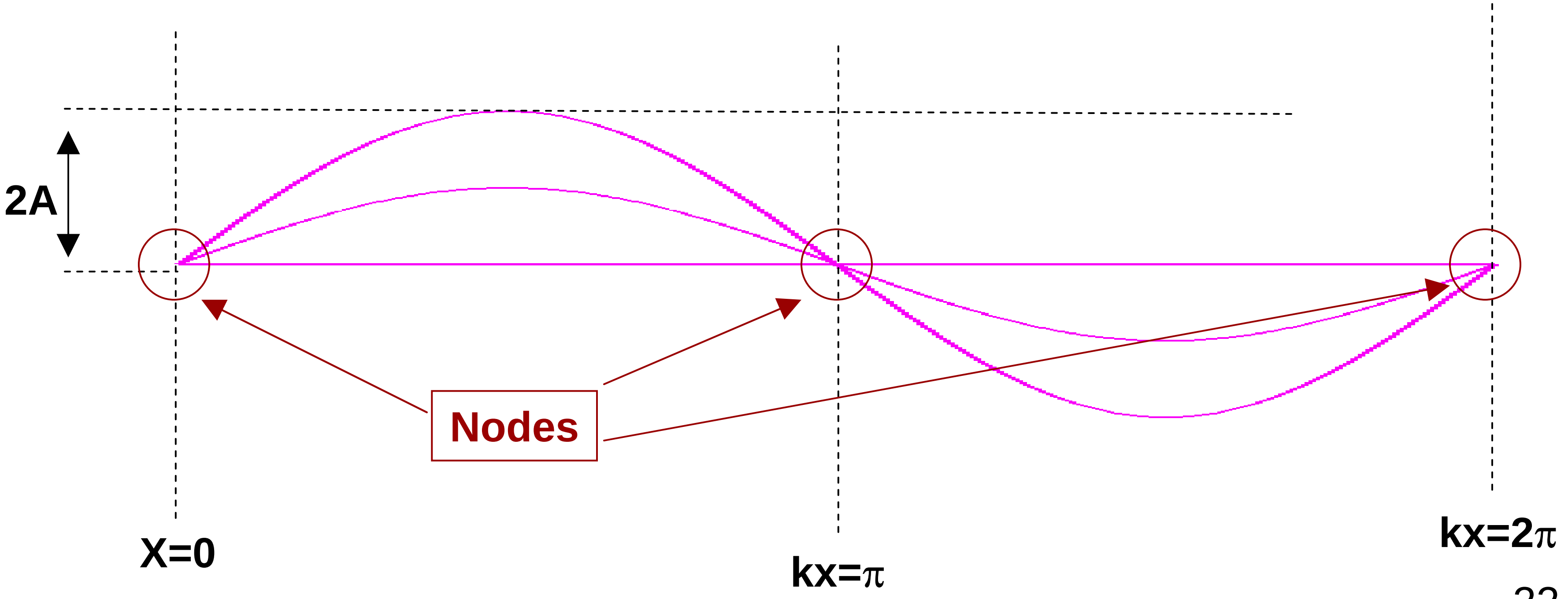
$$y(x,t) = -A\cos(kx+\omega t) + A\cos(kx-\omega t)$$

$$y(x,t) = -A\cos(kx+\omega t) + A\cos(kx-\omega t)$$

$$\cos(kx+\omega t) = \cos kx\cos\omega t - \sin kx\sin\omega t$$

$$\cos(kx-\omega t) = \cos kx\cos\omega t + \sin kx\sin\omega t$$

$$y(x,t) = (2\cdot A\cdot\sin kx)\sin\omega t$$



- Imagine that string is held at both ends.
- L=length of the string
- Nodes at x=0 and x=L

$$y(x,t) = A \cdot \sin kx \cdot \sin \omega t$$

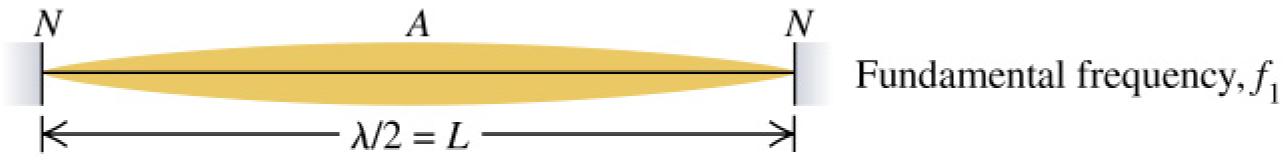
$$kL = n\pi \qquad \text{But } k = \frac{2\pi}{\lambda}$$

$$\rightarrow \lambda = \frac{2L}{n}$$

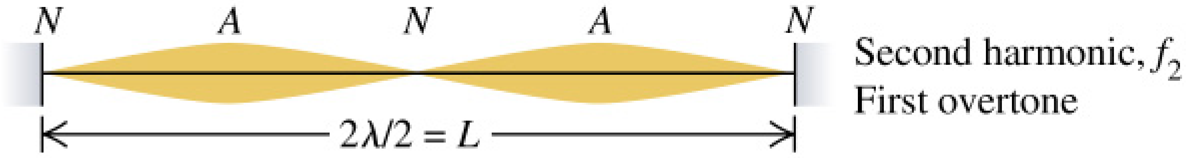
- Only standing waves of very definite wavelengths (and frequencies) are allowed

