Fall 2004 Physics 3
Tu-Th Section

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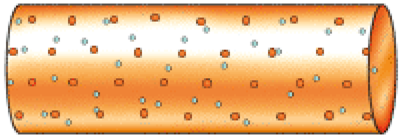
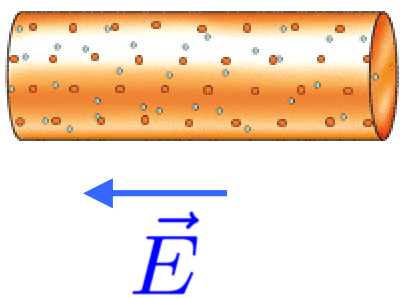
Electrical Current

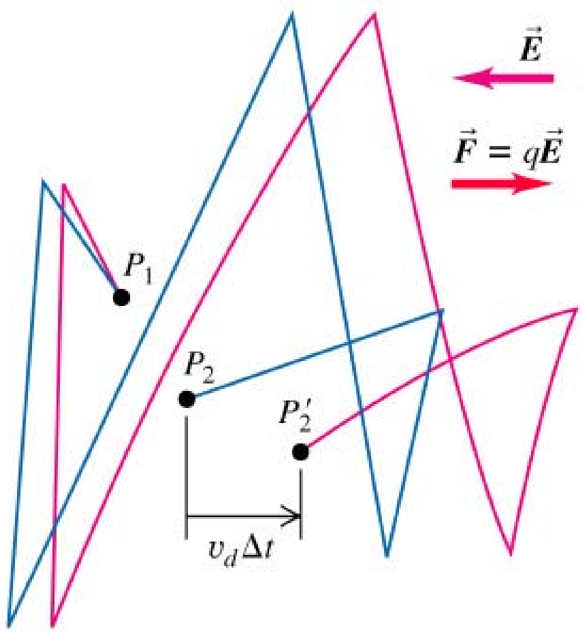
- Electrical current is the net flow of electric charge in a material
  - e.g., a wire
- Remember: a conductor contains free charges (electrons)
- The electrons are in constant motion
  - In fact they move very fast ~ 10^6 m/sec
  - They bounce off the atoms of the lattice
  - Ordinarily, they move in random directions
  - Ordinarily, no net flow of charge



- Now imagine we set up an electric field <u>inside</u> the conductor
- The free charges (electrons) will feel a force F=qE
- They get accelerated in the direction opposite to the electric field
  - Opposite because electrons have –ve charge
- You would think that they should gain more and more velocity
- But they don't because they tend to quickly collide with the atoms of the lattice and their direction gets randomized

- The net effect is that electrons in a conductor in the presence of an electric field tend to <u>drift</u> in the direction opposite the electric field





- The drift velocity (= net velocity of the electrons) is quite small, typically less than mm/sec

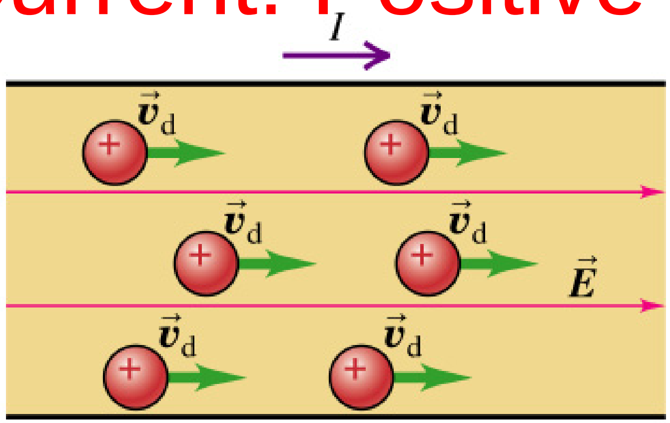
Careful about electric field in a conductor!

