Fall 2004 Physics 3
Tu-Th Section

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Today: Electric Potential Energy

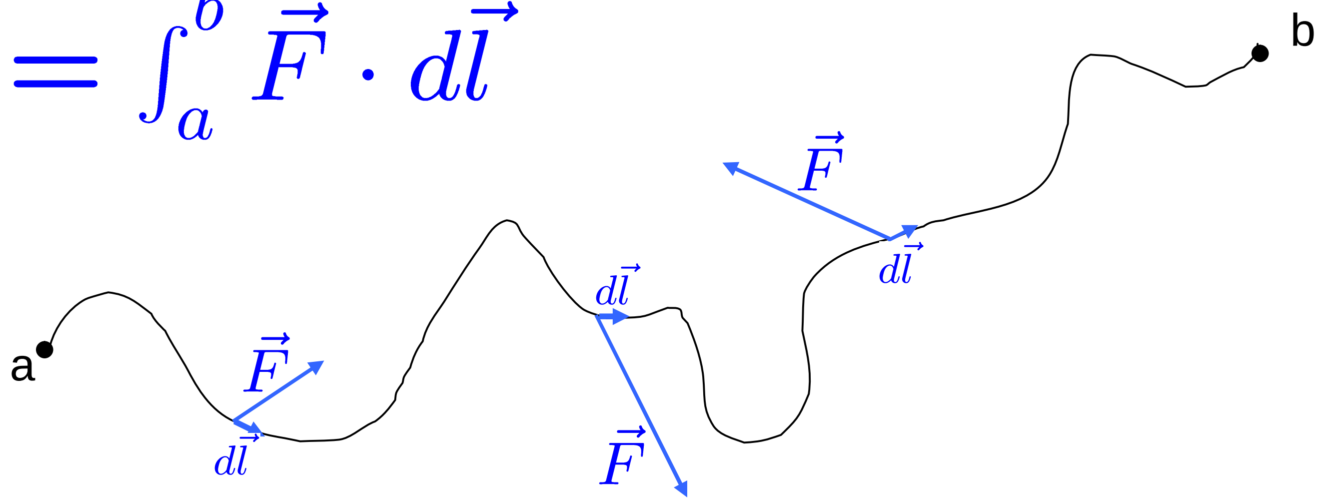
- You should be familiar with the concept of gravitational potential energy from Physics 1
- Let's review
- If a force $\vec{F}$ acts on a particle as the particle moves from a→b, then

$$W_{a \to b} = \int_a^b \vec{F} \cdot d\vec{l}$$

is the work done by the force

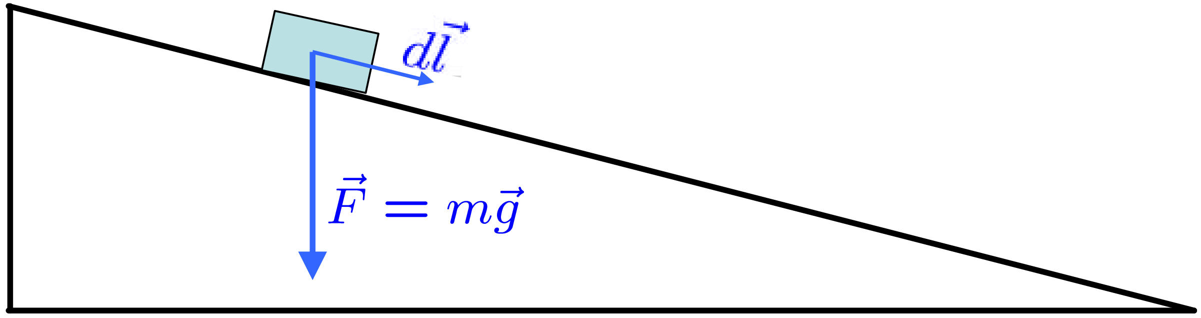
($d\vec{l}$ is the infinitesimal displacement along the path)

