Fall 2004 Physics 3
Tu-Th Section

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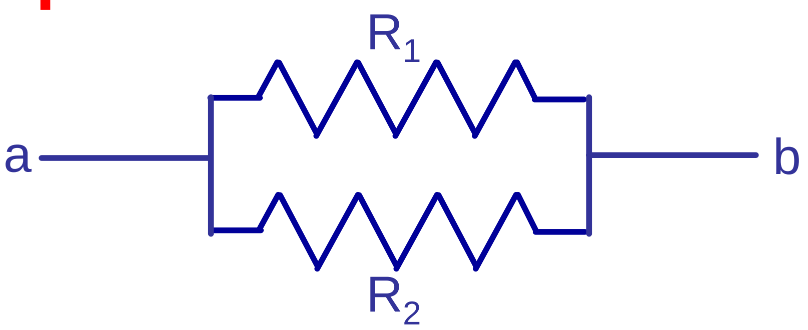
Lecture 16: 22 Nov. 2004

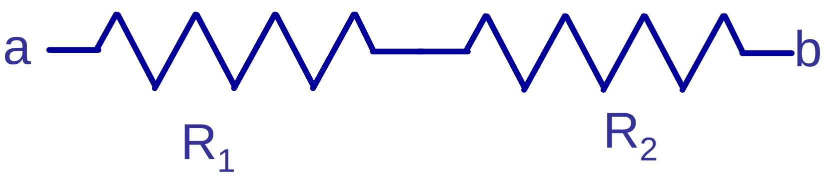
Web page:
http://hep.ucsb.edu/people/claudio/ph3-04/

Resistor in parallel or in series

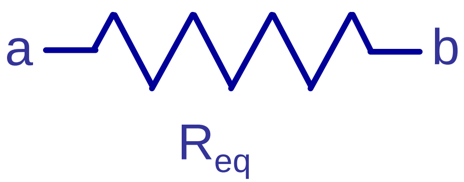
- In parallel:



- In series

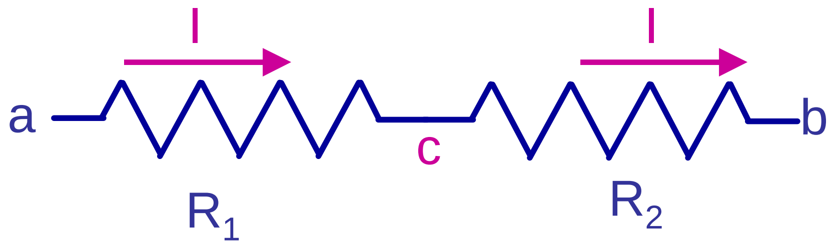


- In both cases these pieces of a circuit can be thought of as "equivalent" to a single resistor

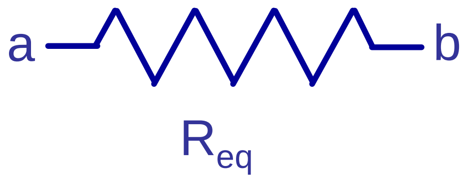


  - Equivalent means that for a given V_{ab} the total current flowing would be the same as if it was a single resistor of resistance R_{eq}, i.e., $I=V_{ab}/R$

R_{eq} for resistors in series



- The current flowing through the two resistors is the same
- The voltage drops are
  - $V_{ac} = I\,R_1$ and $V_{cb} = I\,R_2$
  - $V_{ab} = V_a - V_b = V_a - V_c + V_c - V_b = V_{ac} + V_{cb}$
  - $V_{ab} = I\,R_1 + I\,R_2 = I\,(R_1 + R_2)$
- Same as the current flowing through equivalent resistance $R_{eq} = R_1 + R_2$

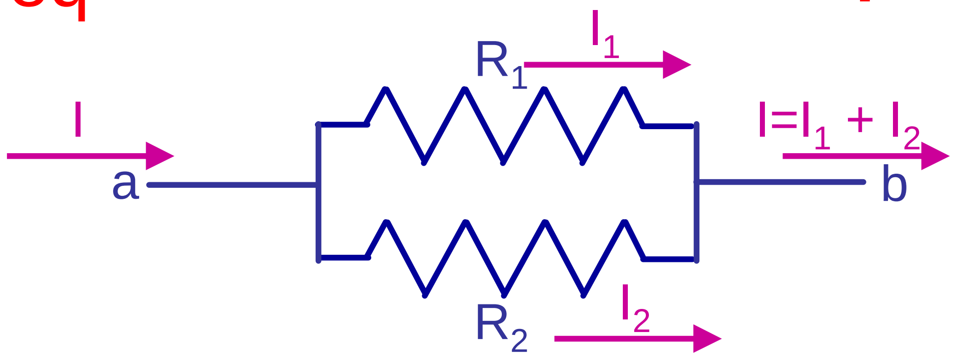


$$R_{eq} = R_1 + R_2$$

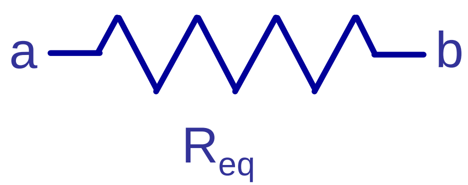
Resistors in series, comments

- It makes sense that the resistances add
- Remember, resistance is something that impedes the flow of current
- If the current has to go through both resistors, the current has to overcome two "obstacles" to its flow

