Fall 2004 Physics 3
Tu-Th Section

Claudio Campagnari

Web page:
http://hep.ucsb.edu/people/claudio/ph3-04/

Chapter 15: <u>Waves</u>
What is a wave?

From www.dictionary.com:

Physics.

- A disturbance traveling through a medium by which energy is transferred from one particle of the medium to another without causing any permanent displacement of the medium itself.
- A graphic representation of the variation of such a disturbance with time.
- A single cycle of such a disturbance.

Close, but not quite right!!!

"disturbance" and "medium"

Disturbance =

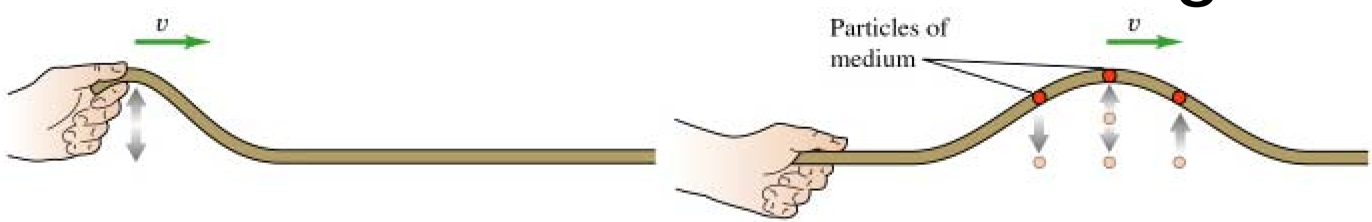
displacement from equilibrium

Medium =

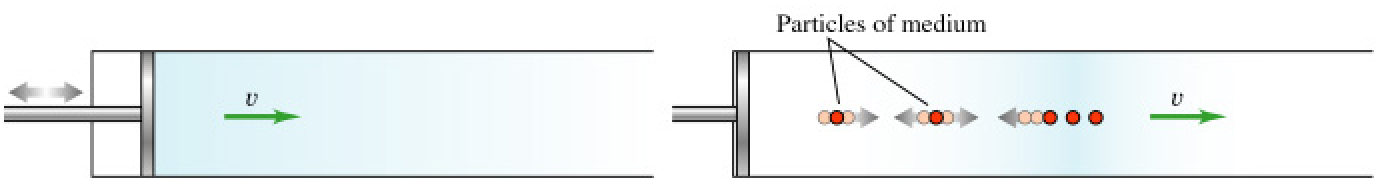
material in which displacement occurs

Examples

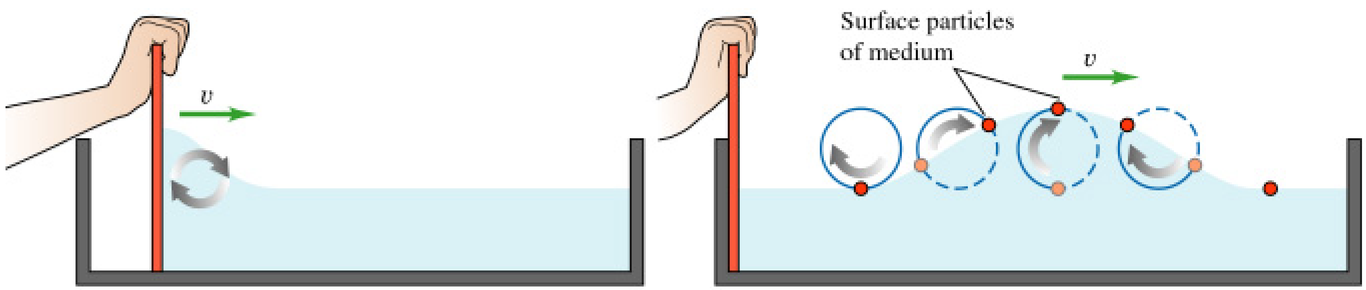
Transverse wave on a string



Longitudinal wave in a fluid (sound)



Combined long. and transv. wave on liquid surface



So what's wrong with dictionary.com?