$$W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l}$$



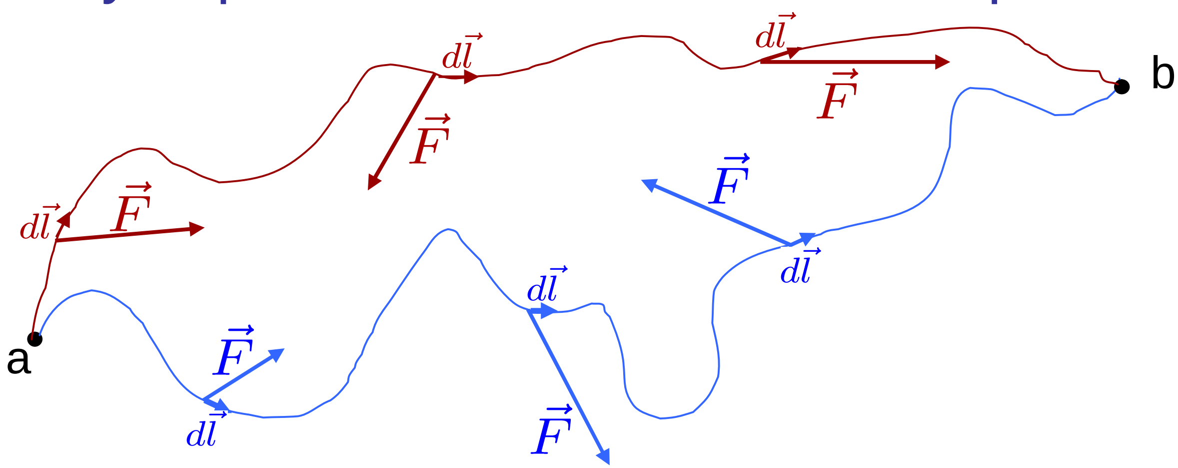
Careful: the force does not necessarily line up with the displacement

For example, a block sliding down an inclined plane under the influence of gravity:



Conservative force

- A force is conservative if the work done by the force is <u>independent of path</u>
  - Only depends on the initial and final points



- Then the work done can be written as function of the difference between some properties of the begin and final point

$$W_{a\to b} = \int_a^b \vec{F} \cdot d\vec{l} = -[U(b) - U(a)]$$

$$W_{a\to b} = \int_a^b \vec{F} \cdot d\vec{l} = -[U(b) - U(a)]$$

- U is the potential energy
- W = -ΔU
- Work energy theorem:

  work = change in kinetic energy

  $W_{a\to b}$ = K(b) – K(a)

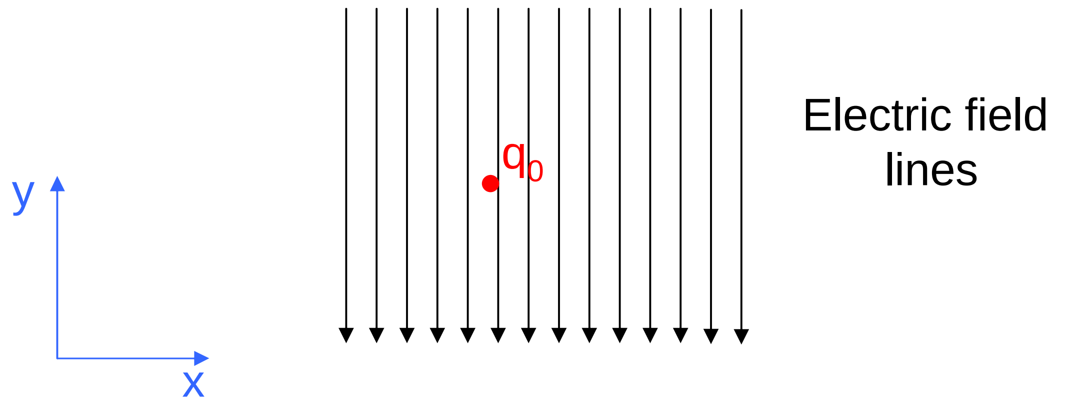
  K(a) + U(a) = K(b) + U(b)
- Potential energy defined up to additive constant

Remember gravitational field, force, potential energy

- Near the surface of the earth, constant force $\vec{F} = m\vec{g}$
- Think of it as mass times constant gravitational field $\vec{g}$
- Then gravitational potential energy U=mgh

Now imagine charge q_0 in constant electric field

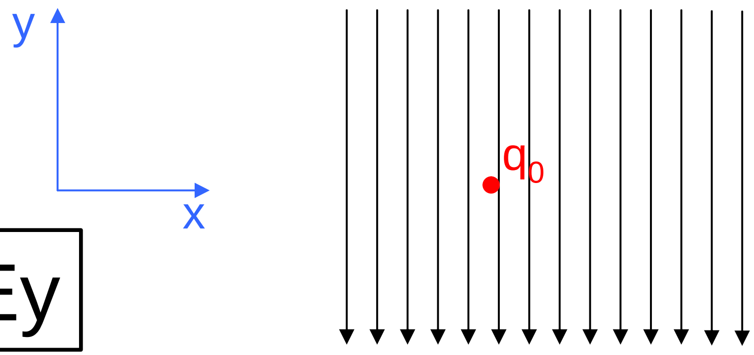
- Constant force $\vec{F} = q_0 \vec{E}$
- By analogy with gravity $U = q_0 E y$