R_{eq} for resistors in parallel

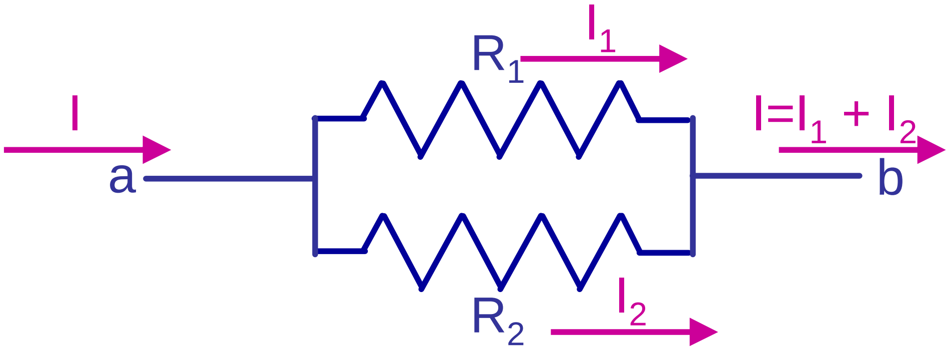


- The currents in the two resistors are different
- But the voltage drops across the two resistors are the same
  - $V_{ab} = I_1 R_1 = I_2 R_2$
  - $I = I_1 + I_2 = V_{ab} (1/R_1 + 1/R_2)$
- Same as the current flowing through equivalent resistance R_{eq}



$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2}$$

How does the current split between two resistors in parallel?

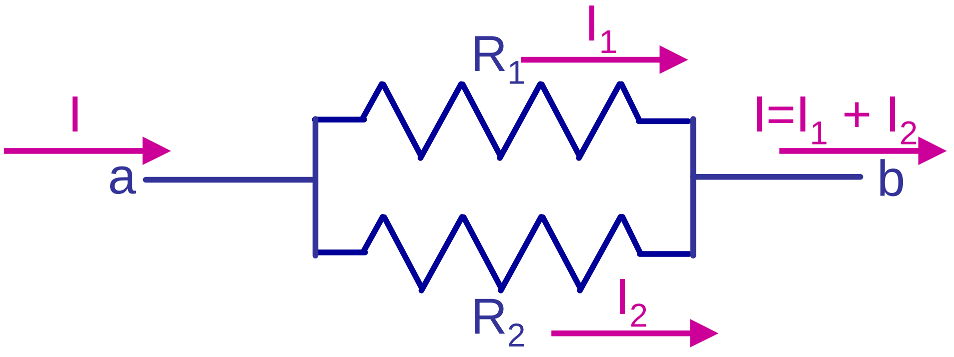


- $V_{ab} = I_1 R_1 = I_2 R_2$ → $\frac{I_1}{I_2} = \frac{R_2}{R_1}$
- The current wants to flow through the <u>smaller</u> resistor
  - Makes sense!

Resistors in parallel, comments

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \rightarrow R_{eq} = \frac{R_1 R_2}{R_1 + R_2}$$

- R_{eq} is always < R_1 and < R_2
- This also makes sense
- When the current encounters two resistors in parallel, there are two possible paths for the current to flow



- It makes sense that the current will have an easier time going past the "obstacle" than it would have if only one resistance was present

Summary and contrast with capacitors

Resistors in series:

$$R_{\text{eq}} = R_1 + R_2 + R_3 + ...$$

Resistors in parallel:

$$\frac{1}{R_{\text{eq}}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + ...$$

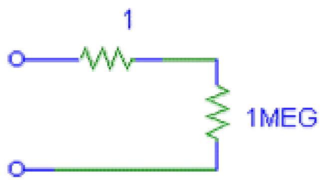
Capacitors in series:

$$\frac{1}{C_{\text{eq}}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + ...$$

Capacitors in parallel:

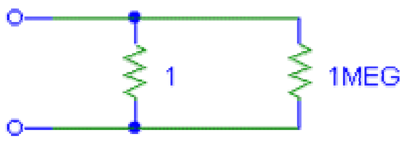
$$C_{\text{eq}} = C_1 + C_2 + C_3 + ...$$

A big resistor and a small resistor...



$R_{eq} = 1\,\Omega + 1\,M\Omega = 1{,}000{,}001\,\Omega$

The big resistor wins.....

