Fall 2004 Physics 3
Tu-Th Section

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Lecture 13: 9 Nov. 2004

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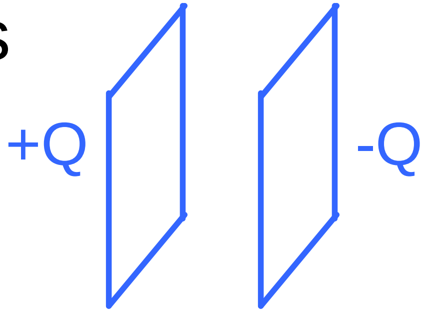
Last Time: Capacitors

- A capacitor is a system of two conductors
- The "classical" mental picture of a capacitor is two parallel conductive planes



- If we have charge +Q on one conductor and –Q on the other conductor, then there will be a potential difference V between the two
- The capacitance is defined as $C = \frac{Q}{V}$
  - ➢ depends on the geometry
    - shape of conductors
    - distance between the conductors
    - material (if any) between the conductors

More on capacitors (last time)

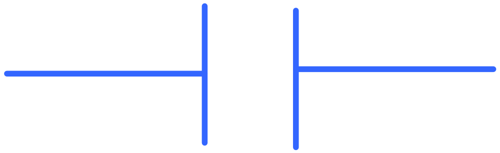
- Capacitors store charge and electrical potential energy
- The energy stored on a capacitor is equal to the work done to charge the capacitor

$$U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

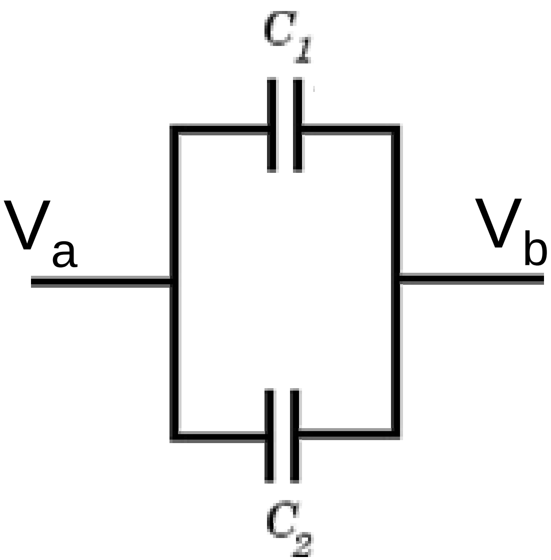
- Units of capacitance:

  [C]=[Charge]/[Potential] = Coulomb/Volt = Farad (F)

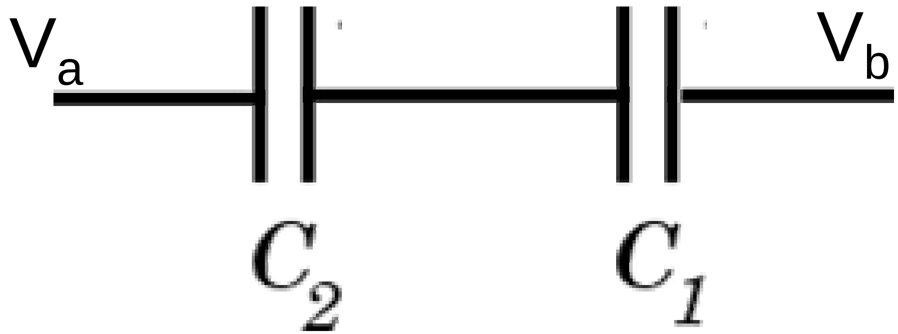
- Symbol of capacitance in circuits



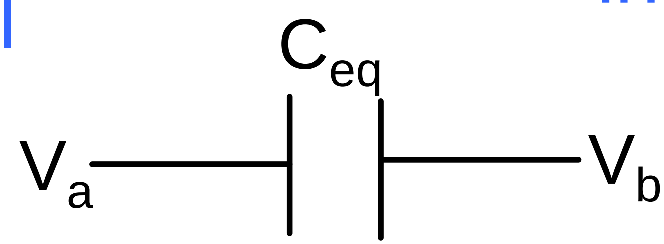
Connecting them together (last time)



in parallel



in series



$$C_{eq} = C_1 + C_2$$

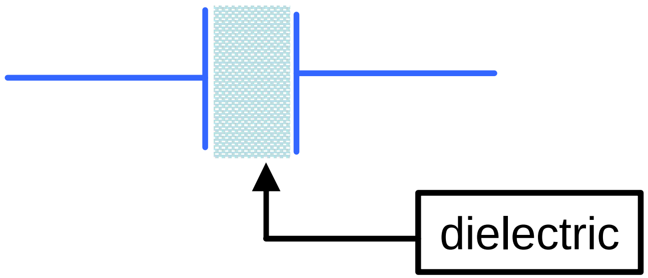
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2}$$

Calculations of C (last time)