- U = electric potential energy of the charge q_0

Careful: the y-axis points opposite to the E field

$U = q_0Ey$



Electric field lines

$$\vec{F} = q_0\vec{E}$$

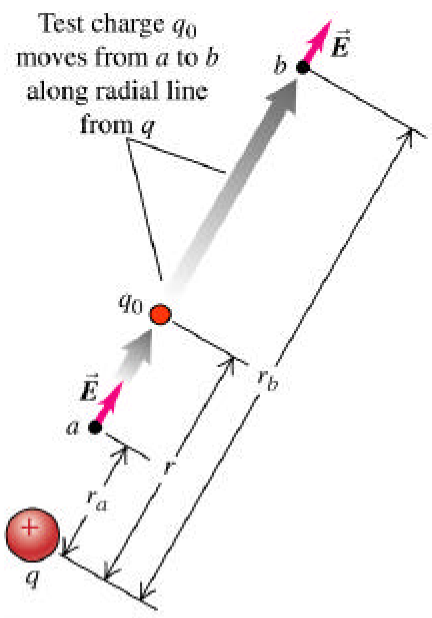
- If q_0 is positive
  - The force is downwards
  - The force "pushes" the charge downwards, towards smaller y
  - The force tends to make U smaller
- If q_0 is negative
  - The force is upwards
  - The force "pushes" the charge upwards towards larger y
  - This also tends to make U smaller
    - because of the –ve sign of the q_0 in the expression $U=q_0Ey$

Potential energy of two point charges

- Remember the definition

$$W_{a\to b} = \int_a^b \vec{F}\cdot d\vec{l} = -[U(b) - U(a)]$$

- Consider displacement along line joining the two charges ("radial displacement



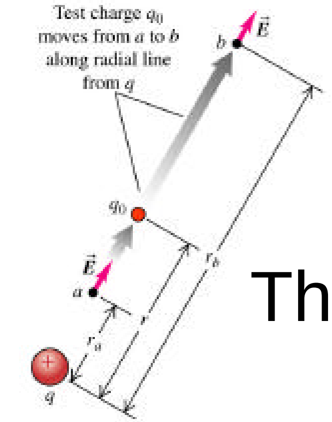
$$W_{a\to b} = \int_{r_a}^{r_b} \vec{F}\cdot d\vec{r}$$

$$W_{a\to b} = \int_{r_a}^{r_b} k\frac{qq_0}{r^2}dr$$

$$W_{a\to b} = kqq_0(\frac{1}{r_a} - \frac{1}{r_b})$$

Tempting to identify

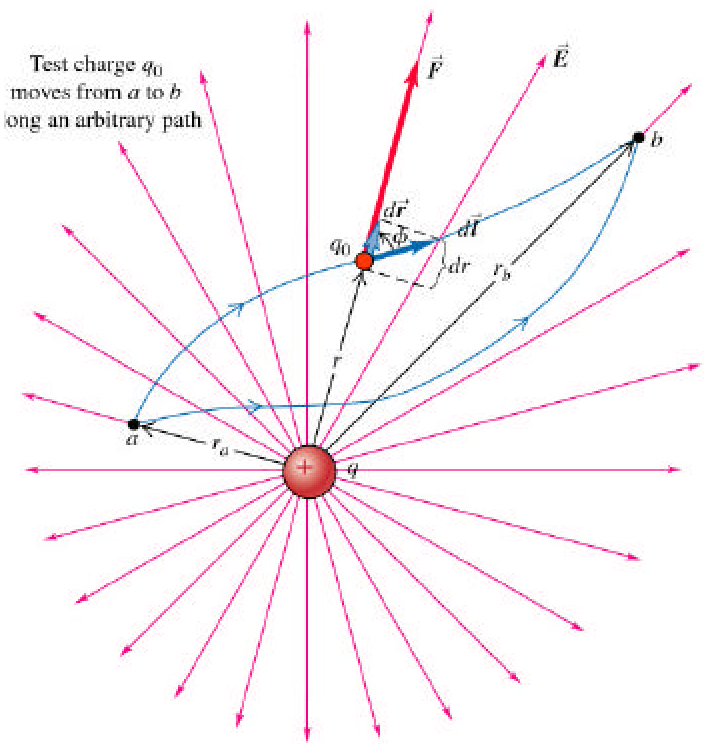
$$U(r) = k\frac{qq_0}{r} + \text{Const.}$$