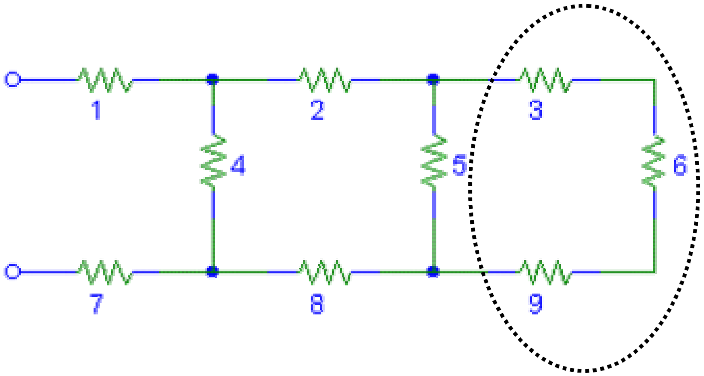


$$\frac{1}{R_{\text{eq}}} = \frac{1}{1\Omega} + \frac{1}{1\text{M}\Omega}$$

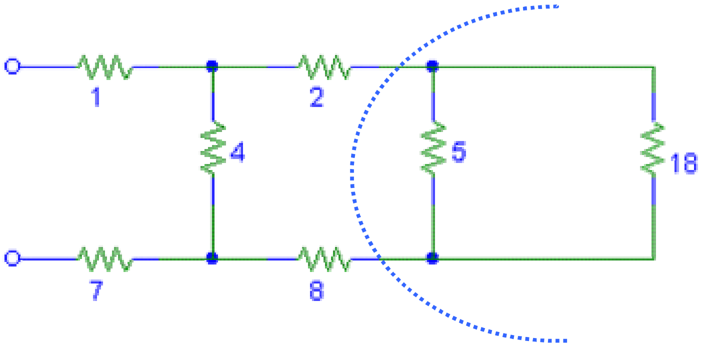
$R_{eq} = 0.999999\,\Omega$

The small resistor wins.....

Find the equivalent resistance



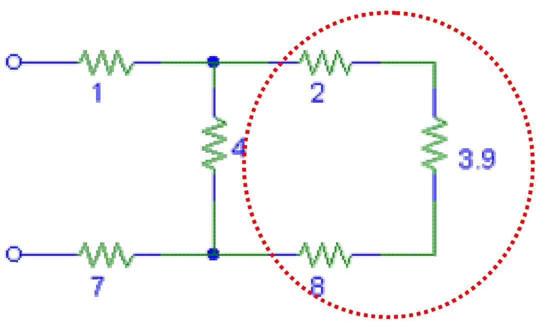
These three are in series.
R = 3 + 6 + 9 = 18 Ω



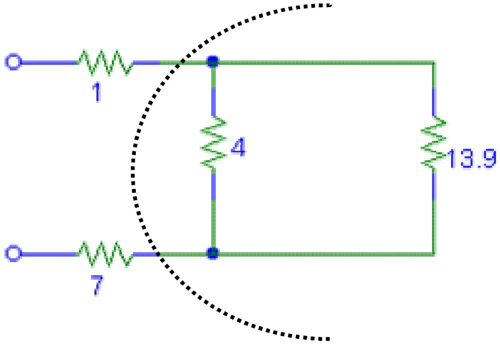
These two are in parallel

$$\frac{1}{R} = \frac{1}{5\Omega} + \frac{1}{18\Omega}$$

R = 3.9 Ω



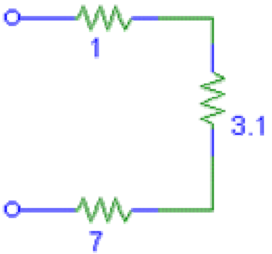
These three are in series
R + 2 + 3.9 + 8 = 13.9 Ω



These two are in parallel

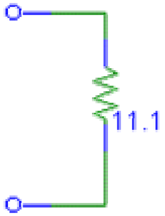
$$\frac{1}{R} = \frac{1}{4\Omega} + \frac{1}{13.9\Omega}$$

R = 3.1 Ω



Three resistors in series

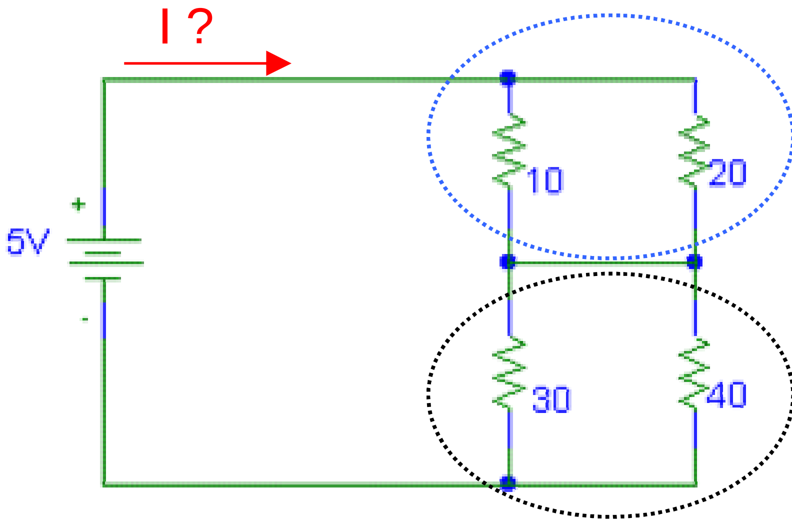
R = 1 + 3.1 + 7 = 11.1 Ω



R_{eq} = 11.1 Ω

Note – for two resistances in parallel, R_{eq} < than each individual one.
For two resistances in sereis, R_{eq} is greater