- We calculated capacitances for a few configuration of conductors
  - Parallel plate $C = \epsilon_0 \frac{A}{d}$
  - Concentric spherical shells
  - Concentric cylinders
- General method for these calculations is
  - Calculate the field between the two conductors
  - From the field, calculate V_{ab} from $V_{ab} = \int E dl$
  - Take C=Q/V

Dielectrics

- If we insert an insulating material (*dielectric*) between the two conductors the capacitance increases



- Let C_0 = capacitance with no dielectric (vacuum)
- Let C = capacitance with dielectric
- Definition of dielectric constant (K)

$$K = \frac{C}{C_0} > 1$$

Dielectric constants

- A property of the material
- Some typical values

Vacuum	1
Air (1 atm)	1.0006
Glass	5-10
Teflon	2.1
Mylar	3.1
Water	81
Strontium titanate	310

Glycerin	43
Plexiglass	3.4
Mica	3-6
Tantalum	11
Ceramics	35-6,000

Capacitor with and without dielectric

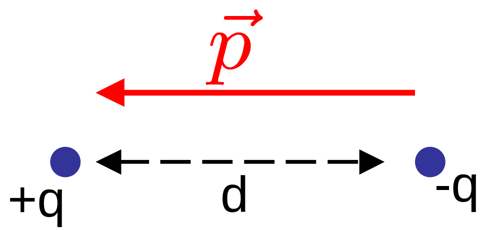
- Capacitance in vacuum C_0
- Capacitance with dielectric $C=KC_0$
- Suppose we put the same charge Q on the two capacitors (with and without dielectric)
- Voltages with and without V and V_0
- $Q=C_0V_0$ and $Q=CV$

$$C_0V_0 = CV = KC_0V \quad \rightarrow \quad \boxed{V = \frac{V_0}{K}}$$

For a given charge, the voltage across a capacitor is reduced in the presence of dielectric

What is going on?

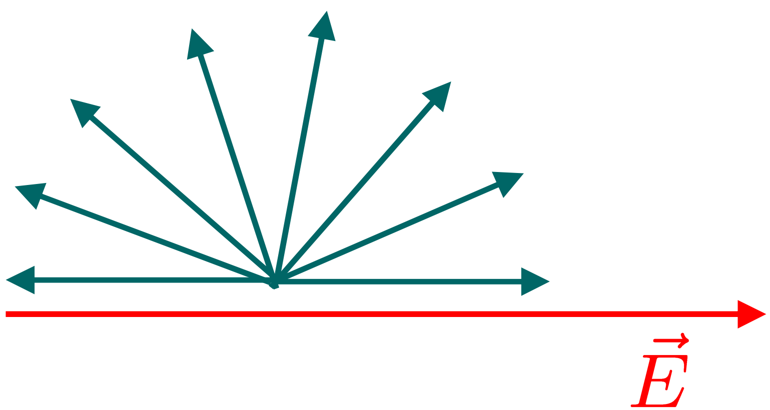
- Remember <u>electric dipole</u>



$$p = qd$$

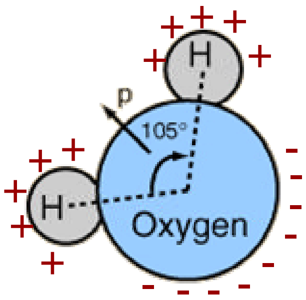
- In electric field, there is a torque on the dipole which tends to align it with the field

$$\vec{\tau} = \vec{p} \times \vec{E}$$

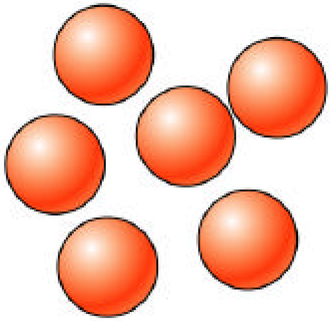


- Some molecules, despite being neutral, have a "natural" imbalance of charge
  - They are dipoles

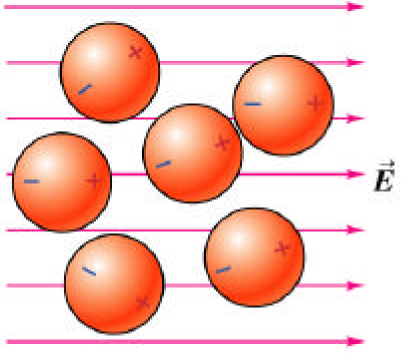
H_20 molecule:



- But even molecules with totally symmetric charge distributions become <u>polarized</u> in electric field

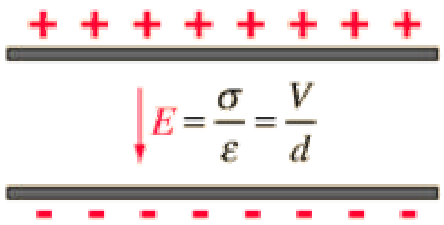


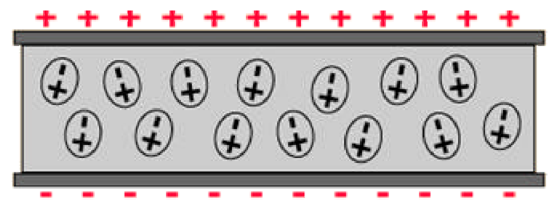
(a) Nonpolar molecules, no applied electric field



(b) Nonpolar molecules with applied electric field

What are the consequences of the presence of dipoles in the dielectric?