$$U(r) = k\frac{qq_0}{r} + \text{Const.}$$

This holds if the work is independent of path

→ Look at a different path r_a → r_b



$$W_{a\to b} = \int_{r_a}^{r_b} \vec{F}\cdot d\vec{l}$$

$$\vec{F}\cdot d\vec{l} = F\,dl\cos\phi = F\,dr$$

→ The work depends only on the radial displacement

It does not depend on the amount of "sideways" displacement

Work only depends on initial and final values of r

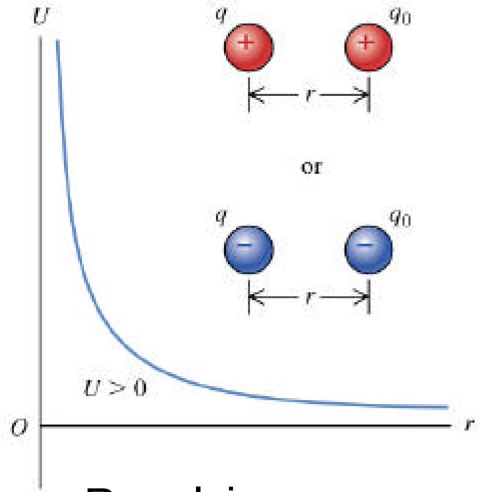
Summary:

Potential energy of two point charges

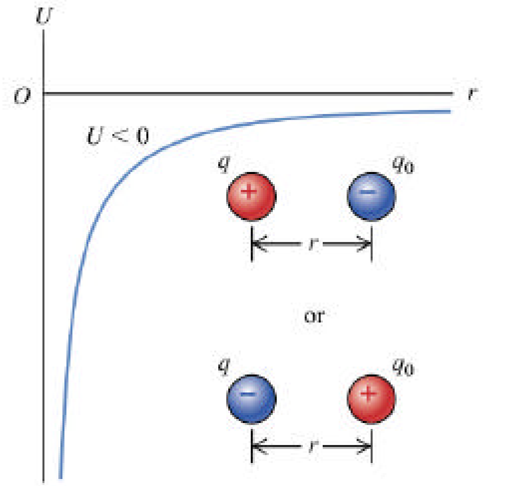
$$U(r) = k\frac{qq_0}{r} + \text{Const.}$$

where *r* is the distance between the two charges

- Most often we take Const=0 for simplicity
- Then U → 0 as *r* →8



Repulsion:
charges want to move apart

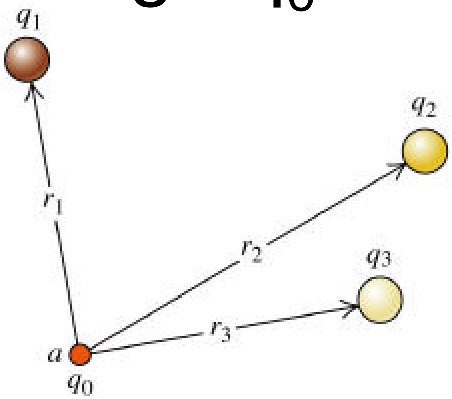


Attraction:
charges want to move together

Always tendency to reduce potential energy

If we have many charges...

- Consider electric field caused by a bunch of charges q_1, q_2, q_3,...
- Bring a test charge q_0 into the picture



- Potential energy associated with q_0

$$U = \frac{q_0}{4\pi\epsilon_0}\left(\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + \cdots\right) = \frac{q_0}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

(point charge q_0 and collection of charges q_i)

Potential energy is an additive quantity

Many charges (cont.)

- If I have a collection of charges, the interaction of each pair will contribute to the <u>total potential energy of the system</u>
- A compact way of writing it is

$$U_{\text{total}} = \frac{1}{4\pi\epsilon_0} \sum_{i<j} \frac{q_i q_j}{r_{ij}}$$

- Where
  - q_i and q_j are the i-th and j-th charge
  - r_{ij} is the distance between i-th and j-th charge
  - i<j insures no double counting

$$U_{\text{total}} = \frac{1}{4\pi\epsilon_0} \sum_{i<j} \frac{q_i q_j}{r_{ij}}$$

You should try to get used to this kind of compact notation!

Let's see an example. Three charges. What are the terms?

Possibilities are

i=1 and j=1	i=2 and j=1	i=3 and j=1
i=1 and j=2	i=2 and j=2	i=3 and j=2
i=1 and j=3	i=2 and j=3	i=3 and j=3

Only some of these satisfy the i<j condition

Then the sum becomes

$$U_{\text{total}} = \frac{1}{4\pi\epsilon_0} \left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} \right]$$

You see that each pair of charges enters once and only once

Work to and from infinity

$$U(r) = k\frac{qq_0}{r} + \text{Const.}$$

= 0

Convention!

