Fall 2004 Physics 3
Tu-Th Section

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Lecture 17: 30 Nov. 2004

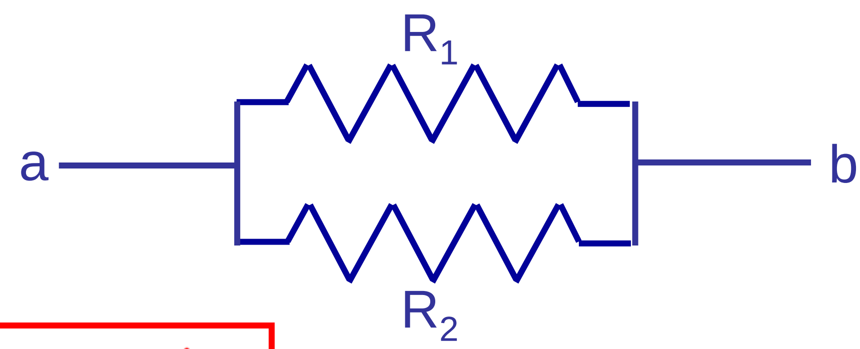
Web page:
http://hep.ucsb.edu/people/claudio/ph3-04/

Reminder

- This is the last lecture
- The final is Thursday, December 9, 12-3 pm
- The final is open book and open notes
- There will be a review session on Thursday at lecture time
  - I will do problems from the old finals and midterms
  - I had posted the old final and midterms on the midterm info page
  - http://hep.ucsb.edu/people/claudio/ph3-04/midterm.html

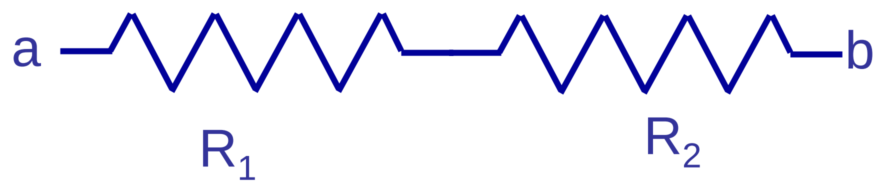
Last Time

- Resistors in parallel:



$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2}$$

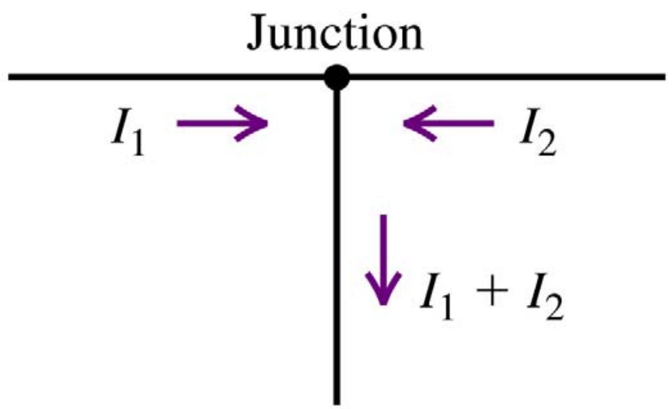
- Resistors in series



$$R_{eq} = R_1 + R_2$$

Last Time: Kirchoff's rule for current

- At a node (or junction) $\Sigma I = 0$

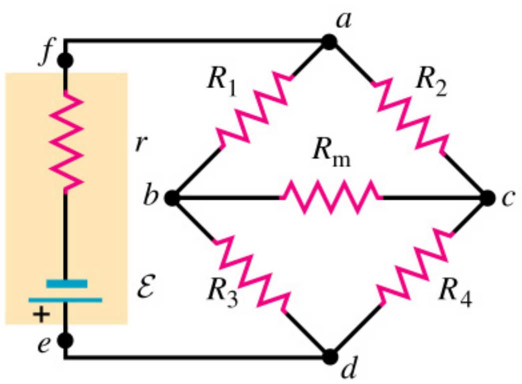


Careful about the signs!
It is a good idea to always draw the arrows!

- This is basically a statement that charge is conserved

Last Time: Kirchoff's rule for voltage

The <u>total</u> voltage drop across a closed loop is zero



For example:

$V_{ab} + V_{bc} + V_{ca} = 0$

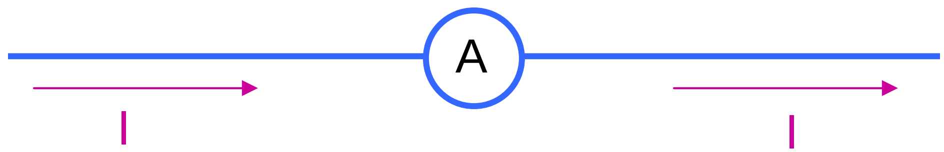
$(V_a - V_b) + (V_b - V_c) + (V_c - V_a) = 0$

But this holds for any loop,

e.g. a-b-d-c-a or b-a-f-e-d-b, ...