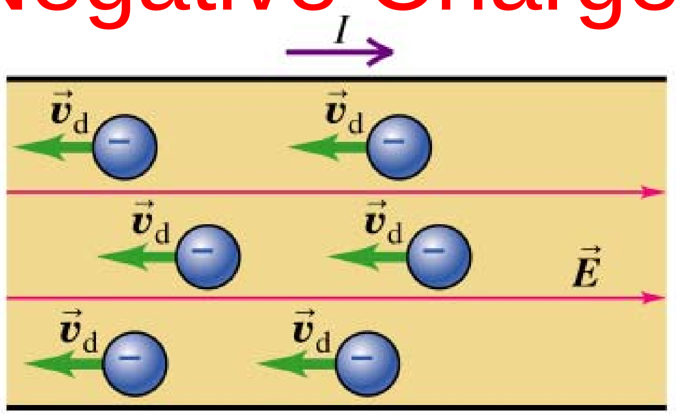
- Up until today, we always said that there is no electric field inside a conductor
- But now we arguing about what happens when there is an electric field inside a conductor!
- Up until today, we have been concerned with electrostatic situations (= the charges do not move)
- Today we start to discuss electrical current, i.e., charges in motion

E-field in conductors (cont.)

- Our statement "no E-field inside a conductor" was based on the argument that if the E-field is not zero then the charges will move and rearrange themselves in such a way as to make E=0

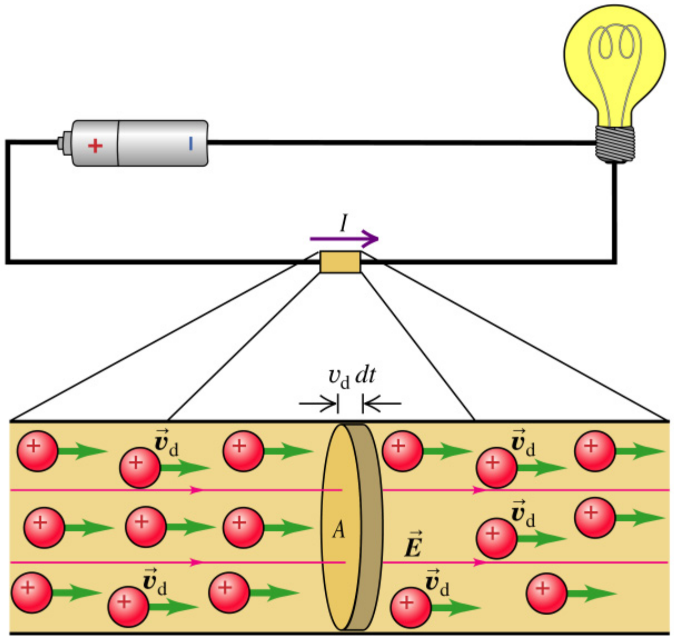
Current: Positive vs Negative Charges





- Convention: current is defined in the direction of drift of positive charges
- In a metal, the charges that drift are electrons, so current is in the opposite direction as the drift of electrons
  - ➢ a bit awkward, and mostly historical
- In a chemical solution the charges can be both positive and negative (ions)

Definition of Current



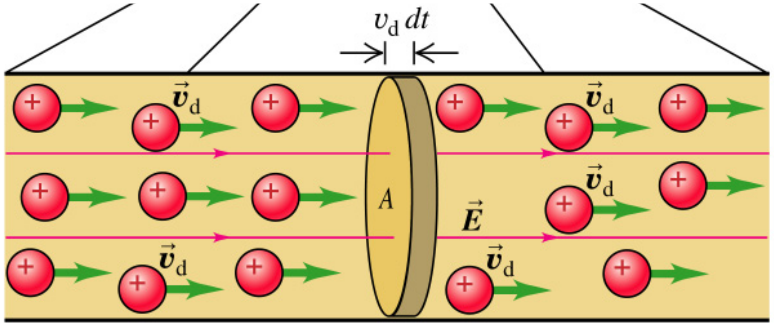
- Net charge flowing through the <u>total area</u> per unit time