Work done by the electric field in going from a→b

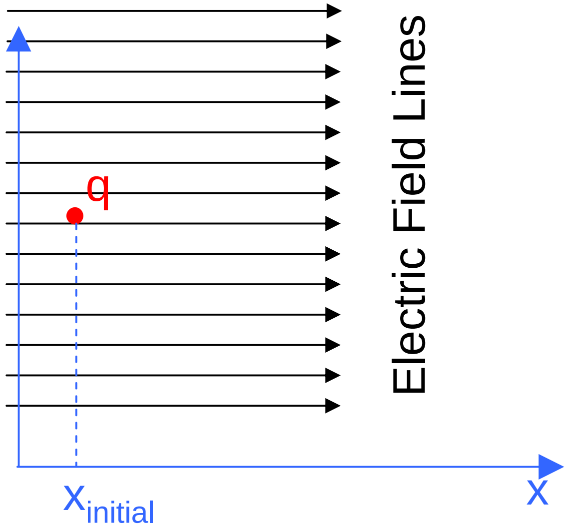
$$W_{a\to b} = U(a) - U(b)$$

$U(\infty)=0$ → U(r) can be thought of as the work that the electric field would do in moving the test charge q_0 from its position to ∞

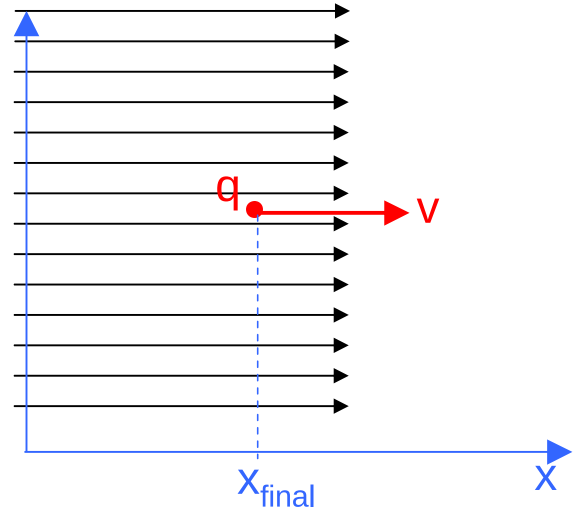
Conversely, the work that an external force would need to do to bring the charge from ∞ to its current position is –U

Example

- A particle of charge q and mass m is accelerated from rest by a constant electric field E. What is the velocity after the particle travelled a distance L?



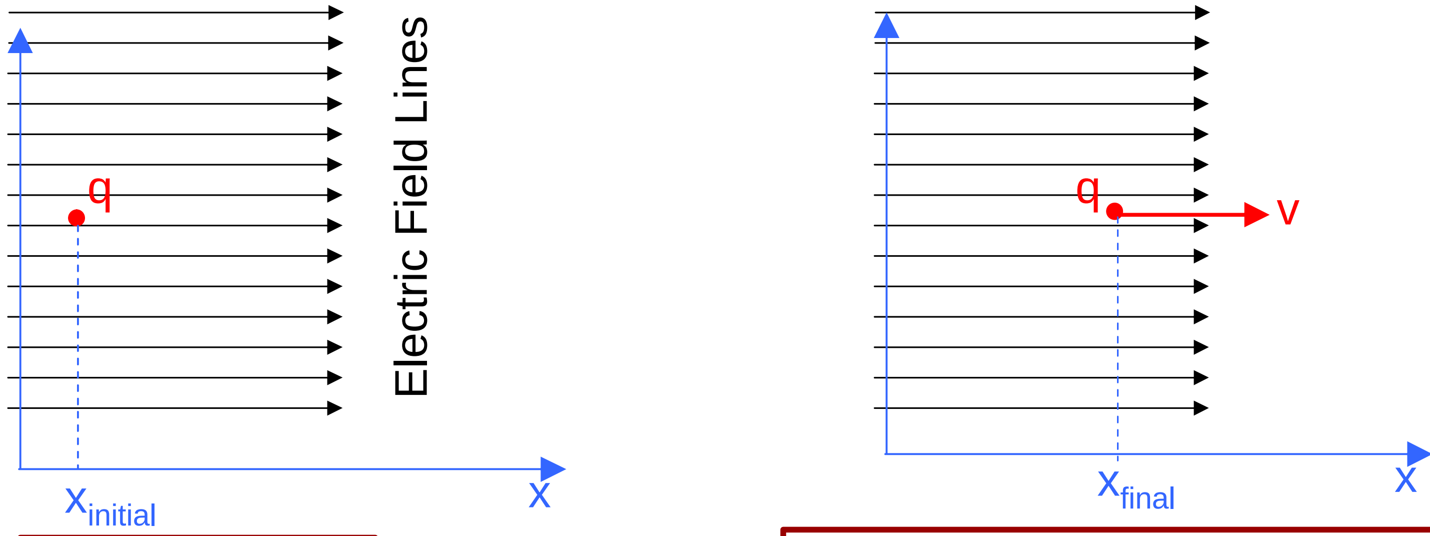
Before: v=0



After: $x_{final}-x_{initial}$ = L, v=???