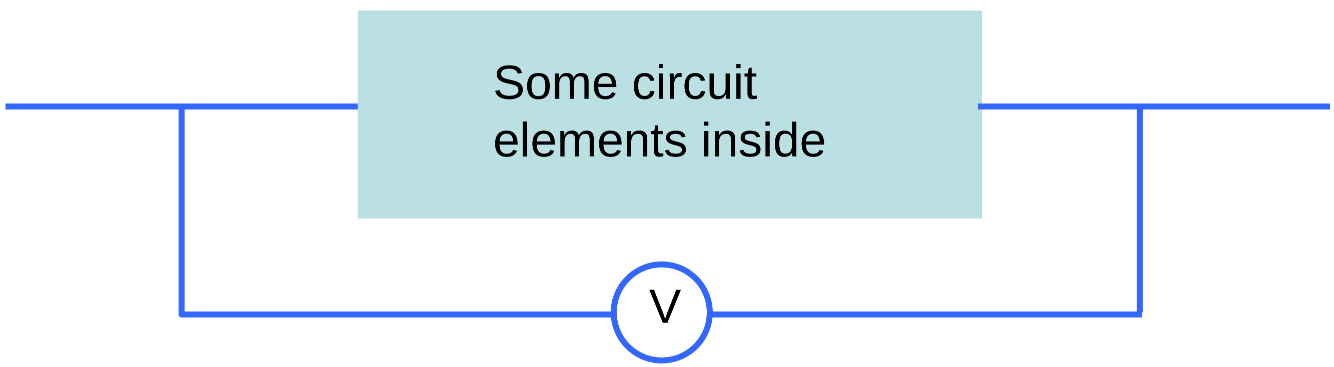
Measuring current, voltage, resistance

- If you want to measure current "somewhere" in your circuit you put a current-measuring-device (ammeter) where you care to know the current



- Ideally, the presence of the ammeter should not influence what is going on in your circuit
  - ➢ Should not change the current or the voltages
- The ideal ammeter has zero resistance

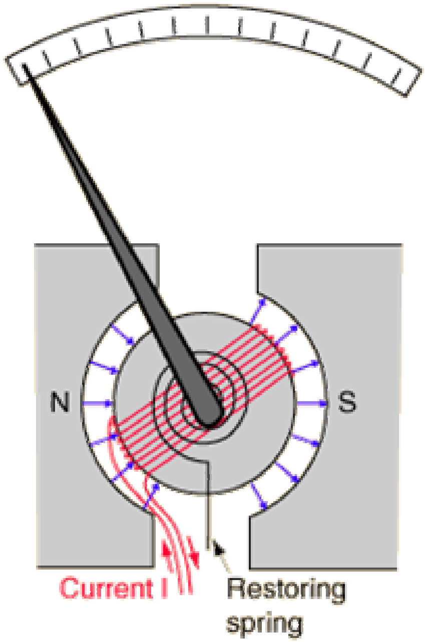
- If you want to measure the voltage difference between two points in your circuit you connect a voltage-measuring-device (voltmeter) to the two points



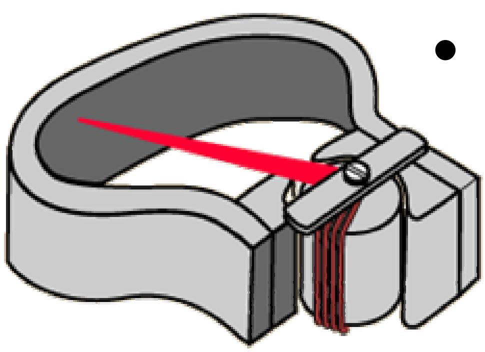
- Ideally, the voltmeter should not influence the currents and voltages in your circuit
- An ideal voltmeter has infinite resistance

Galvanometer

- The galvanometer is the "classic" device to measure current
- Based on the fact that a wire carrying current in a magnetic field feels a force
  - You'll see this next quarter in Physics 4

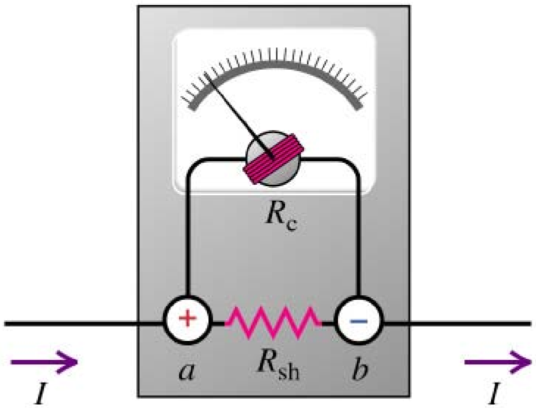


- The current flows through a coil in a magnetic field
- The coil experiences a torque proportional to current
- The movement of the coil is "opposed" by a spring
- The deflection of the needle is proportional to current



Galvanometer (cont.)