$$I = \frac{dQ}{dt}$$

Units of Current

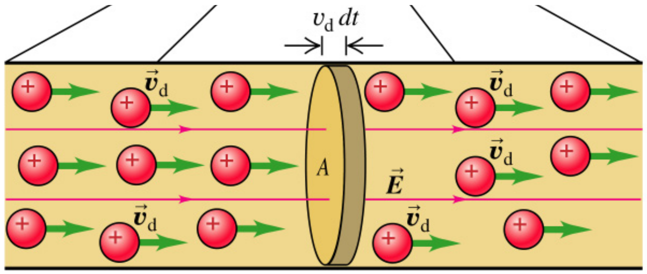
- I=dQ/dt → [I] = Coulomb/sec
- 1 Coulomb/sec = 1 A (Ampere)
- The Ampere is one of the four fundamental units of the international system of units (SI)
  - meter
  - Kg
  - sec
  - Ampere
- It is formally defined in terms of the force between two parallel wires
  - You'll see it in Physics 4

Relationship between I and v_d



- $I = dQ/dt$
- In time dt, every charge moves $dx = v_d dt$
- All the charges in a volume $dV=Adx$ will flow through the area
- $dQ = n\,q\,dV$
    - n = number of charges/unit volume
- $dQ = n\,q\,A\,v_d\,dt$
- $I = dQ/dt = n\,q\,v_d\,A$

Current Density



- $I = n\,q\,v_d\,A$
- Definition of current density: current per unit area
- $J = I/A = n\,q\,v_d$
- This can also be defined vectorially as

$$\vec{J} = nq\vec{v}_d$$

- Note, if $q<0$ the vector current density and the vector drift velocity point in opposite direction
  - **as they should!**

What is n?

- n = number of charges / unit volume
- In metals, charges = electrons
- $n = n' N \rho$
  - N = number of atoms per Kg
  - ρ = density in Kg/m^3
  - n' = number of free electrons per atom
- Example, Cu
  - n' = 1
  - $\rho = 9\ 10^3\ Kg/m^3$
  - Mass of Cu atom = 63.6 amu = 63.6 ($1.7\ 10^{-27}$ Kg)

    → 1 Kg of Cu → $N = 9.2\ 10^{24}$ atoms
- Putting it together: $n = 8\ 10^{28}\ /\ m^3$

Typical value of v_d

- $I = n\, q\, A\, v_d$
- Take I=1A, and 1 mm diameter wire

$$v_d = \frac{I}{nqA}$$

$$v_d = \frac{1A}{(8\cdot 10^{28}\mathrm{m}^{-3})(1.6\cdot 10^{-19}\mathrm{C})\pi(0.5\cdot 10^{-3}\mathrm{m})^2}$$

$$\boxed{v_d = 0.1\ \mathrm{mm/sec}}$$

Resistivity

- Current density $J = I/A = n\ q\ v_d$
- It is not surprising that the drift velocity depends on the electric field
  - ➢ Higher drift velocity → higher E-field
- For many materials and in many situations the drift velocity is proportional to electric field. Then

$$E = \rho J \quad \text{(Ohm's Law)}$$

- ρ = resistivity

Resistivity (cont.)

- $E = \rho J$ or $J = (1/\rho)E$
- ρ is a property of the material
- For a given field, the smaller ρ the larger the current J
- ρ is a measure of how easy it is for a material to conduct electricity
  - small ρ, good conductor
  - large ρ, poor conductor

Units of Resistivity

- $\rho = E/J$
- $[\rho] = (V/m) / (A/m^2) = (V/A)\ m$
- $1\ V/A = 1\ \text{Ohm} = 1\ \Omega$