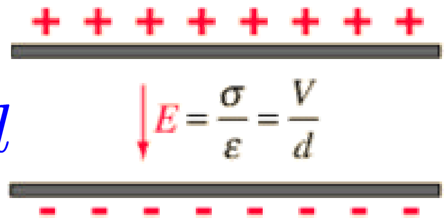


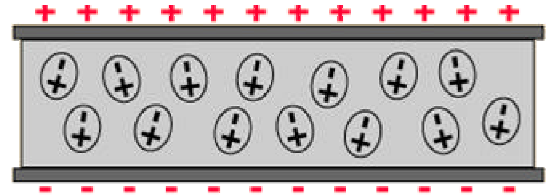


- Some of the original electric field is neutralized
- In this picture the dielectric effectively puts net negative charge on the top plate, net positive charge on the negative plate
- Call σ_i the surface charge density due to the dielectric
    - ➢ the subscript i here stands for "induced"

$$E = \frac{\sigma - \sigma_i}{\epsilon_0}$$

$$E_0 = \frac{\sigma}{\epsilon_0}$$
$$V_0 = E_0 d$$





$$E = \frac{\sigma - \sigma_i}{\epsilon_0}$$
$$V = Ed$$

But we also had $V_0 = KV$

$$\frac{\sigma}{\epsilon_0} d = K \frac{\sigma - \sigma_i}{\epsilon_0} d$$

$$\sigma_i = \sigma\left(1 - \frac{1}{K}\right)$$

Note: as K gets very large $\sigma_i \rightarrow \sigma$ and the induced density (almost) cancels the original density

$$Ed = \frac{V_0}{K} = \frac{E_0}{K} d = \frac{\sigma}{\epsilon_0} d$$

$$E = \frac{\sigma}{K\epsilon_0} = \frac{\sigma}{\epsilon}$$

$\varepsilon = K\varepsilon_0$ = permittivity of dielectric

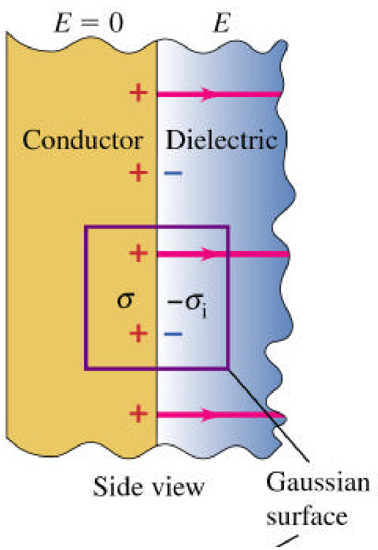
Parallel plate capacitor revisited

$$C = KC_0 = K\epsilon_0 \frac{A}{d} = \epsilon \frac{A}{d}$$

(parallel-plate capacitor, dielectric between plates)

This is what (almost) always happens with a dielectric.
Replace ε_0 with $\varepsilon = K\varepsilon_0$

Gauss's Law in dielectric



$$\Phi = \frac{Q_{\text{enclosed}}}{\epsilon_0}$$

$$EA = \frac{(\sigma - \sigma_i)A}{\epsilon_0}$$

$$\sigma_i = \sigma(1 - \frac{1}{K})$$ from before

$$\sigma - \sigma_i = \frac{\sigma}{K}$$

$$KEA = \frac{\sigma A}{\epsilon_0}$$

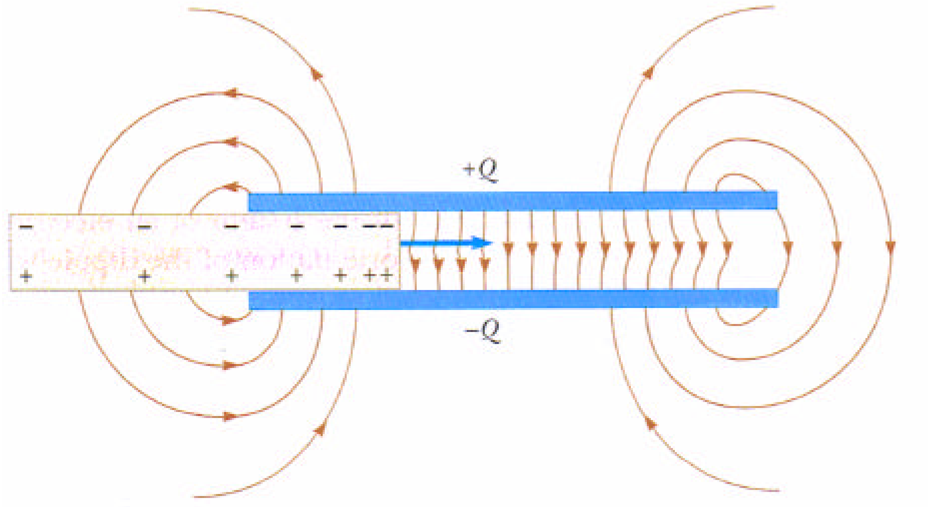
We showed this in
a very special situation
but it holds generally

$$\oint K\vec{E} \cdot d\vec{A} = \frac{Q_{\text{encl-free}}}{\epsilon_0}$$

Example 1

- Parallel plate capacitor, capacitance C_0. Charged to charge Q. Battery is removed. Slab of material dielectric constant K is inserted. What is the change in energy of the system?