- A typical galvanometer has a "full-scale-current" (I_{fs}) of 10 μA to 10 mA
- The resistance of the coil is typically 10 to 1000 Ω.
- How can we use a galvanometer to measure currents higher than its full scale current?
  1. Divide the current, so that only a well understood fraction goes through the coil
  2. Measure how much goes through the coil
  3. Rescale by the known fraction



- R_{sh} = "shunt" resistance
- The current I divides itself between the coil and the shunt
  - $I = I_C + I_{sh}$

- By Ohms's law, $V_{ab} = I_C R_C = I_{sh} R_{sh}$
- $I_{sh} = I_C (R_C/R_{sh})$
- $I = I_C + I_{sh} = I_C (1 + R_C/R_{sh})$

→ If R_C and R_{sh} are known, measuring I_C is equivalent to measuring I

- Furthermore, I is still proportional to I_C, which is proportional to the deflection of the needle
- Thus, by "switching in" different shunt resistances I can effectively change the "range" of my current measurement

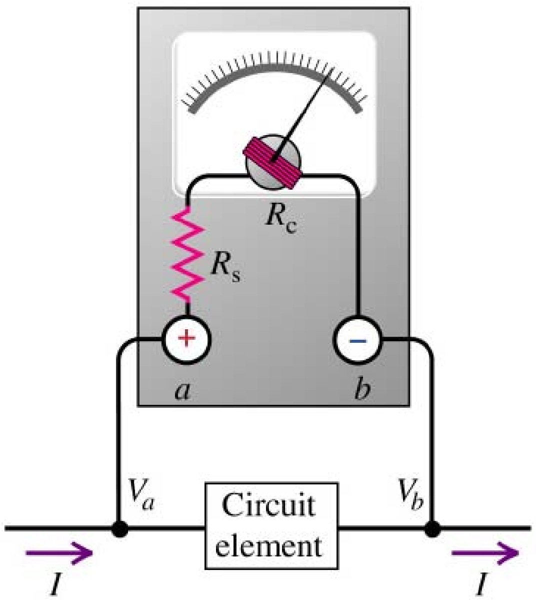
Example

- Galvanometer, R_C=10 Ω, I_{fs}=1 mA
- What shunt resistance should I use to make the full scale deflection of the needle 100 mA?

- $I = I_C (1 + R_C/R_{sh})$
- Want the "multiplier" to be 100 (i.e. 1 mA → 100 mA)
- $1 + R_C/R_{sh} = 100 \rightarrow R_{sh} = 0.101\ \Omega$

- Bonus: R_C and R_{sh} in parallel
- Equivalent resistance $R_{eq} = R_C R_{sh}/(RC+Rsh) = 0.1\ \Omega$
- Small, much closer to ideal ammeter (R=0)

Galvanometer as a Voltmeter



- Move the shunt resistance to be in series (rather than in parallel) with the coil

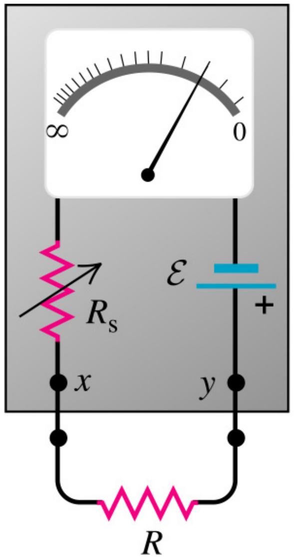
- Remember that an ideal voltmeter has infinite resistance, so we want to make the resistance of the device large!
- $I_C = V_{ab}/(R_C + R_{sh})$
- The needle deflection measures I_C and, knowing R_C and R_{sh}, measures V_{ab}

Example

- Galvanometer, R_C=10 Ω, I_{fs}=1 mA
- What shunt resistance should I use to make a voltmeter with full scale deflection of the needle V_{fs} = 10 V?

- $I_C = V_{ab}/(R_C + R_{sh})$
- $R_C + R_{sh} = V_{fs}/I_{fs}$ = 10 V / 1 mA = $10^4\ \Omega$
- R_{sh} = 9,990 Ω

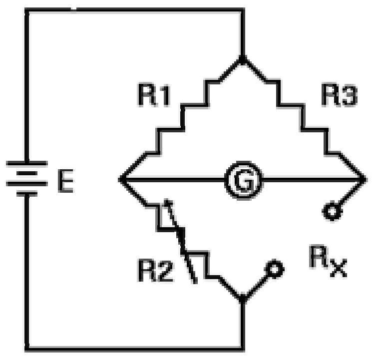
- Bonus: R_C and R_{sh} in series
- Equivalent resistance of voltmeter = $R_C + R_{sh} = 10^4\ \Omega$ (large!)