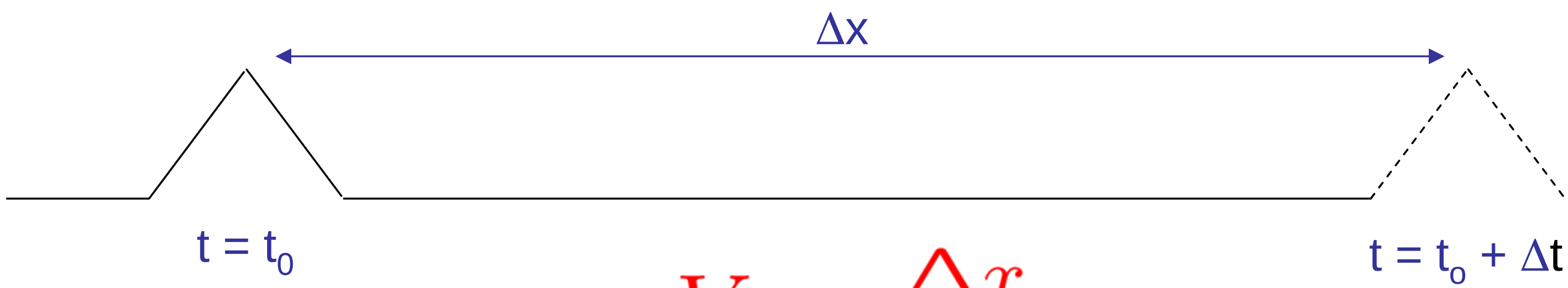
- There are waves that do not need a "medium" to propagate.
- e.g., Electromagnetic Waves (light) can propagate in vacuum.
  - This was a hard concept to swallow at the end of the 19^{th} century, but it is the way it is.
- For now we concentrate on mechanical waves that propagate in a medium.

Longitudinal vs. Transverse

- Longitudinal: the disturbance is parallel to the direction of propagation.
- Transverse: the disturbance is perpendicular to the direction of propagation.

Wave Velocity (Speed)

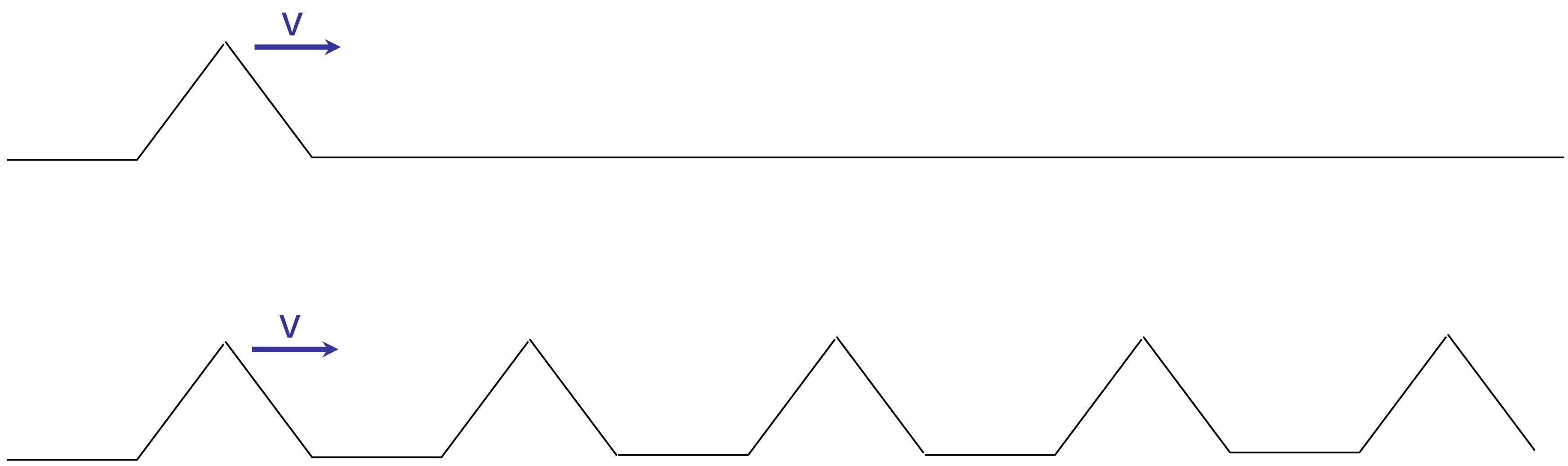
The "disturbance" propagates in space



$$V = \frac{\Delta x}{\Delta t}$$

Do not confuse the velocity of the wave with the velocity of the particles in the medium