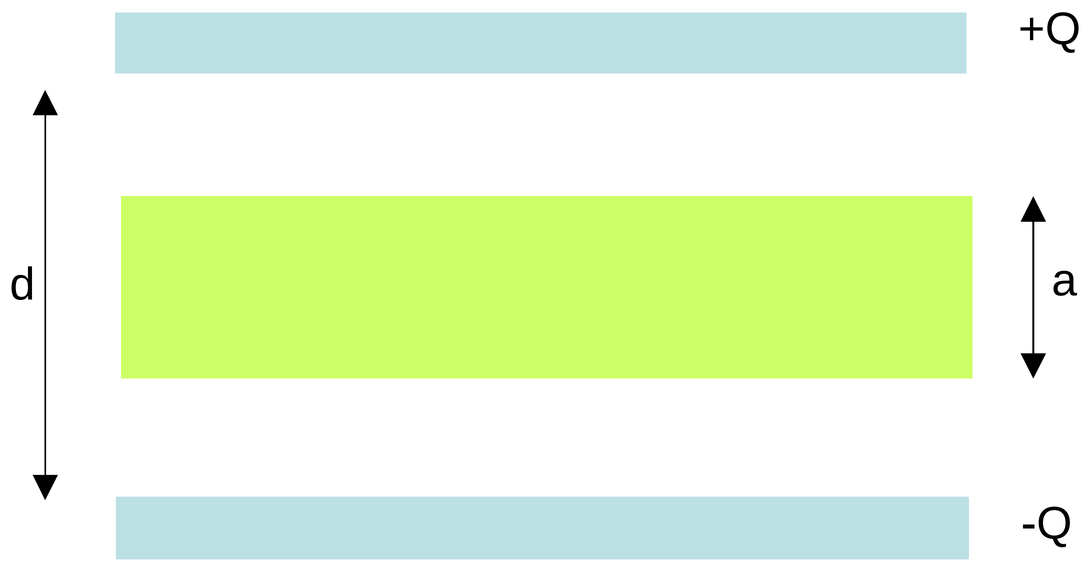
- Initial energy $U_{init} = Q^2/2C_0$
- Key concept: <u>the charge does not change</u>
- Final energy $U_{final} = Q^2/2C = Q^2/2KC_0 = U_{init}/K$
- $U_{final} < U_{init}$ because $K>1$
- What happened to the energy?



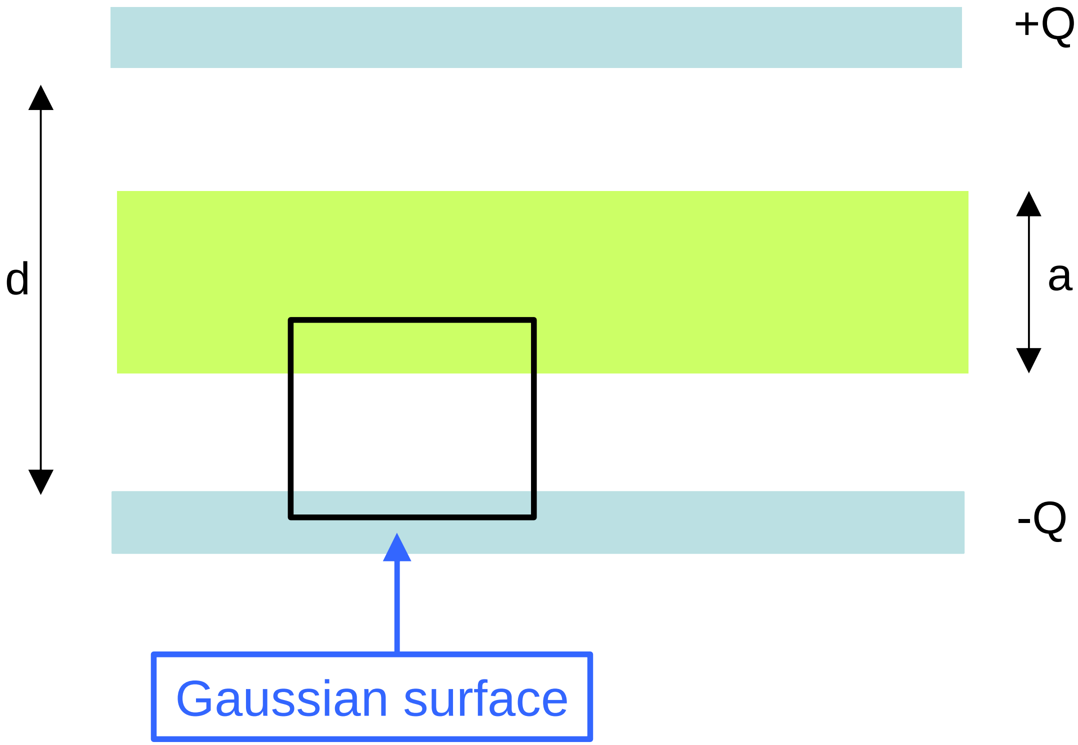
- The dielectric is being pulled into the capacitor by the fringe (edge) field
- Work is done by the capacitor
- This is reflected by a lower potential energy after the dielectric is completey inside