$$v = f\lambda \quad \rightarrow \quad f = \frac{v}{\lambda} = n\frac{v}{2L}$$

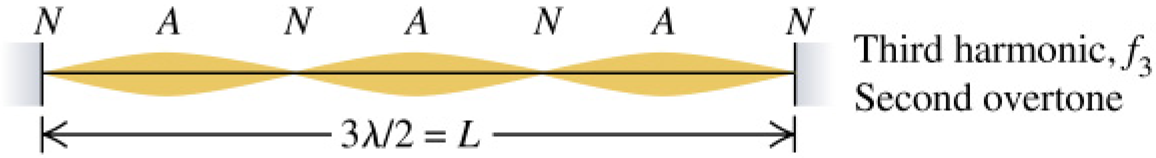
Normal Modes



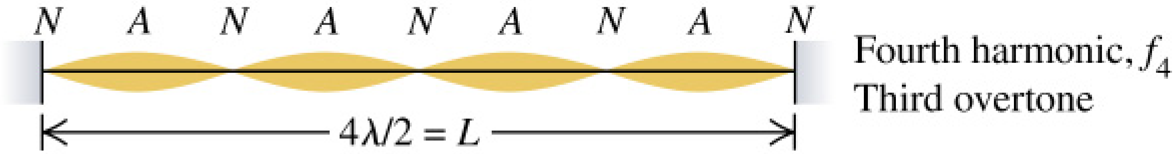
(a) $n = 1$



(b) $n = 2$



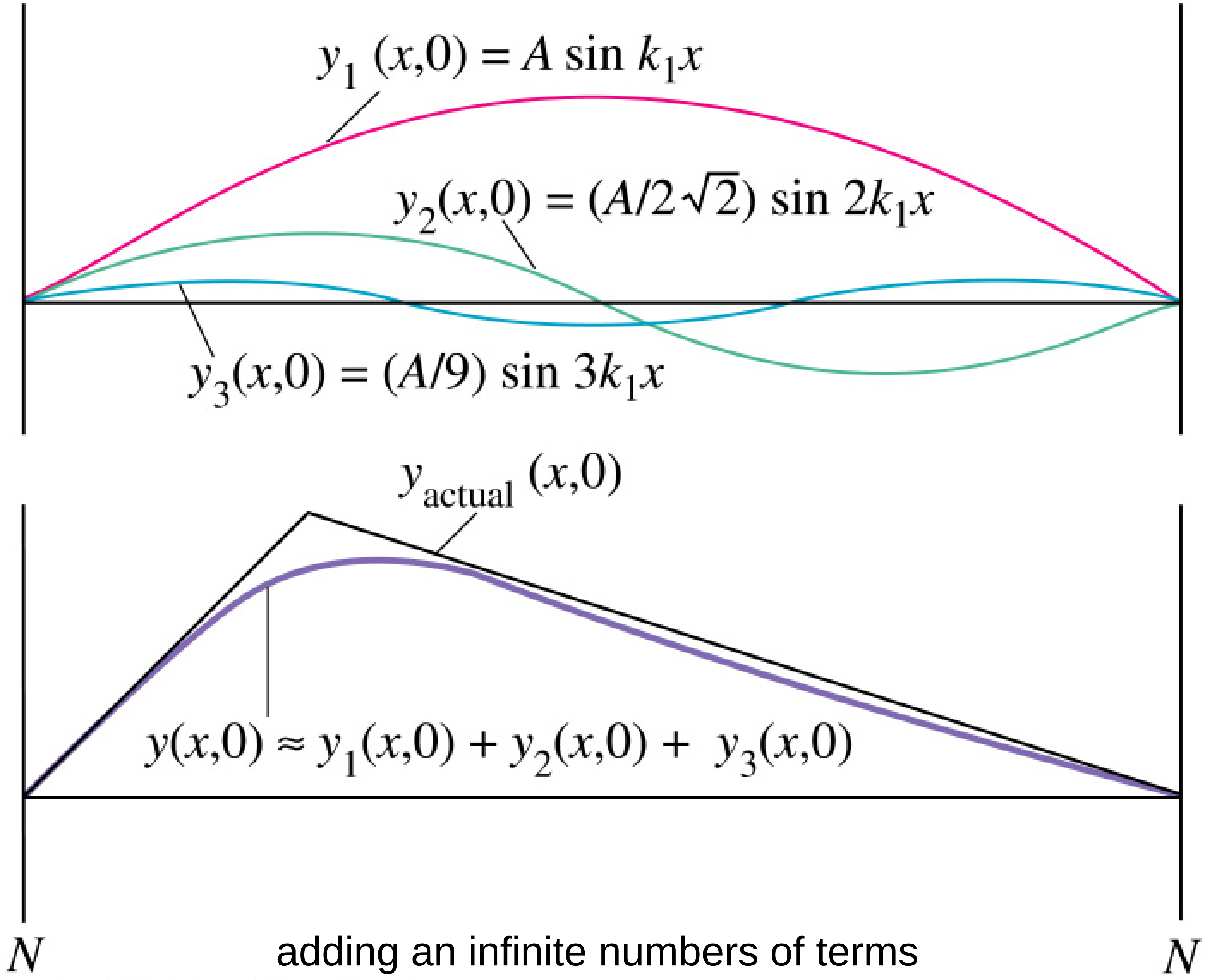
(c) $n = 3$



(d) $n = 4$

- If you could displace a string in a shape corresponding to one of the normal modes, then the string would vibrate at the frequency of the normal mode
  - Surrounding air would be displaced at the same frequency producing a pure sinusoidal sound wave of the same frequency.
- In practice when you pluck a guitar string you do not excite a single normal mode.
  - Because you do not displace the string in a perfectly sinusoidal way

- The displacement of the string can be represented as a sum over the normal modes (Fourier series).



adding an infinite numbers of terms
you can get the exact shape

How to control the frequency of the normal modes

$$f = n\frac{v}{2L} = \frac{n}{2L} \cdot \sqrt{\frac{F}{\mu}}$$

- Longer strings → lower frequencies.
  - Cello vs violin
- Higher tension (F) → higher frequencies.
- More messive strings → lower frequencies.