Resistivity for some materials

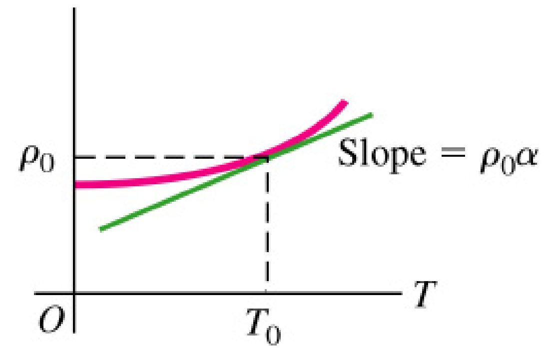
metals (conductors)	Al	$2.8\ 10^{-8}\ \Omega$-m
	Cu	$1.7\ 10^{-8}\ \Omega$-m
	Au	$2.4\ 10^{-8}\ \Omega$-m
semiconductors	Ge	$0.6\ \Omega$-m
	Si	$2300\ \Omega$-m
insulators	Quartz	$8\ 10^{17}\ \Omega$-m
	Teflon	$> 10^{13}\ \Omega$-m
	Glass	10^{10}-$10^{14}\ \Omega$-m

Conductivity

- Simply defined as the inverse of resistivity
- $\sigma = 1/\rho$
- High conductivity = good conductor
- Low conductivity = bad conductor
- Measured in $(\Omega\text{-m})^{-1}$

Resistivity vs Temperature (1)

- In a conductor the "resistance" to the flow of electrons occurs because of the collisions between the drifting electrons and the lattice
- When T increases, lattice atoms vibrate more violently
- Collisions more frequent
- Resistivity increases



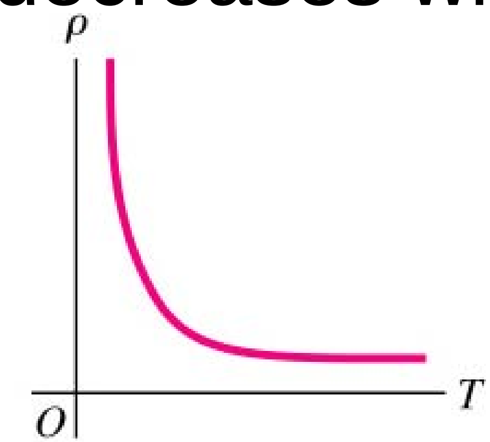
- Approximate linear dependence near room temperature

$$\rho(T) = \rho_0[1 + \alpha(T - T_0)]$$

α depends on material, typically fraction of per-cent per degree

Resistivity vs Temperature (2)

- In a semiconductor as T increases more electrons are shaken loose from the atoms in the lattice
- The number of charge carriers increases with temperature
- The resistivity decreases with temperature



Resistivity vs Temperature (3)

- In some materials (superconductors) the resistivity becomes ZERO below some "critical temperature" T_C

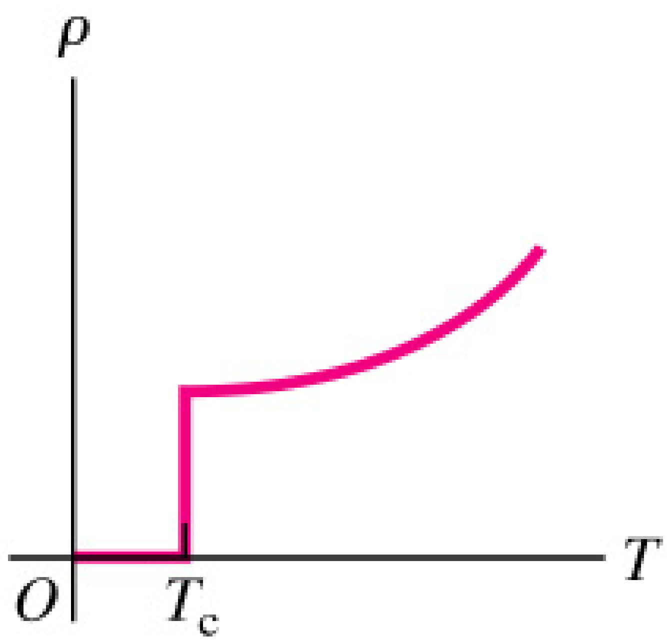


Table of T_C

Table 4. Critical temperatures of some superconductors.