Example 2

- Parallel plate capacitor. Plate separation d and plate area A. An uncharged metallic slab of thickness a is inserted midway between the plates. Find the capacitance.

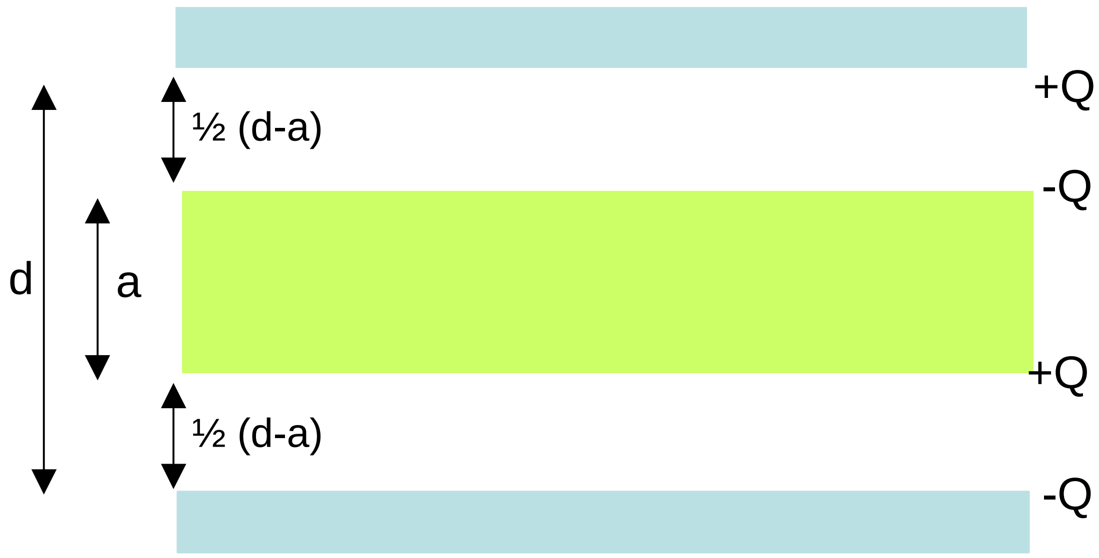


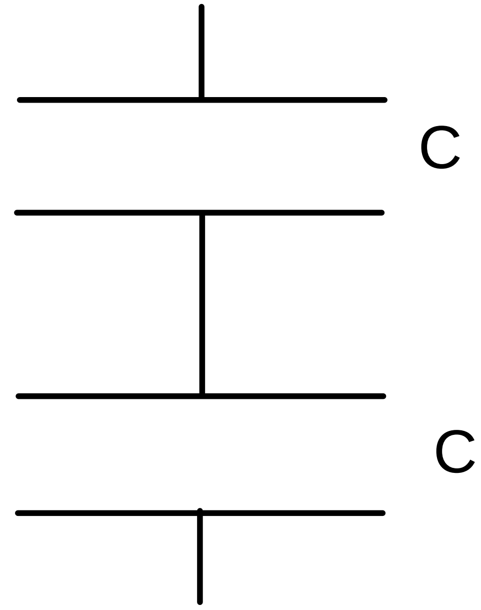
Let's find the charge on the surfaces of the metal slab



The flux through the gaussian surface is zero.
This is because on the surfaces inside the conductors there is no field
And on surfaces in-between the conductors field is parallel to surfaces

Thus by Gauss's Law the total enclosed charge is zero.
The charges on the two sides of the slab must exactly balance
the charges on the plates of the capacitor