Galvanometer as a resistance meter (aka Ohmmeter)

- $I_C = \varepsilon/(R_S + R)$
- From the needle deflection, measure I_C
- Then, knowing the emf and R_S infer R
- In practice R_S is adjusted so that when R=0 the deflection is maximum, i.e. $I_{fs} = \varepsilon/R_S$

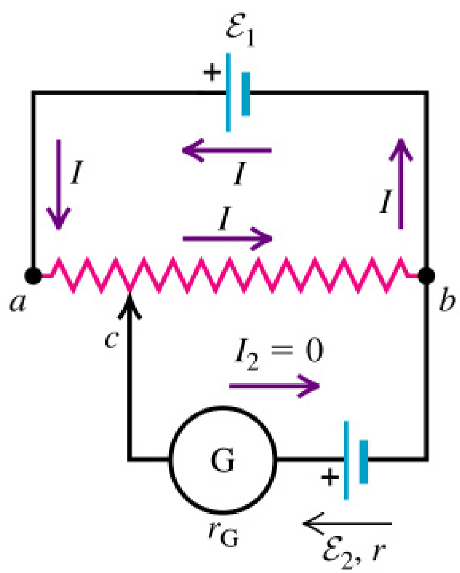
Wheatstone Bridge (Problem 26.77)



- A clever method to accurately measure a resistance
- R_1 and R_3 are known
- R_2 is a variable resistor
- R_x is an unknown resistor
- R_2 is varied until no current flows through the galvanometer G
- Let I_1, I_2, I_3 and I_x be the currents through the four resistors.
- $I_1 = I_2$ and $I_3 = I_x$
- No current through G: no voltage difference across it
- $I_1R_1 = I_3R_3$ and $I_2R_2 = I_xR_x$ → $R_x = R_3R_2/R_1$

Potentiometer

- A circuit used to measure an unknown emf by comparing it with a known emf

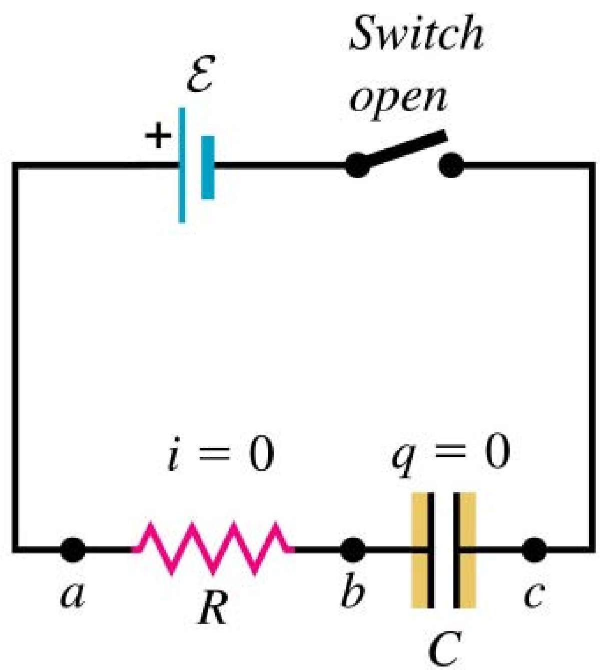


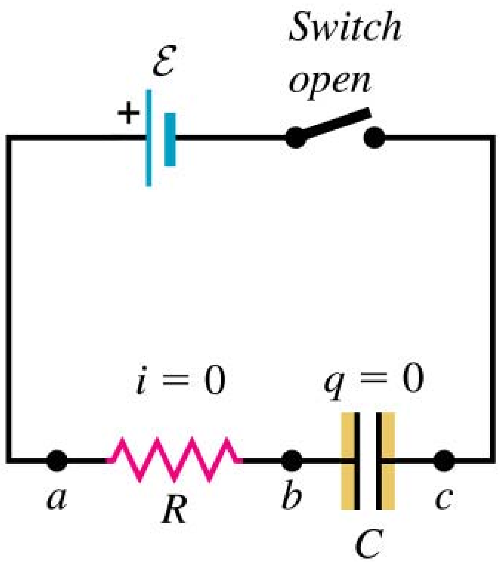
- ε_1 is known, ε_2 is unknown
- Slide the point of contact "c", i.e., change the resitance R_{ac}, until the galvanometer shows no current
- Then $\varepsilon_2 = V_{cb} = I\ R_{cb}$
- But $I = \varepsilon_1 / R_{ab}$

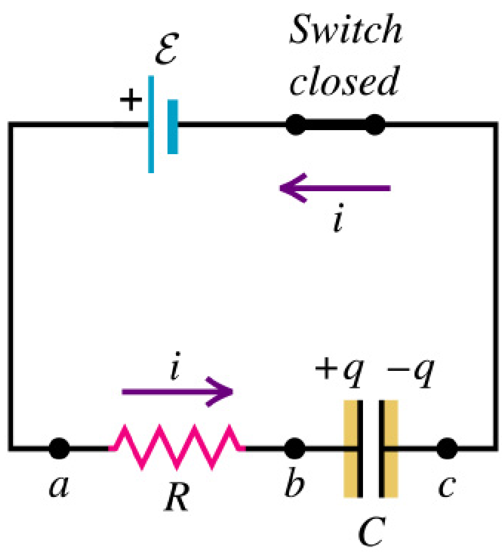
$\rightarrow \varepsilon_2 = \varepsilon_1 R_{cb}/R_{ab}$