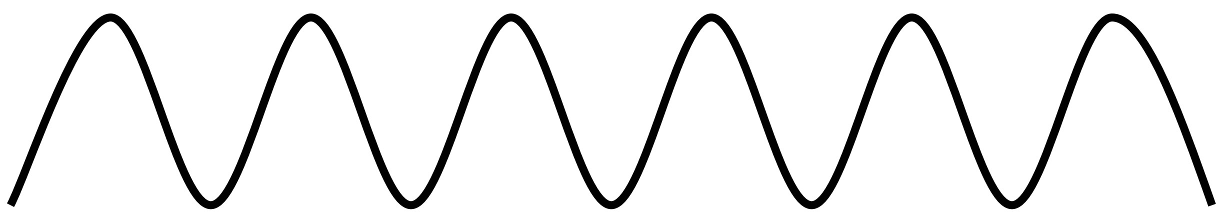
Wave pulses vs. periodic waves



In a periodic wave the disturbance repeats itself and the motion of the particles of the medium is periodic

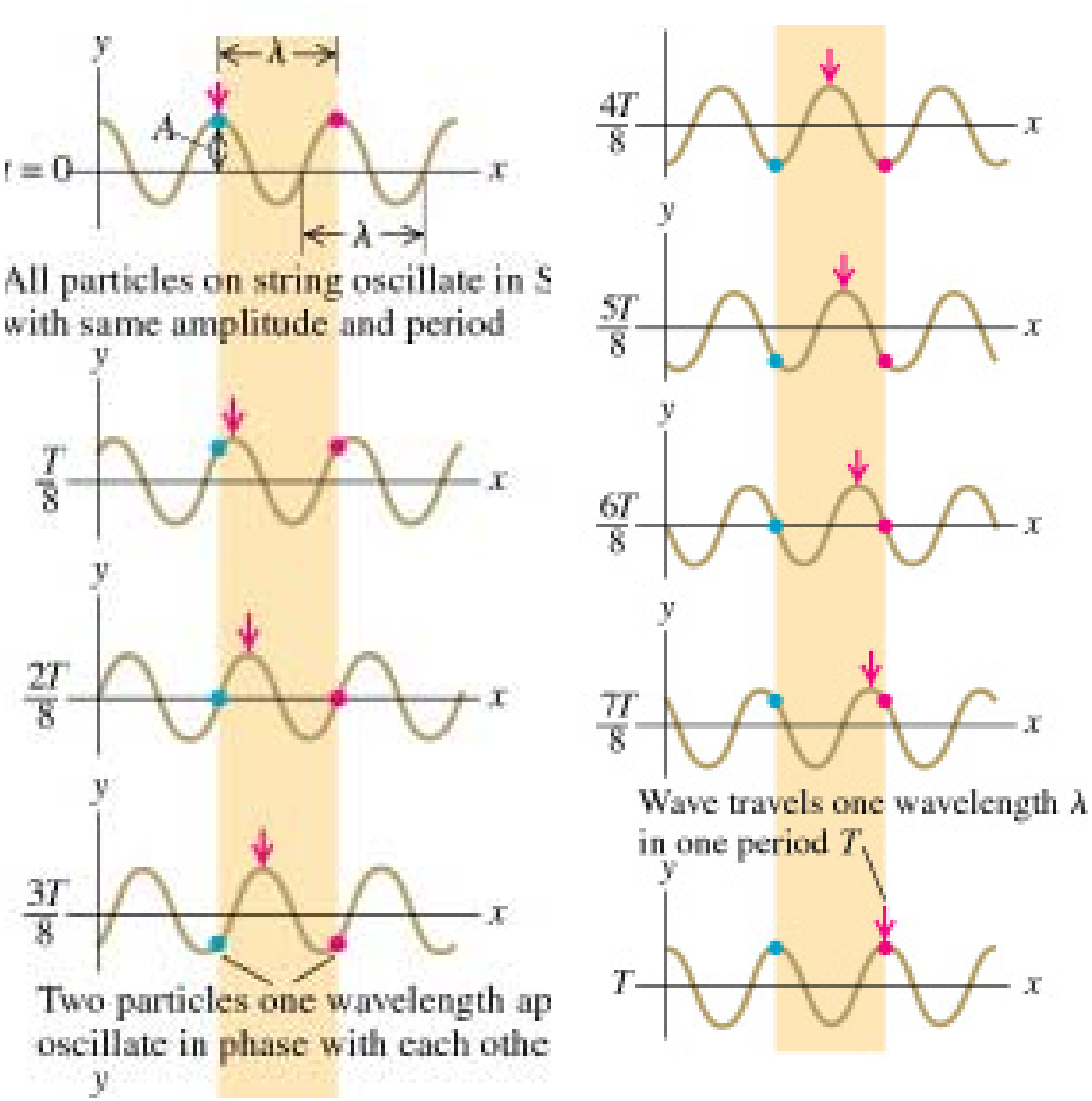
Sinusoidal waves

- Special case of periodic wave where the motion of each particle in the medium is simple harmonic with the same amplitude and the same frequency.
- Resulting wave is a symmetrical sequence of crests and troughs:



- Important because any periodic wave can be represented as the sum of sinusoidal waves (Fourier decomposition)

Sinusoidal waves (cont.)



__Wavelength (λ):__

Distance over which displacement repeats

__Period (T):__

Time over which displacement repeats

__Frequency (f):__

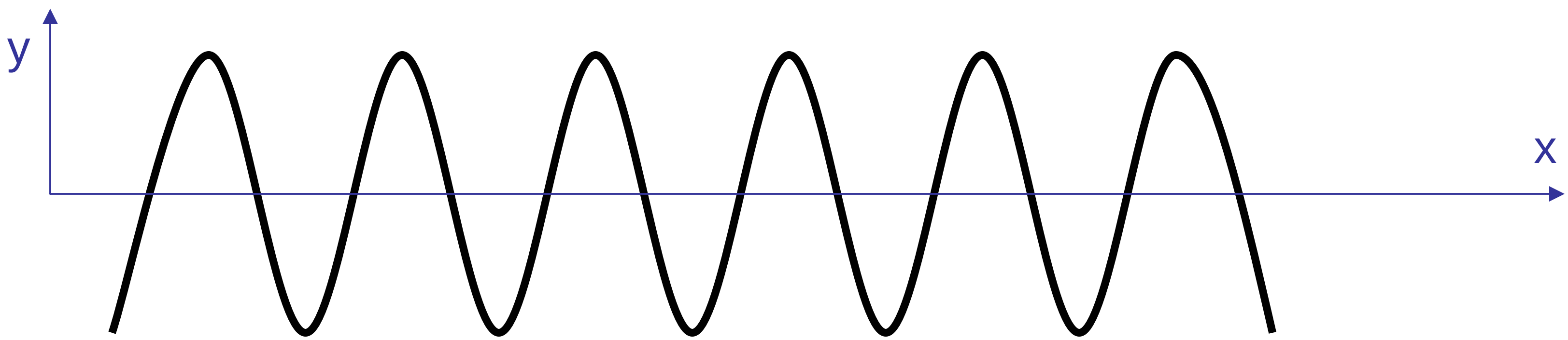
$f=1/T$

__Velocity (v):__

$v = \lambda/T = \lambda f$

Mathematical Description

- Waves on a string for concreteness



- Wave Function: a function that describes the position of any particle at any time:
- y(x,t)
- Take y=0 as equilibrium position, i.e., unperturbed, stretched string

Mathematical description (sinusoidal wave)

- Take particle at x=0. Its motion is described by y(x=0,t).
- It undergoes Simple Harmonic Motion (SHM)
  - $y(x=0,t) = A\cos(\omega t+\delta) = A\cos\omega t$ (set $\delta=0$)
- What is then y(x,t), i.e., the displacement at any other position along the string?
- The disturbance travels with velocity v.
- The motion at any x at a given time t is the same as it was at x=0 at a time t-x/v

$$y(x,t) = A\cos[\omega(t - \frac{x}{v})]$$
$$y(x,t) = A\cos[\omega(\frac{x}{v} - t)]$$

Wave number: $k = \frac{2\pi}{\omega}$

$$\lambda = \frac{2\pi}{k},\ f = \frac{\omega}{2\pi},\ v = \lambda f$$

$$\rightarrow \omega = vk$$

$$\rightarrow y(x,t) = A\cos(kx - \omega t)$$

Nothing fundamental here: just different ways of writing the same thing.........