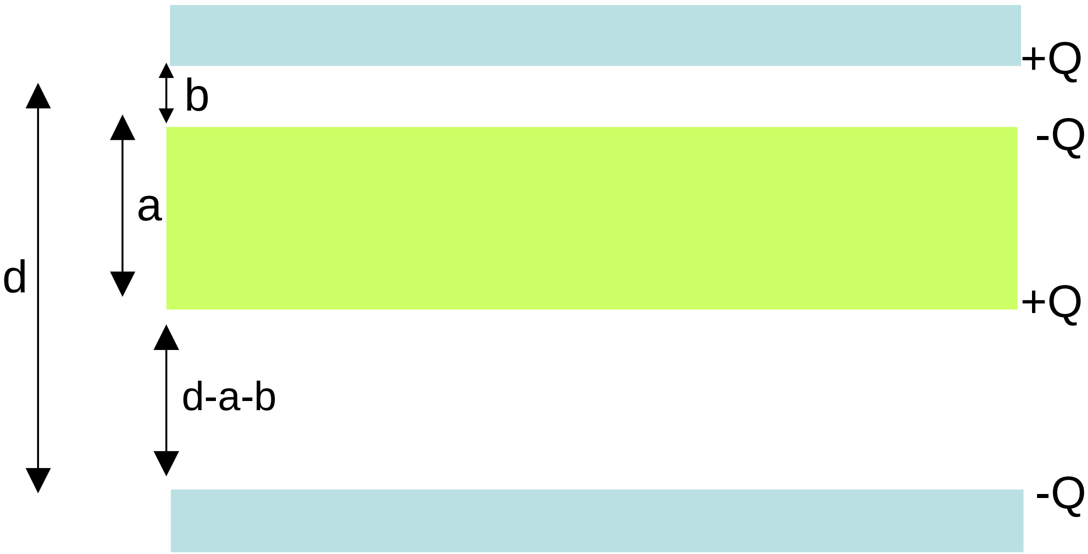
This is just like two capacitors in series! $C = \epsilon_0 \frac{A}{\frac{1}{2}(d-a)}$

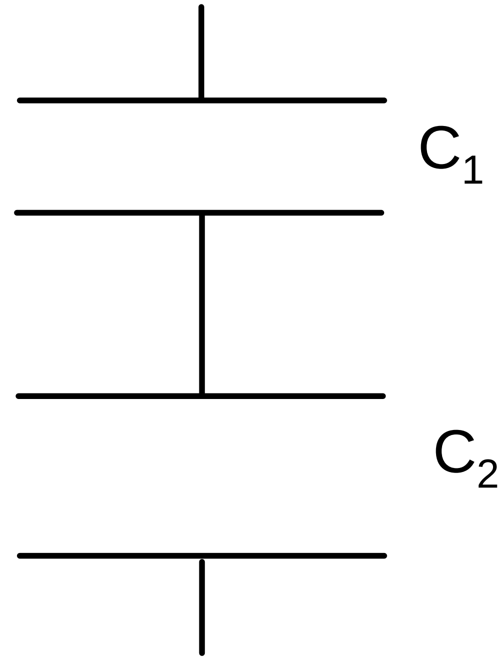
$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{C} = \frac{2}{C}$$

$$C_{eq} = \epsilon_0 \frac{A}{d-a}$$

Sanity check: for a=0 should recover standard $C=\varepsilon_0$ A/d OKAY!

- Show that the answer does not depend on where the slab is placed



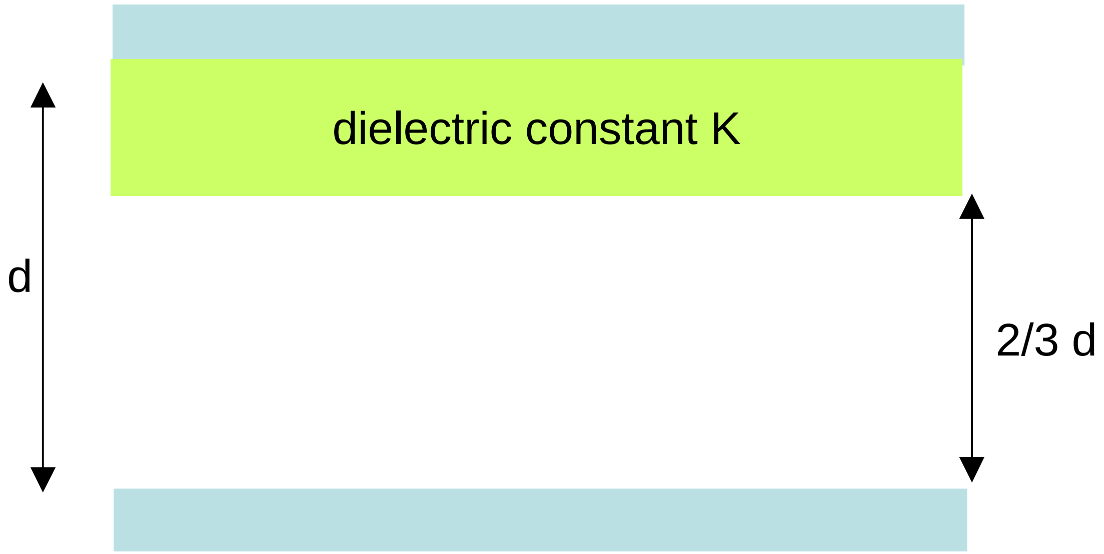


$$C_1 = \epsilon_0 \frac{A}{b} \qquad C_2 = \epsilon_0 \frac{A}{d-a-b}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{b}{A\epsilon_0} + \frac{d-b-a}{A\epsilon_0}$$