RC Circuit: R and C in series

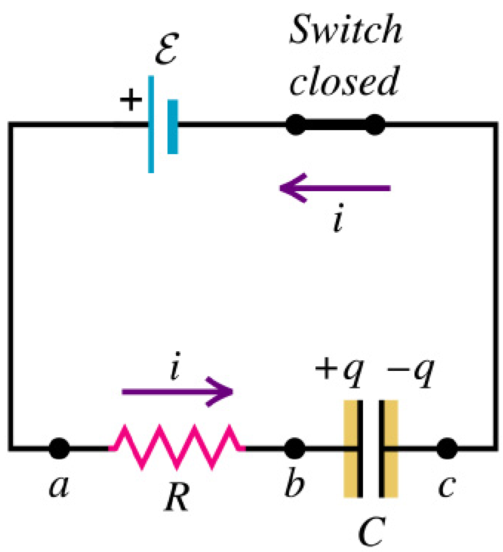
- So far we have only discussed circuits where the currents and potentials never change (DC circuits) (DC = direct current)
- What happens when I close the switch?







- Both i and q are functions of time
- <u>Define</u> t=0 when the switch is being closed
- At t<0, i=0 and q=0
- At t>0 current starts to flow and the capacitor starts to charge up
- V_{ab} = iR and V_{bc} = q/C
- Kirchoff: $\varepsilon = V_{ab} + V_{bc}$ = iR + q/C



$$i(t) = \frac{\epsilon}{R} - \frac{q(t)}{RC}$$

$$\frac{dq}{dt} = \frac{\epsilon}{R} - \frac{q}{RC}$$

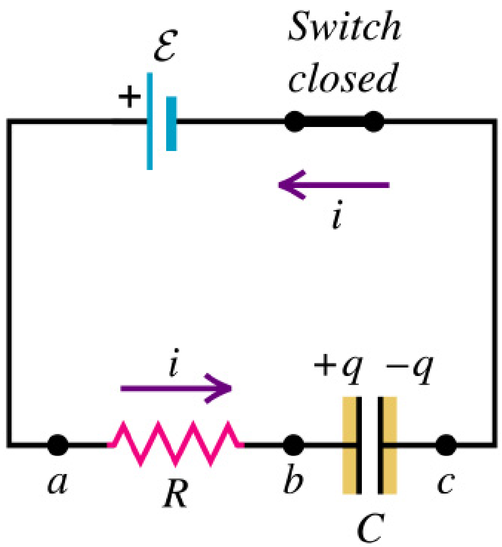
$$\frac{dq}{dt} = -\frac{1}{RC}(q - C\epsilon)$$

$$\frac{dq}{q-\epsilon C} = -\frac{dt}{RC}$$

$$\int_0^q \frac{dq'}{q' - \epsilon C} = -\int_0^t \frac{dt'}{RC}$$

$$\ln\left(\frac{q-\epsilon C}{-\epsilon C}\right) = -\frac{t}{RC}$$

Here we put primes on the integrating variables so that we can use q and t for the limits. The limits of integration are chosen because q=0 at t=0 and charge=q at some later time t



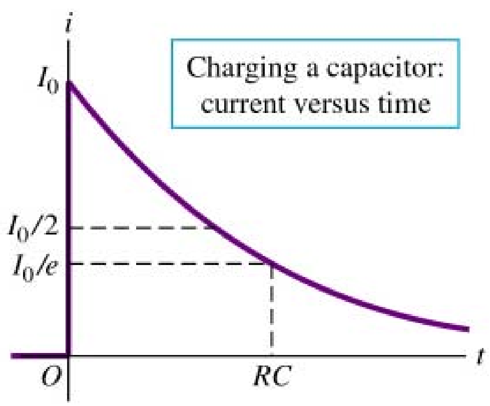
$$\ln\left(\frac{q-\epsilon C}{-\epsilon C}\right) = -\frac{t}{RC}$$

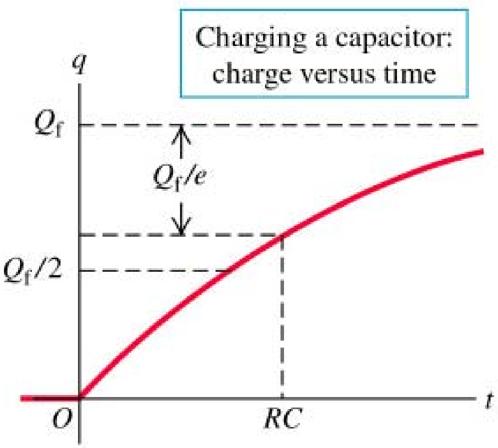
Now take the exponent of both sides

$$\frac{q-\epsilon C}{-\epsilon C} = e^{-\frac{t}{RC}}$$

$$q = \epsilon C\left(1 - e^{-\frac{t}{RC}}\right) = Q_f\left(1 - e^{-\frac{t}{RC}}\right)$$

$$i = \frac{dq}{dt} = \frac{\epsilon}{R}e^{-\frac{t}{RC}} = I_0 e^{-\frac{t}{RC}}$$