Another example

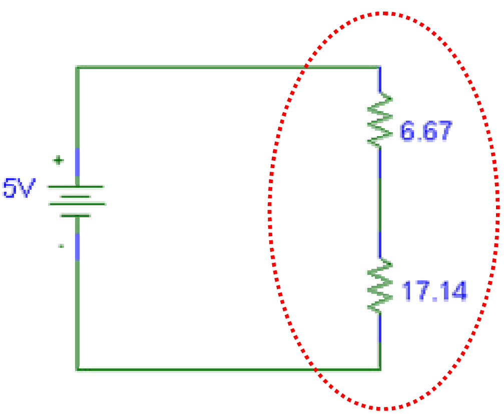


In parallel:

$$\frac{1}{R} = \frac{1}{10\Omega} + \frac{1}{20\Omega}$$
$$R = 6.67\Omega$$

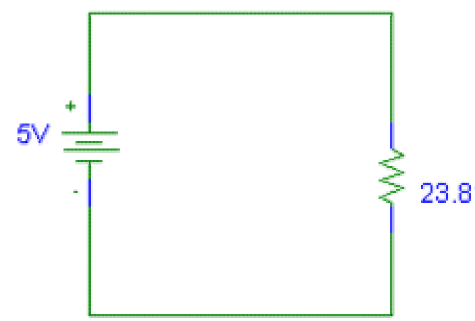
In parallel:

$$\frac{1}{R} = \frac{1}{30\Omega} + \frac{1}{40\Omega}$$
$$R = 17.14\Omega$$



In series:

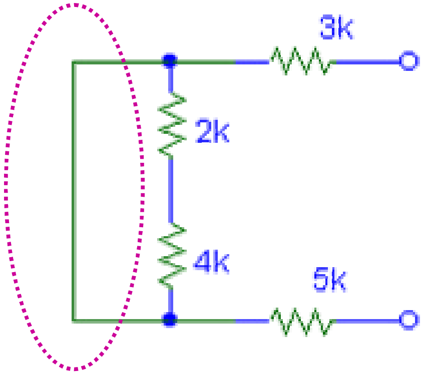
$$R = 6.67 + 17.14$$
$$R = 23.81\Omega$$

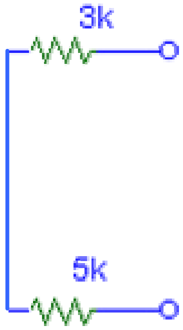


$$I = \frac{V}{R}$$
$$I = \frac{5\ \text{V}}{23.81\ \Omega} = 0.21\text{A}$$

Another example (k means kΩ)

"Short", R=0

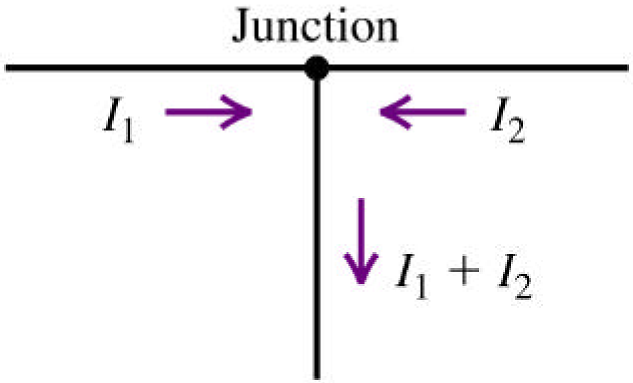




Series: R = 3 + 5 = 8 kΩ

Kirchoff's rules

- We have already applied them, at least implicitely
- First rule: at a node (or junction) ΣI = 0

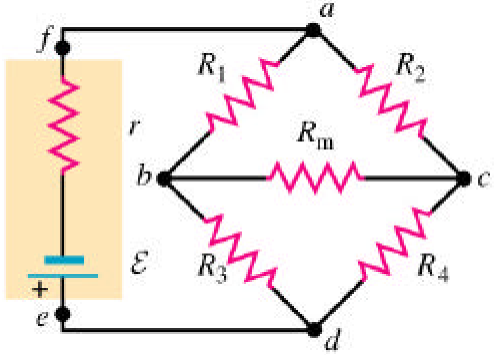


Careful about the signs!
It is a good idea to always draw the arrows!

- This is basically a statement that charge is conserved

Kirchoff's rules (continued)

- Second rule: the <u>total</u> voltage drop across a closed loop is zero



For example:

$V_{ab} + V_{bc} + V_{ca} = 0$

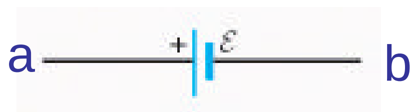
$(V_a – V_b) + (V_b – V_c) + (V_c – V_a) = 0$

But this holds for any loop,

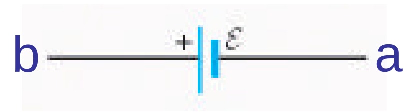
e.g. a-b-d-c-a or b-a-f-e-d-b, ...