Compound or Element	T_C (K)	Compound or Element	T_C (K)
Mercury	4	Nb_3Sn	18
Vanadium	5.4	Nb_3Ge	23
Lead	7.2	$Ba_{0.6}K_{0.4}BiO_3$	30
Technetium	7.8	$Cs_2Rb@C_{60}$	33
Niobium	9.5	MgB_2	39
Sulfur (at 93 Gpa)	10	$La_{1.85}Sr_{0.15}CuO_4$	40
$(CH_3CH_2)_2Cu(NCS)_2$	11.4	$Tl_2Ba_2CuO_6$	80
$LiTi_2O_4$	12	$YBa_2Cu_3O_7$	93
$BaPb_{0.75}Bi_{0.25}O_3$	13	$Tl_2Ba_2CaCu_2O_8$	105
YNi_2B_2C	15.5	BiScCO ($BiSr_2Ca_3Cu_3O_{10}$)	110
NbN	16	$Tl_2Ba_2Ca_3Cu_4O_{12}$	115
V_3Ga	16.5	$Tl_2Ba_2Ca_2Cu_3O_{10}$	125
Sulfur (at 160 Gpa)*	17	$HgBa_2Ca_2Cu_3O_{10}$	134
V_3Si	17	$HgBa_2Ca_2Cu_3O_{10}$ (at 30 Gpa)**	164
Nb_3Al	17.5		

*Highest reported Tc for an element **Highest reported T_C to date

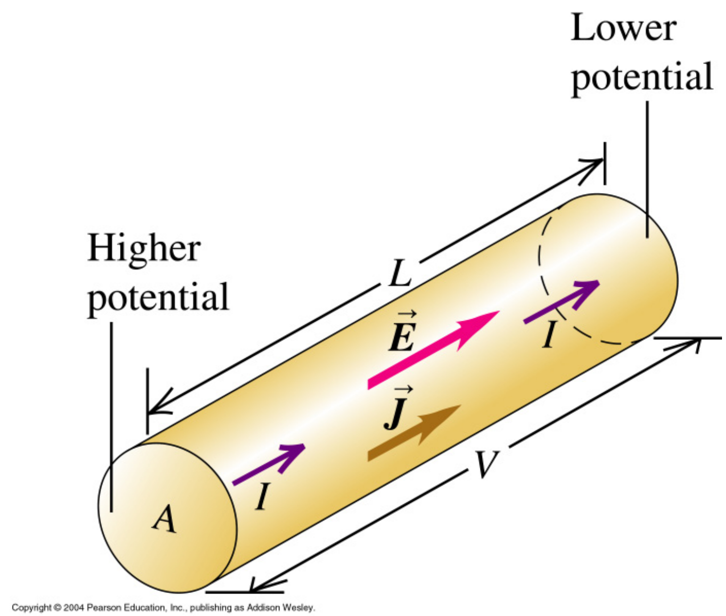
Resistance

- Ohm's Law: $E = \rho J$
- Not very convenient because
  - ➢ We are more often interested in the current I rather than the current density $J=I/A$
  - ➢ It is easier to use potential rather than field
- Consider cylindrical conductor
- $V_{ab} = V = E L$
- $I = J A$
- Ohm's Law:

$(V/L) = \rho\ (I/A)$

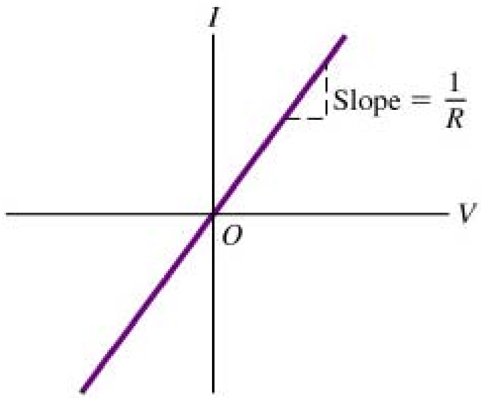
$$V = \frac{\rho L}{A} I = RI$$

R = resistance. Units: Ω

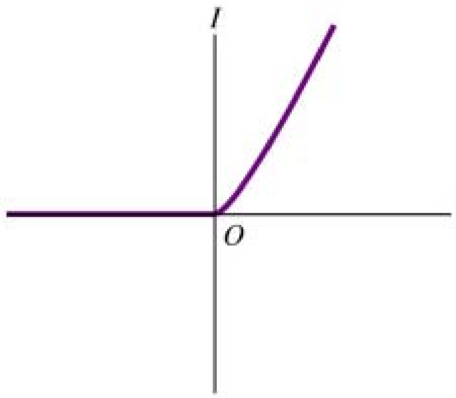


Ohm's Law

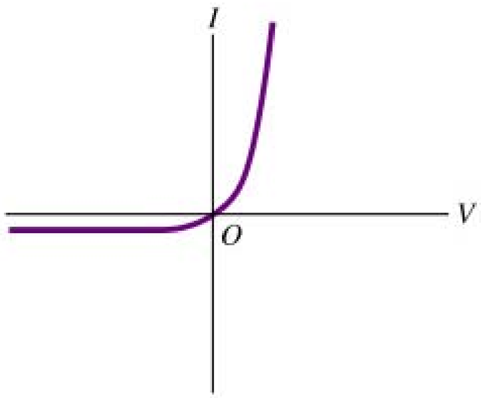
- The most "useful" (common?) way of writing down Ohm's law is I = V/R
- The current is proportional to the voltage
- Applies to many materials, but not all!



(a) Resistor that obeys Ohm's law

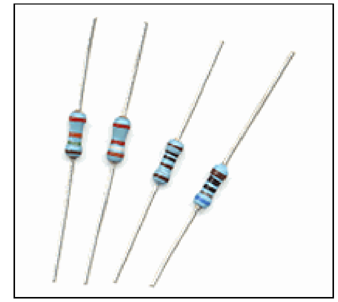


(b) Vacuum diode (does not obey Ohm's law)

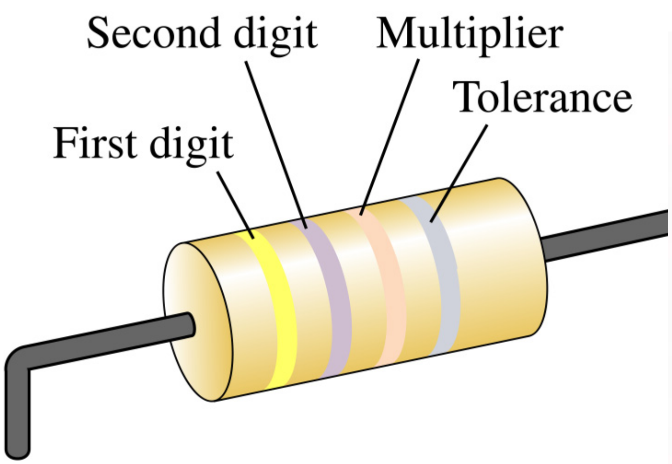


(c) Semiconductor diode (does not obey Ohm's law)

Resistors



- Circuit elements of well-defined resistance
- They almost always have color-coded bands that allow you to read-off the resistance



Color	Value as Digit	Value as Multiplier
Black	0	1
Brown	1	10
Red	2	10^2
Orange	3	10^3
Yellow	4	10^4
Green	5	10^5
Blue	6	10^6
Violet	7	10^7
Gray	8	10^8
White	9	10^9