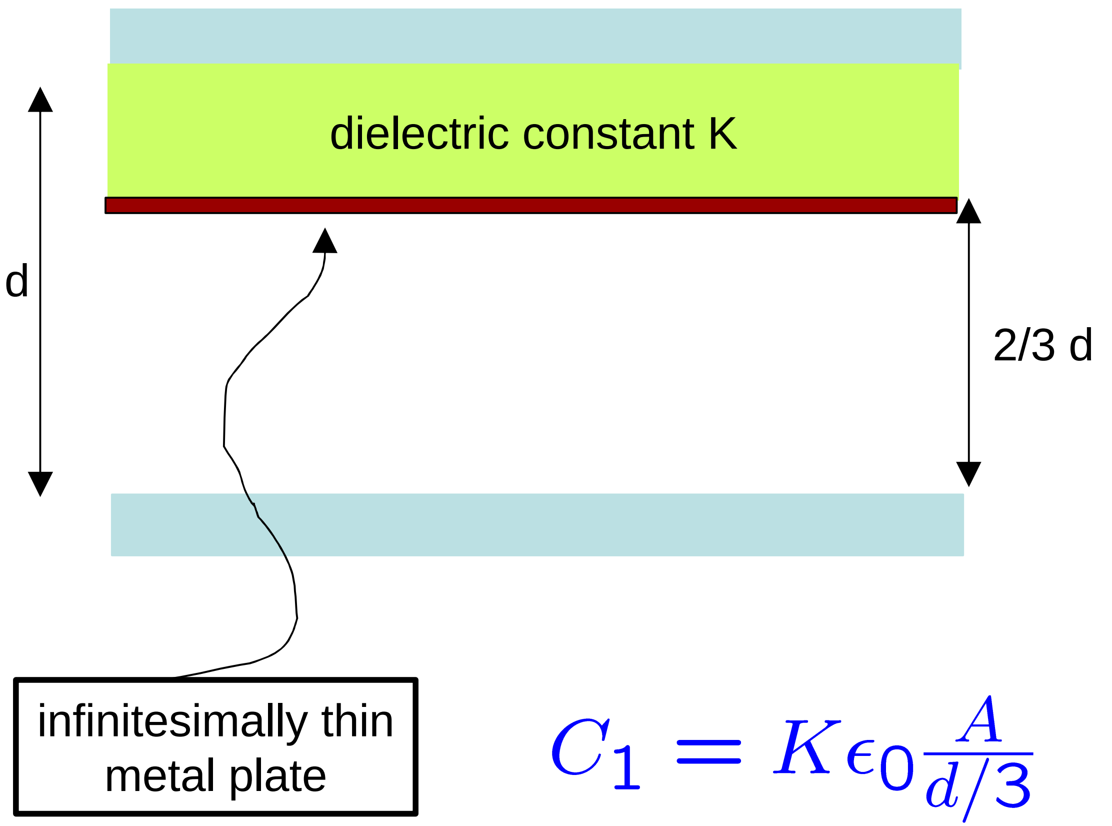
$$\boxed{C_{eq} = \epsilon_0 \frac{A}{d-a}}$$

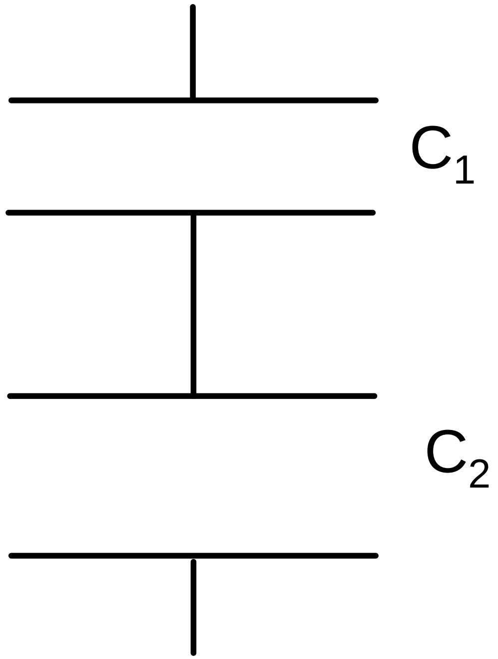
Example 3: find the capacitance



Trick. In previous example we learned

1. Can insert metallic slab through the plates and consider the combination as two capacitors in series
2. If the slab thickness approaches zero, then the capacitance approaches the capacitance as if the slab was absent