Careful about signs:

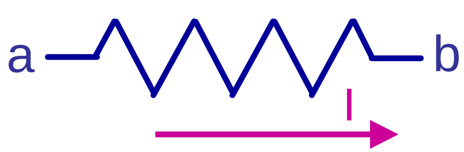
- When you apply Kirchoff's 2nd rule, the absolute value of the voltage drop across a resistor is IR and across a source of emf is ε
- You must keep the sign straight! For emf, it is easy. For resistors, depends on the sign of I



$V_{ab} = + \varepsilon$



$V_{ab} = - \varepsilon$

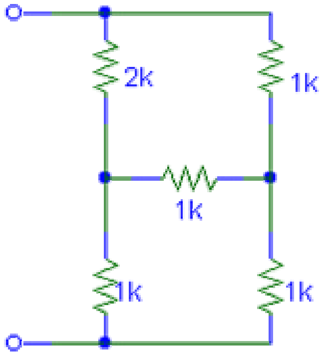


$V_{ab} = IR$

$V_{ab} = -IR$

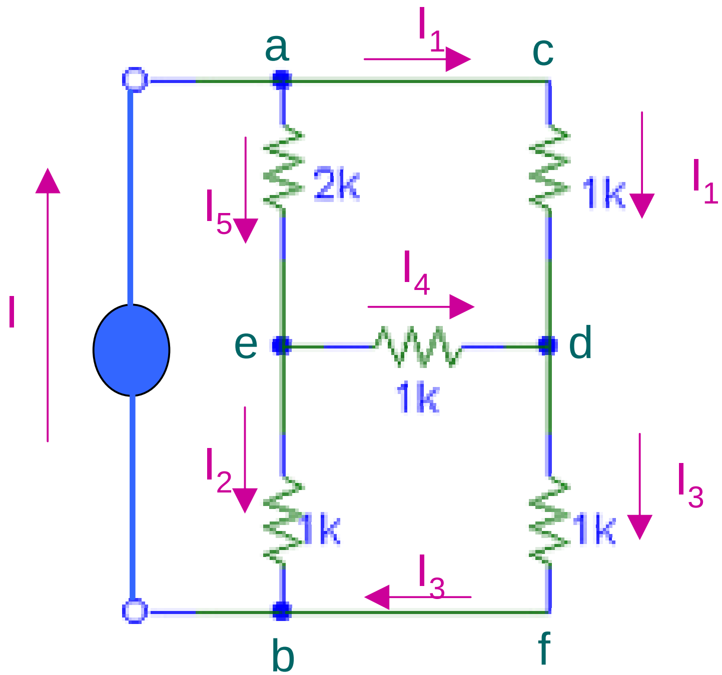
These signs are such that current always runs from high to low potential. Note algebraically I can be +ve or –ve. What matters is your convention, i.e., the direction of the arrow

Another example: find R_{eq}



Add a fictitious source of current, say I=1A, across the terminals. Then if I can calculate the voltage across the terminals, R = V/I.

Also, label everything!!!!!



<u>Kirchoff law for current:</u>

Node a: $I = I_1 + I_5$

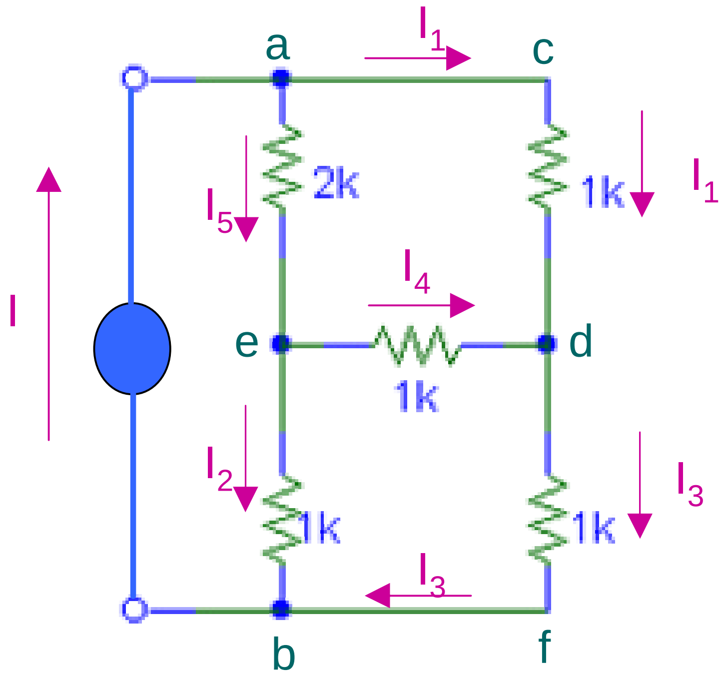
Node e: $I_5 = I_4 + I_2$

Node d: $I_4 + I_1 = I_3$

Node b: $I_2 + I_3 = I$

4 equations, 5 unknowns

(I_1, I_2, I_3, I_4, I_5)