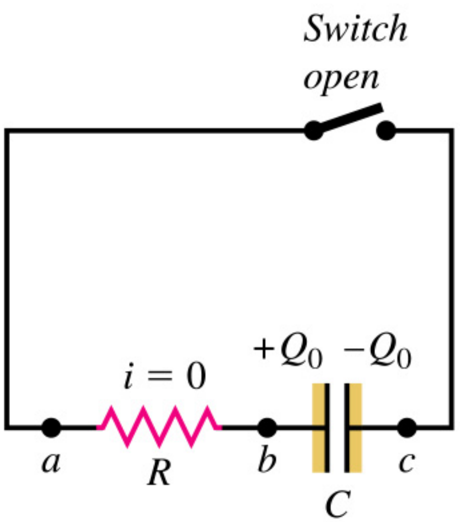


RC = time constant (unit of time)

$$q = \epsilon C(1 - e^{-\frac{t}{RC}}) = Q_f(1 - e^{-\frac{t}{RC}})$$
$$i = \frac{dq}{dt} = \frac{\epsilon}{R} e^{-\frac{t}{RC}} = I_0 e^{-\frac{t}{RC}}$$

- After a long time, i.e., t >> RC, $e^{-t/RC} \sim 0$
- Then i=0 and q=εC
- The charge on the capacitor is the same as if the capacitor had been directly connected to the battery, without the series resistor
- The series resistors "slows" the charging process (larger R → larger time constant RC)

Now the reverse process: discharging a capacitor



Kirchoff: iR + q/C = 0

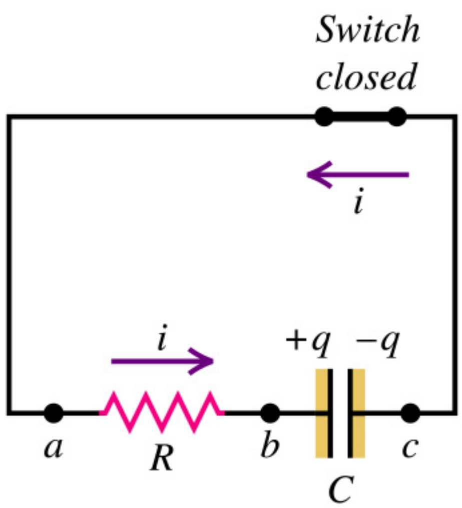
$$i = \frac{dq}{dt} = -\frac{q}{RC}$$

$$\frac{dq}{q} = -\frac{dt}{RC}$$

Switch closed at t=0 when q=Q_0

$$\int_{Q_0}^{q} \frac{dq'}{q'} = \int_0^t -\frac{dt'}{RC}$$

$$\ln \frac{q}{Q_0} = -\frac{t}{RC}$$

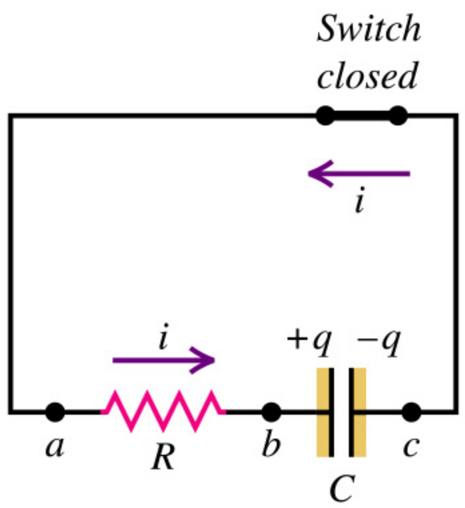


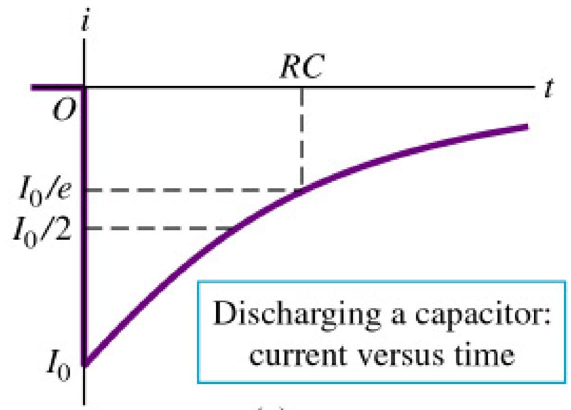
$$q = Q_0 e^{-\frac{t}{RC}}$$

$$i = \frac{dq}{dt} = -\frac{Q_0}{RC} e^{-\frac{t}{RC}} = -I_0 e^{-\frac{t}{RC}}$$

$$q = Q_0 e^{-\frac{t}{RC}}$$

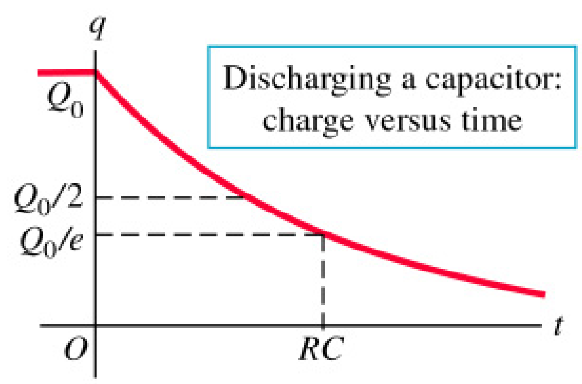
$$i = \frac{dq}{dt} = -\frac{Q_0}{RC} e^{-\frac{t}{RC}} = -I_0 e^{-\frac{t}{RC}}$$