Guiding principle: <u>conservation of energy</u>

$$K_{initial} + U_{initial} = K_{final} + U_{final}$$



Before: v=0

After: $x_{final}-x_{initial}$ = L, v=???

$$K_{initial} + U_{initial} = K_{final} + U_{final}$$

$K_{initial} = 0$ and $K_{final} = ½\, m\, v^2$

The electric potential energy is a function of x

$$U(x) = -\,qEx + \text{Constant}$$

Careful: earlier we defined it without a minus sign
This is because before we had the axis pointing opposite to the electric field. But here the x-axis points in the same direction as the electric field!

$$K_{initial} + U_{initial} = K_{final} + U_{final}$$

$$K_{initial} = 0 \text{ and } K_{final} = ½\, m\, v^2$$

$$U(x) = -\, qEx + \text{Constant}$$

$$U_{initial} = -qEx_{initial} + \text{Constant}$$

$$U_{final} = -qEx_{final} + \text{Constant}$$

$$0 - qEx_{initial} + \text{Const.} = ½\, m\, v^2 - qEx_{final} + \text{Const.}$$

$$½\, m\, v^2 = qE\underbrace{(x_{final} - x_{initial})}_{= L}$$

$$v^2 = 2qEL/m$$

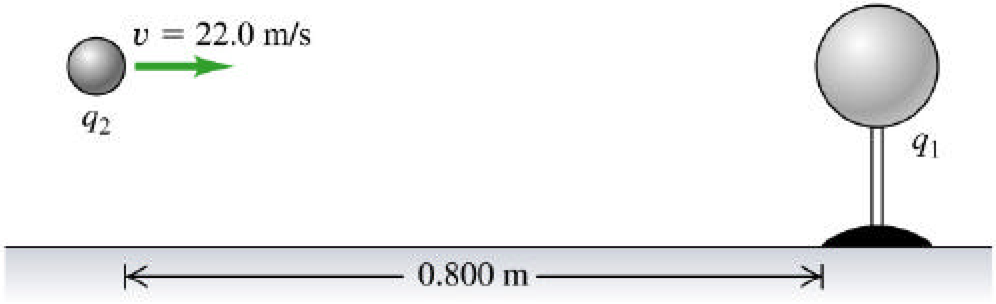
Note that the arbitrary constant dropped out

Another example (Prob. 23.3)

A metal sphere, charge q_1 = -2.8 μC is held stationary by an insulating support. A 2nd sphere, q_2 = -7.8 μC amd m=1.5 g is moving towards q_1. When the two spheres are d=0.8 m apart, q_2 is moving with v=22 m/sec.

(a) What is the speed of q_2 when the spheres are 0.4 m apart?

(b) How close does q_2 get to q_1 before turning back?



Conservation of energy $K_1 + U_1 = K_2 + U_2$

$$K_1 = \tfrac{1}{2} m v_1^2 \qquad U_1 = k\, q_1 q_2 / d_1$$

$$K_2 = \tfrac{1}{2} m v_2^2 \qquad U_2 = k\, q_1 q_2 / d_2$$

Conservation of energy $K_1 + U_1 = K_2 + U_2$

$K_1 = ½\, m\, v_1^2$ $U_1 = k\, q_1q_2/d_1$

$K_2 = ½\, m\, v_2^2$ $U_2 = k\, q_1q_2/d_2$

$$½\, m\, v_1^2 + k\, q_1q_2/d_1 = ½\, m\, v_2^2 + k\, q_1q_2/d_2$$

$$v_2^2 = v_1^2 + 2(k/m)q_1q_2\,(1/d_1 - 1/d_2)$$

$$v_2 = \sqrt{22^2 + 2\frac{9\cdot 10^9}{1.5\cdot 10^{-3}}(-2.8\cdot 10^{-6})(-7.8\cdot 10^{-6})\left(\frac{1}{0.8} - \frac{1}{0.4}\right)}\ \text{m/s}$$

$v_2 = 12.5$ m/s

Next: where does it stop?