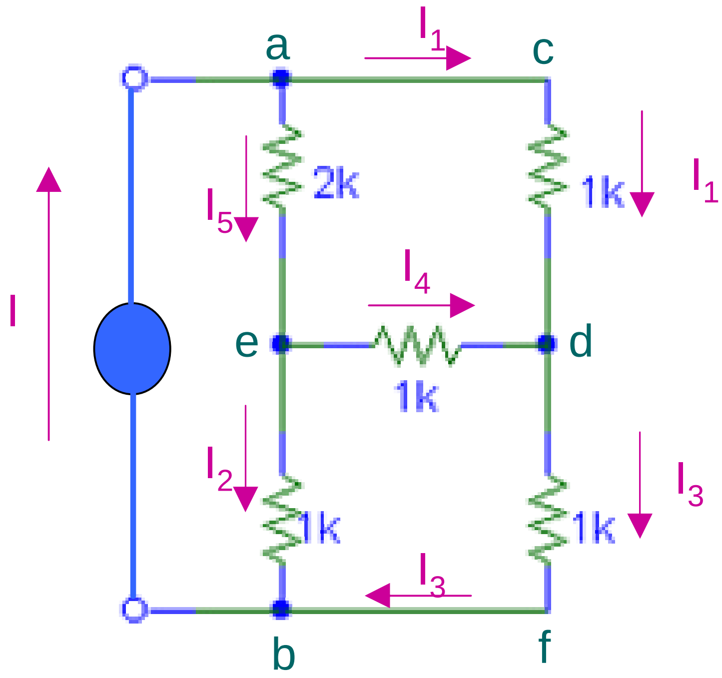
Kirchoff law for current:

Node a: $I = I_1 + I_5$
Node e: $I_5 = I_4 + I_2$
Node d: $I_4 + I_1 = I_3$
Node b: $I_2 + I_3 = I$

Now I apply Kirchoff law for voltage loops.
But let's be smart about it!
I want V_{ab} (or V_{cf}, they are the same)
$V_{ab} = 2I_5 + I_2$
$V_{cf} = I_1 + I_3$
Try to eliminate variables and remain with I_5 , I_2

Node b: eliminates I_3: $I_3 = I - I_2$
Node a: eliminates I_1: $I_1 = I - I_5$
acdea loop: $I_1 - I_4 - 2I_5 = 0$ → $I - I_5 - I_4 - 2I_5 = 0$ → $I_4 = I - 3 I_5$

Node e: $I_5 = (I - 3I_5) + I_2$ → $4I_5 - I_2 = I$
acdfbea loop: $I_1 + I_3 - I_2 - 2I_5 = 0$
$(I - I_5) + (I - I_2) - 2 I_5 = 0$
$2I - 2I_2 - 3I_5 = 0$



$4I_5 – I_2 = I$

$2I – 2I_2 – 3I_5 = 0$

Solution:

$I_2 = 5I/11$ and $I_5 = 4I/11$

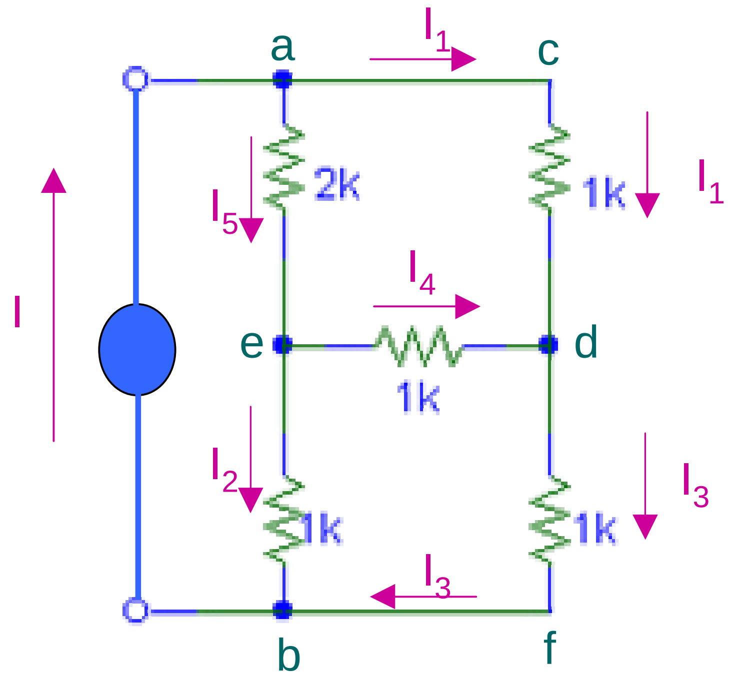
Then:

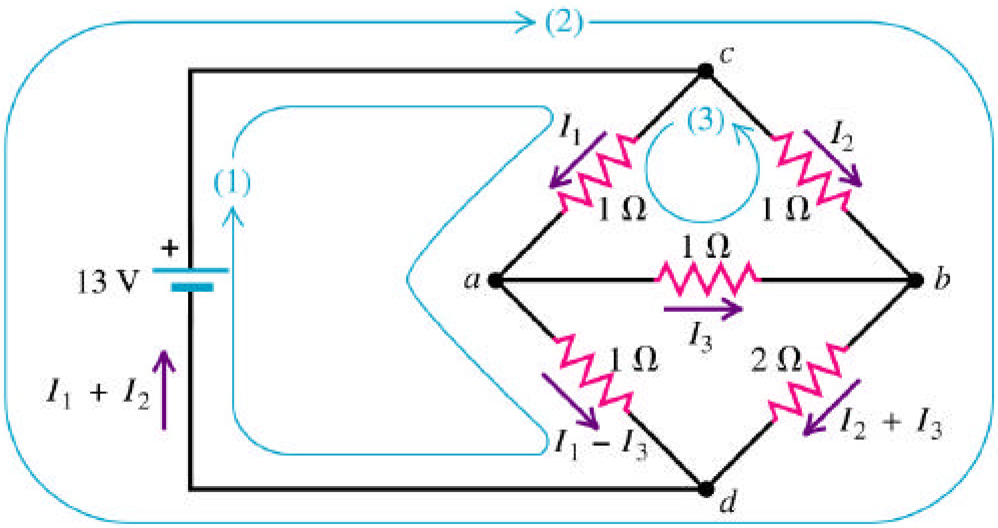
$V_{ab} = 2I_5 + I_2$

$V_{ab} = 13I/11$

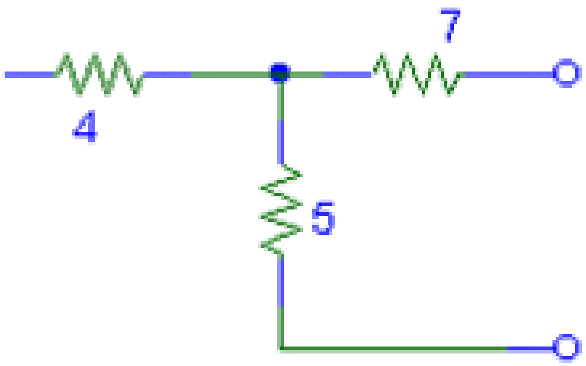
$R_{eq} = 13/11\ k\Omega = 1.2\ k\Omega$

Note: Example 26.6 in the textbook is essentially the same. (It has resistances of Ω instead of kΩ)
The book solves it using a battery rather than a fictitious current source. But the answer is the same, as it should!
Check out the alternative method for yourself!



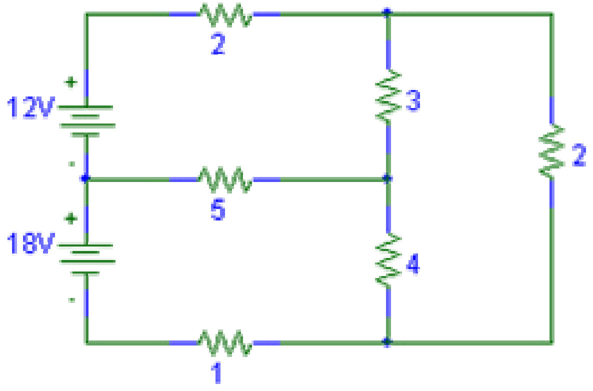


Find R_{eq} of this circuit:



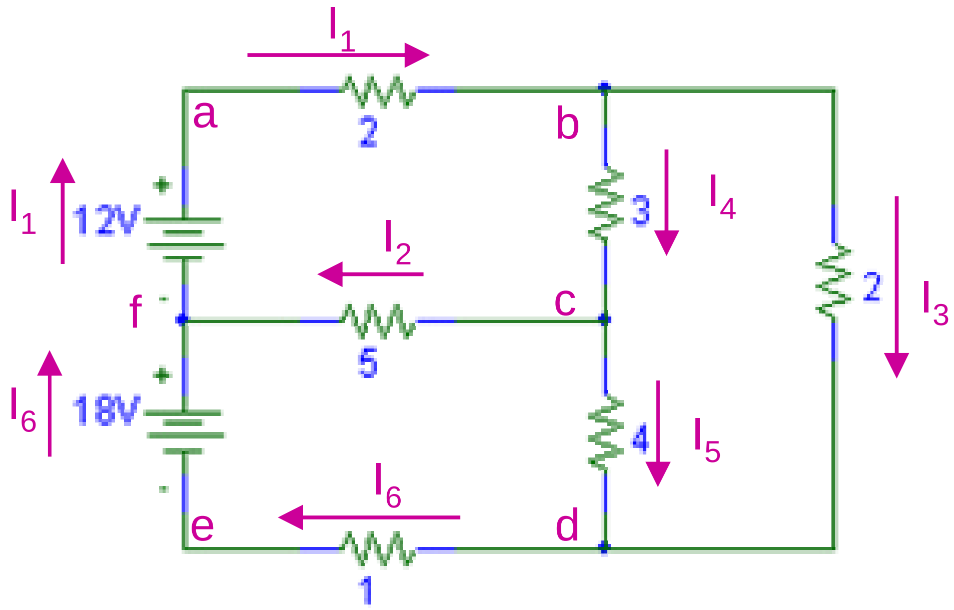
- The other side of the 4 Ω resistor is not connected to anything
- It is as if it was not there!
- Two resistors in series, $R = 7\ \Omega + 5\ \Omega = 12\ \Omega$