Note: this is for waves moving in positive x-direction. For a wave moving in negative x-direction, kx-ωt becomes kx+ωt

Aside: sines and cosines, don't get confused!

- SHM
- $y = A \sin(\omega t+\delta)$

  $= A \cos\delta \sin\omega t + A \sin\delta \cos\omega t$
- If you choose
  - $\delta = 0$ → $y = A \sin\omega t$
  - $\delta = 90^o$ → $y = A \cos\omega t$

The wave equation

Familiar SHM properties:

$$y(x,t) = A\cos(kx - \omega t)$$
$$v_y = \frac{\partial y(x,t)}{\partial t} = \omega A \sin(kx - \omega t)$$
$$a_y = \frac{\partial v_y}{\partial t} = \frac{\partial^2 y(x,t)}{\partial t^2}$$
$$a_y = -\omega^2 A\cos(kx - \omega t) = -\omega^2 y(x,t)$$

But also:

$$\frac{\partial^2 y(x,t)}{\partial x^2} = -k^2 A\cos(kx - \omega t) = -k^2 y(x,t)$$

So:

$$\frac{\partial^2 y(x,t)/\partial t^2}{\partial^2 y(x,t)/\partial x^2} = \frac{\omega^2}{k^2} = v^2$$

Wave Equation:

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\,\frac{\partial^2 y(x,t)}{\partial t^2}$$

One of the most important equations in physics. Whenever we encounter it we can say that a disturbance ("y") can propagate along the x-axis with velocity v.

- Any function y(x,t)=y(x±vt) is a solution of the wave equation.

For example:

$$y(x,t) = A \cdot \frac{(x \pm vt)^3}{\cos \alpha (x \pm vt)}$$

or

$$y(x,t) = B \cdot e^{\gamma (x \pm vt)} + \frac{C}{(x \pm vt)^2}$$

etc..........

- <u>All</u> these functions represent disturbances propagating to the right or the left with speed v

Proof that y(x±vt) is solution of wave equation

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\ \frac{\partial^2 y(x,t)}{\partial t^2}$$

Let $z = x \pm vt$ so that $y(x,t)=y(x\pm vt)=y(z)$

$$\frac{\partial y}{\partial x} = \frac{dz}{dx}\cdot\frac{dy}{dz} = \frac{dy}{dz}$$

$$\frac{\partial^2 y}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{dy}{dz}\right) = \frac{dz}{dx}\cdot\frac{d}{dz}\left(\frac{dy}{dz}\right) = \frac{d^2 y}{dz^2}$$

So left-hand side of wave equation:

$$\frac{\partial^2 y}{\partial x^2} = \frac{d^2 y}{dz^2}$$

Right-hand side of wave equation $\left(\frac{\partial^2 y}{\partial t^2}\right)$

$$\frac{\partial y}{\partial t} = \frac{dz}{dt} \cdot \frac{dy}{dz} \qquad z = x \pm vt \;\; \rightarrow \frac{dz}{dt} = \pm v$$

$$\frac{\partial y}{\partial t} = \pm v \cdot \frac{dy}{dz}$$

$$\frac{\partial^2 y}{\partial t^2} = \frac{\partial}{\partial t}\left(\pm v \cdot \frac{dy}{dz}\right) = \pm v \cdot \frac{d}{dz}\left(\pm v \cdot \frac{dy}{dz}\right)$$