(a)

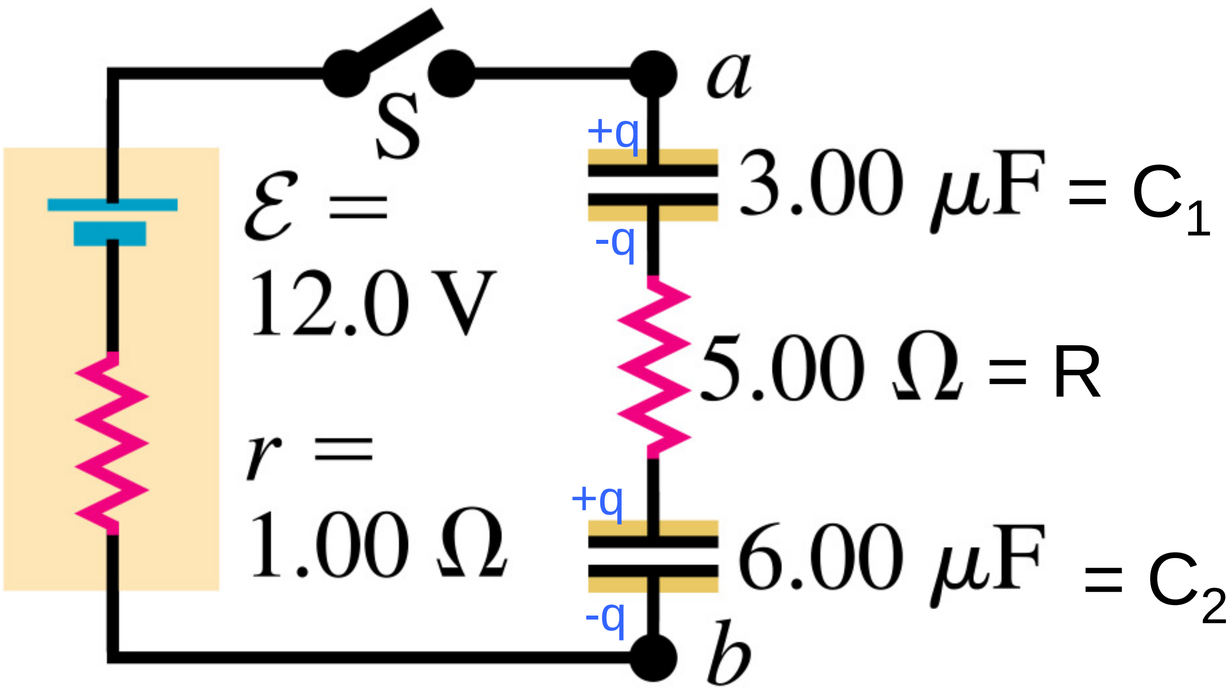
Again,
time constant RC



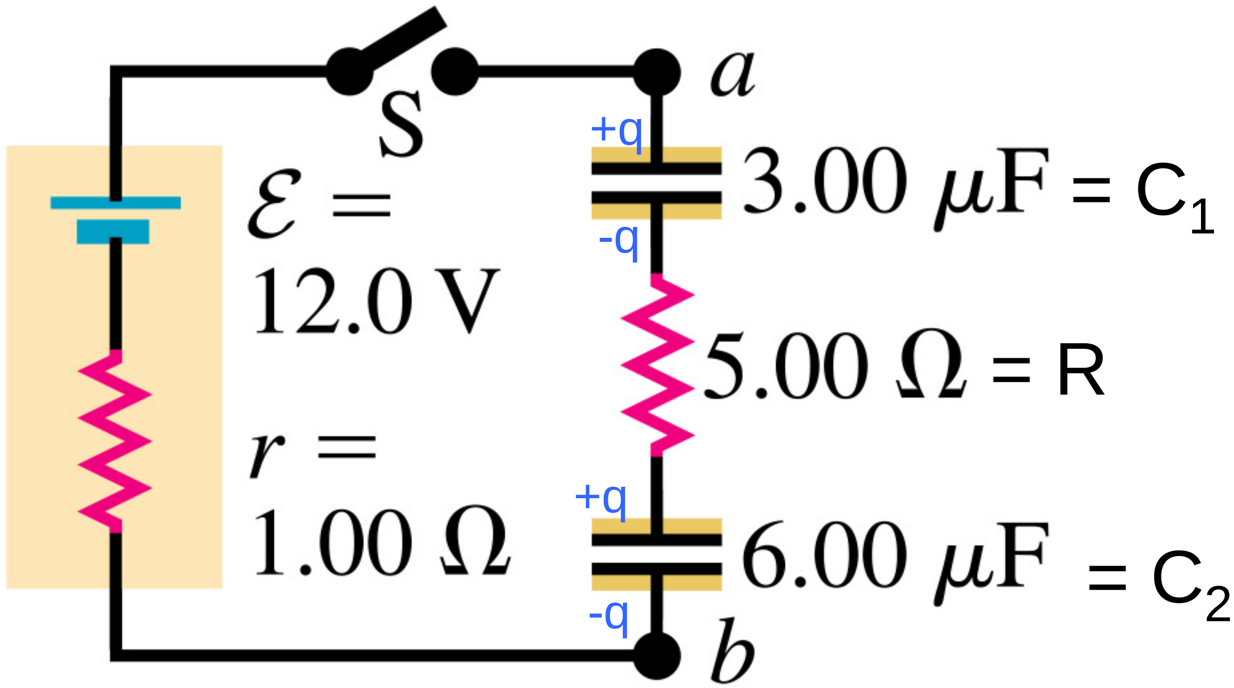
(b)