It stops when $v_2 = 0$; the conservation of energy equation:

$$½\, m\, v_1^2 + k\, q_1q_2/d_1 = k\, q_1q_2/d_2$$

$$d_2 = d_1\frac{kq_1q_2}{mv_1^2 + 2kq_1q_2} = 0.32\ \text{m}$$

Electric potential

- Definition: if a charge q_0 in an electric field has electric potential energy U, then the electric potential is defined as

$$V = \frac{U}{q_0}$$

- Think of electric potential as "potential energy per unit charge"
- Much as electric field is "force per unit charge"

Electric Potential $V = \frac{U}{q_0}$

- Electric potential is a property of the electric field and varies as a function of position in space
- Since U is defined up to an arbitrary constant, V is also defined up to an arbitrary constant.
- Only differences in potential between two points are meaningful
- Jargon: potential of a with respect to b

$$V_{ab} = V_a - V_b$$

Electric Potential $V = \frac{U}{q_0}$

- Units: [V] = [U]/[Q] = Joule/Coulomb
- Definition 1 Volt = 1 J/C
  - Abbreviation: V
- Potential of a w.r.t. b (V_{ab}) also called voltage

$$V_{ab} = V_a - V_b = \frac{U_a - U_b}{q_0} = \frac{W_{a\to b}}{q_0}$$

- V_{ab} = work done <u>by electric force</u> in moving unit charge <u>from a to b</u>
- V_{ab} work done <u>against electric</u> force in moving unit charge <u>from b to a</u>

How to not get confused by the signs!

Just remember one general principle

- The electric force does positive work in moving from high electric potential energy to low electric potential energy
- Just like gravity does positive work in moving a body down towards the surface of the earth
  - Body is high → potential energy is high
  - Body is low → potential energy is low

Using previous results for U....

$$V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0}\frac{q}{r} \qquad \text{(potential due to a point charge)}$$

$$V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

(potential due to a collection of point charges)

And also an (obvious) generalization:

$$V = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r}$$

(potential due to a continuous distribution of charge)

V from E

- Given a charge distribution, it is straight forward (in principle!) to find V

$$V = \frac{U}{q_0} = \frac{1}{4\pi\epsilon_0}\sum_i \frac{q_i}{r_i}$$

- Sometimes you can get V starting from E

$$W_{a\rightarrow b} = U_a - U_b$$

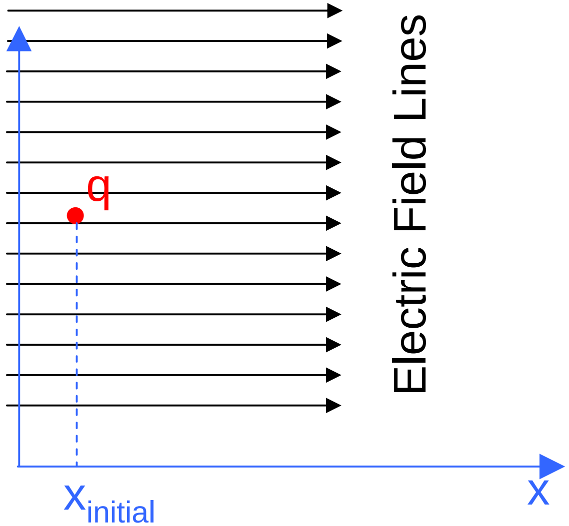
$$W_{a\rightarrow b} = \int_a^b \vec{F}\cdot d\vec{l} = \int_a^b q_0\vec{E}\cdot d\vec{l}$$

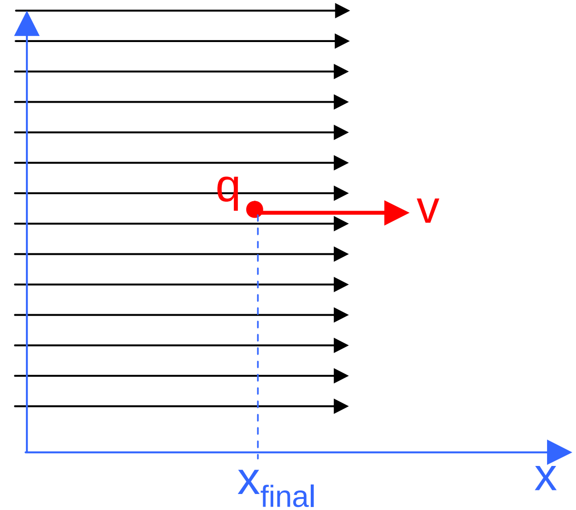
$$V_a = \frac{U_a}{q_0}$$

$$V_a - V_b = \int_a^b \vec{E}\cdot d\vec{l} = \int_a^b E\cos\phi\, dl$$

Example

Back to charge q in constant electric field





Before: v=0

After: x_{final}-$x_{initial}$ = L, v^2=2qEL/m

What is $\Delta V = V_{initial} - V_{final}$?