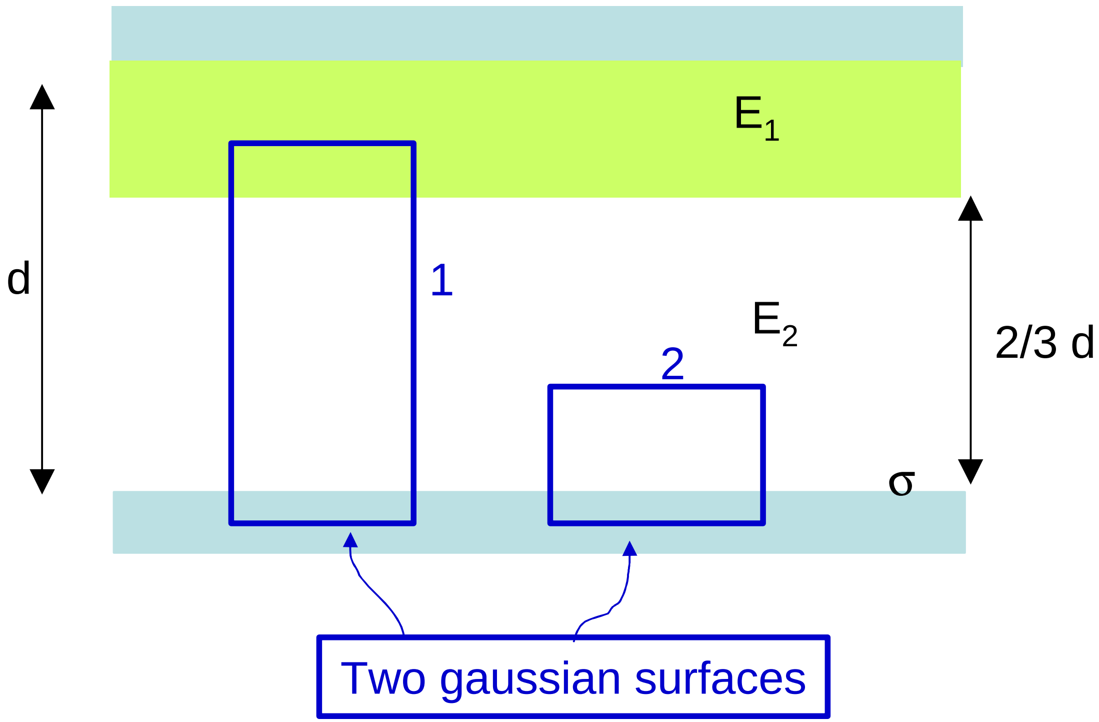


$$C_1 = K\epsilon_0 \frac{A}{d/3} \qquad C_2 = \epsilon_0 \frac{A}{2d/3}$$

$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{d/3}{KA\epsilon_0} + \frac{2d/3}{A\epsilon_0}$$

$$C_{eq} = \frac{3K}{2K+1}\frac{\epsilon_0 A}{d}$$

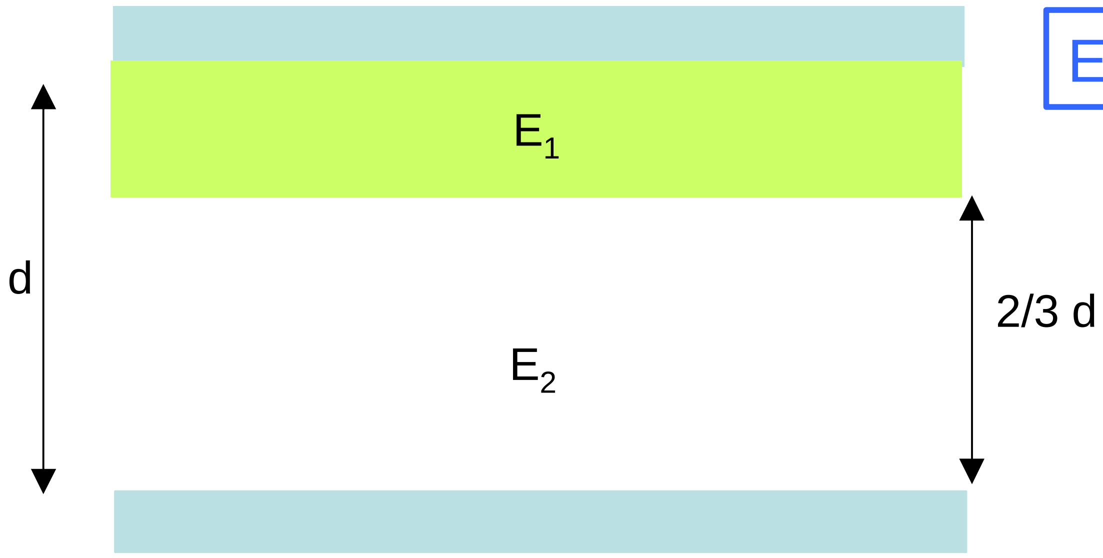
Second approach...no tricks!



2nd Gaussian surface, (S_2 is area of horizontal side): $E_2S_2 = \sigma\, S_2/\varepsilon_0$

1st Gaussian surface, (S_1 is area of horizontal side): $KE_1S_1 = \sigma\, S_1/\varepsilon_0$

$$E_1 = \sigma/(K\varepsilon_0) \quad \text{and} \quad E_2 = \sigma/\varepsilon_0$$



$E_1 = \sigma/(K\varepsilon_0)$ and $E_2 = \sigma/\varepsilon_0$

Potential difference: $V = E_1(\frac{1}{3}d) + E_2(\frac{2}{3}d)$

$$V = \left(\frac{1}{3K} + \frac{2}{3}\right)\frac{d\sigma}{\epsilon_0} = \frac{2K+1}{3K}\frac{Qd}{A\epsilon_0}$$

$$C = \frac{Q}{V}$$

$$C = \frac{3K}{2K+1}\frac{\epsilon_0 A}{d}$$