Example: Problem 26.86

Find the time constant for this circuit



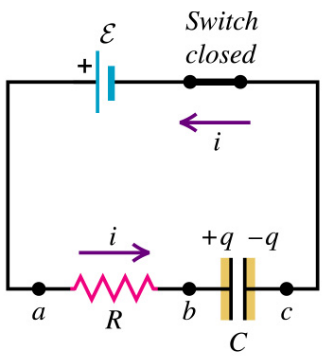
Looks complicated, but notice that charges on C_1 and C_2 must be the same!!



Kirchoff: $q/C_1 + iR + q/C_2 + ir = \varepsilon$

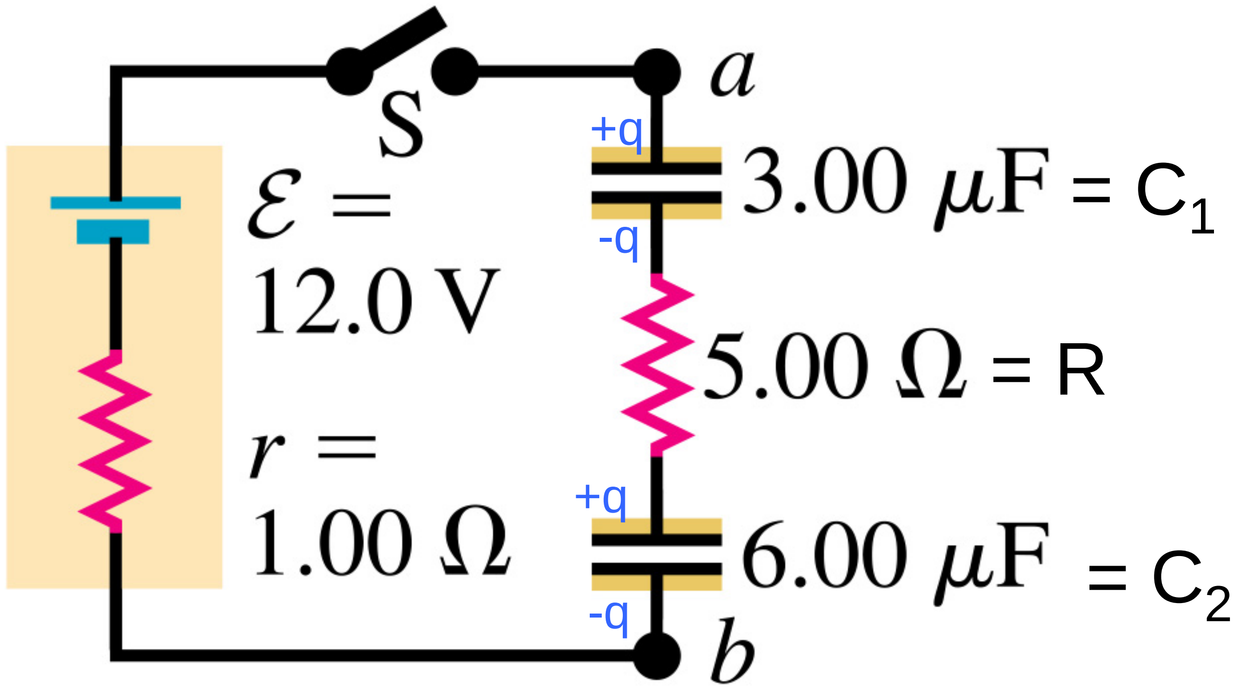
$q(1/C_1 + 1/C_2) + i\,(R+r) = \varepsilon$

Compare with what we had for the simple RC circuit



Here we had Kirchoff: $iR + q/C = \varepsilon$

Thus, our circuit is equivalent to a simple RC circuit with series resistance (r+R) and capacitors C_1 and C_2 in series!



Equivalent capacitance: $1/C_{eq} = 1/C_1 + 1/C_2 \rightarrow C_{eq} = 2\ \mu F$
Series resistance $R_{eq} = r + R = 6\ \Omega$

Time constant = $R_{eq}C_{eq} = 12\ \mu sec$