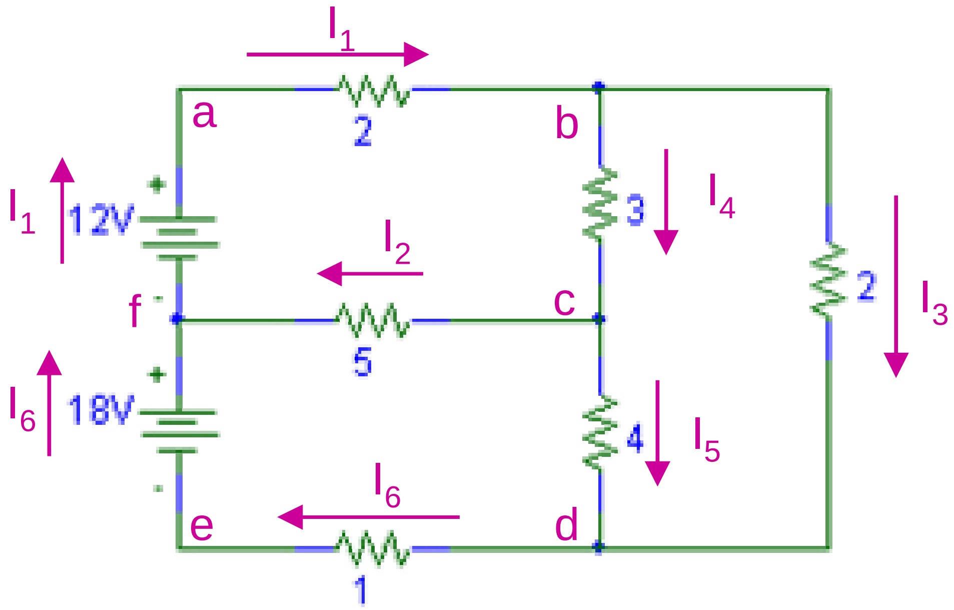
Another problem.....



Find the current through the 5 Ω resistor

First step: Label everything!





Use Kirchoff law for current nodes to eliminate some variables.

Node c: $I_4 = I_2 + I_5$ → eliminate I_4

Node d: $I_6 = I_5 + I_3$ → eliminate I_6

Node b: $I_1 = I_3 + I_4 = I_3 + I_2 + I_5$ → eliminate I_1

Node f : $I_6 + I_2 = I_1$

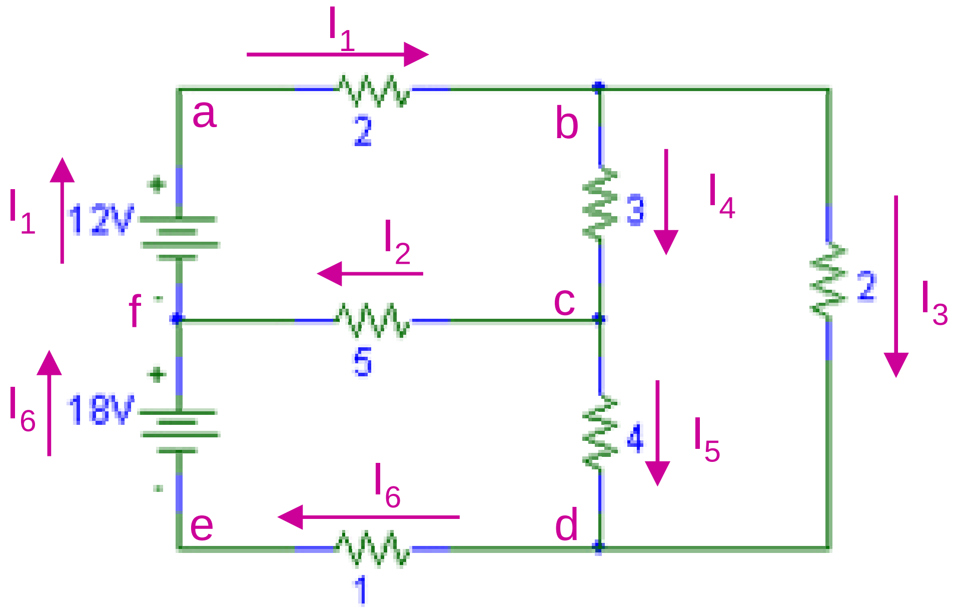
$(I_5 + I_3) + I_2 = I_3 + I_2 + I_5$ → no extra information !

Now everything is in terms of I_2, I_3 and I_5

It is easy to eliminate I_3:

Loop bdb: $3I_4 + 4I_5 - 2I_3 = 0$

$3(I_2+I_5) + 4I_5 - 2I_3 = 0$ → $I_3 = ½ (3I_2 + 7I_5)$



All currents in terms of I_2 and I_5:

$I_1 = ½ (5I_2 + 9I_5)$

$I_3 = ½ (3I_2 + 7I_5)$

$I_4 = I_2 + I_5$

$I_6 = ½ (3I_2 + 9I_5)$

Loop fabcf:

$-12 + 2I_1 + 3I_4 – 5I_2 = 0$

$-12 + 5I_2 + 9I_5 + 3I_2 + 3I_5 – 5I_2 = 0$

$12I_5 + 3I_2 = 12$

Loop fcdef

$-5I_2 + 4I_5 + I_6 – 18 = 0$

$-5I_2 + 4I_5 + 3/2\ I_2 + 9/2\ I_2 – 18 = 0$

$13I_5 – 7I_2 = 36$

Two equations, two unknowns, can solve, get $I_2 = - 2.24$ A