$$\Delta V = \int_{x_{\text{initial}}}^{x_{\text{final}}} \vec{E} d\vec{x} = EL$$

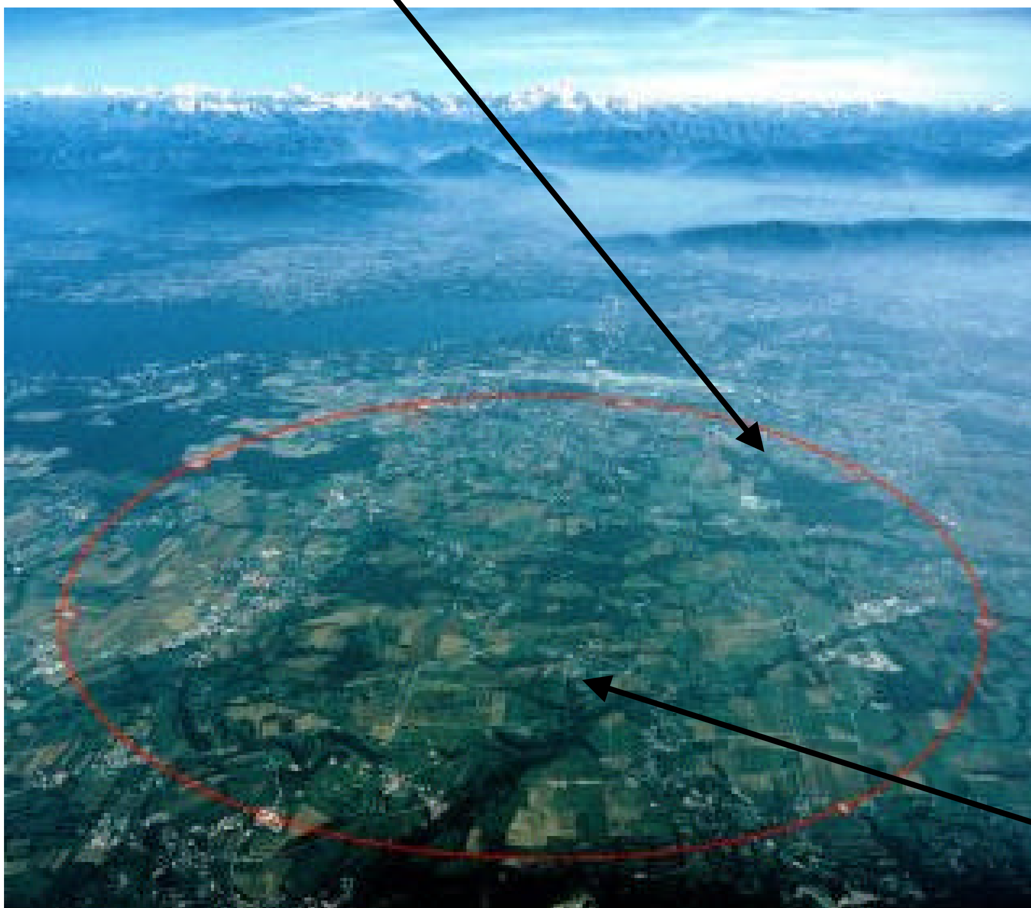
The electron volt (eV)

- Consider an electron accelerated through a potential difference of $\Delta V = 1$ V.
- Change in potential energy $\Delta U = -e\Delta V$
- This must be compensated by a change in kinetic energy $\Delta K = e\Delta V = 1.6\ 10^{-19}$ J
- Definition of electron volt (eV):

The kinetic energy gained by an electron accelerated through a $\Delta V = 1$ Volt

- $1\text{eV} = 1.6\ 10^{-19}$ J
- Useful unit of energy in atomic physics, chemistry, etc.
  - This is a <u>unit of energy</u>, not potential
  - Don't get confused

The largest accelerator



CERN LHC
(Geneva, Switzerland)

Underground tunnel
26 Km circumference

Accelerates protons to
7 TeV = 7 10^{12} eV

One more word about units

$$\vec{E} = \frac{\vec{F}}{q} \Longrightarrow [E] = \frac{\mathsf{N}}{\mathsf{C}}$$

$$\Delta V = \int_{x_{\text{initial}}}^{x_{\text{final}}} \vec{E} d\vec{x} \Longrightarrow [E] = \frac{\mathsf{V}}{\mathsf{m}}$$

These are the same