$$\frac{\partial^2 y}{\partial t^2} = v^2 \cdot \frac{d^2 y}{dz^2}$$

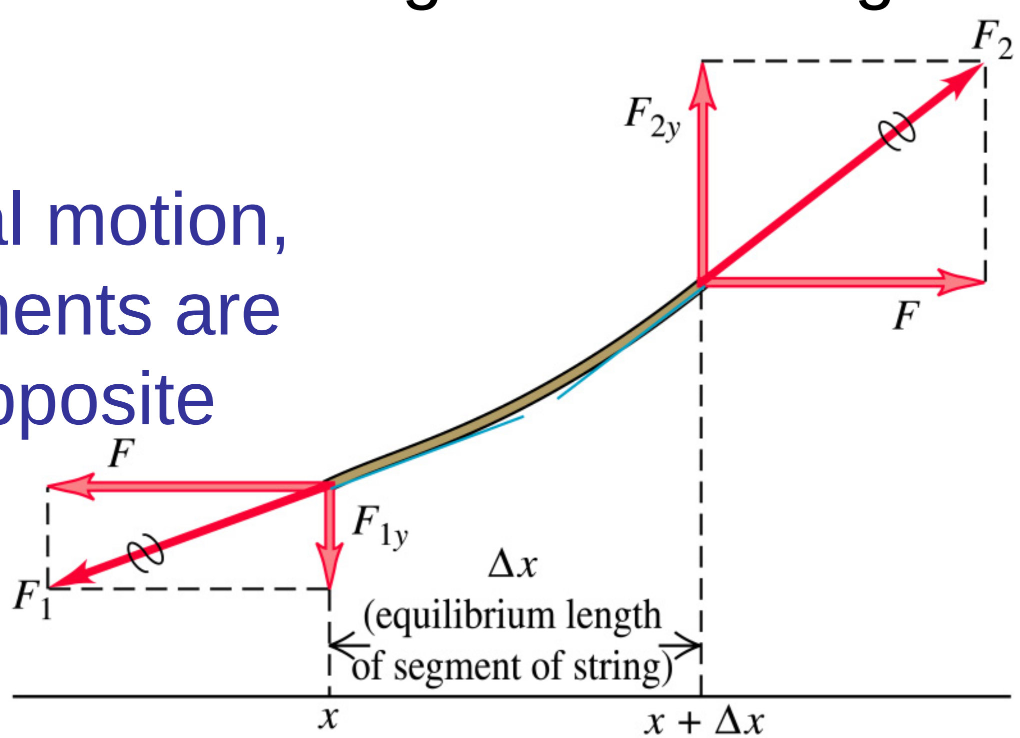
Left-hand side was $\frac{\partial^2 y}{\partial x^2} = \frac{d^2 y}{dz^2}$

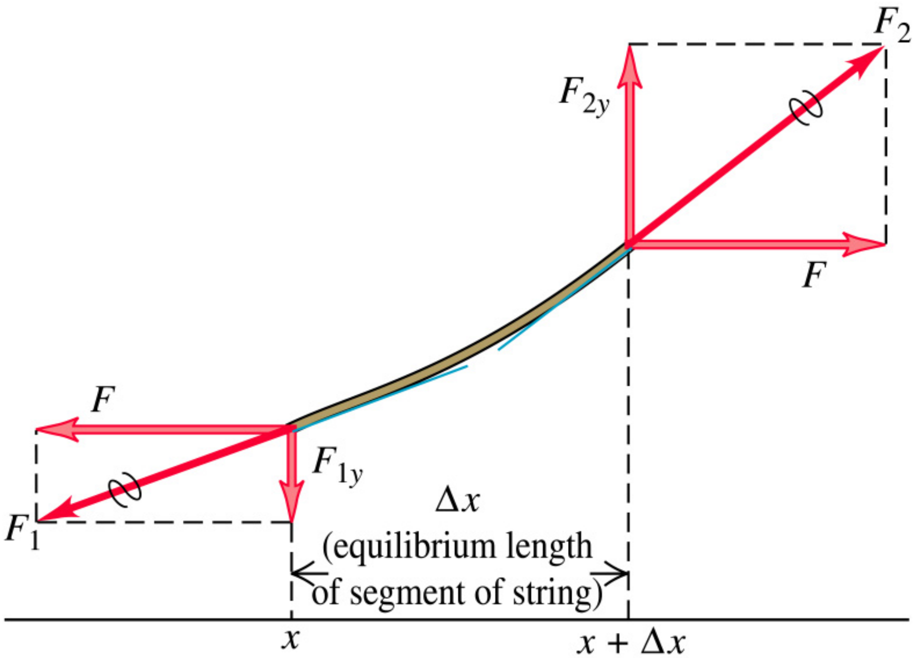
$$\rightarrow \frac{\partial^2 y(x \pm vt)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y(x \pm vt)}{\partial t^2}$$

Wave equation for a string

- F = string tension
- μ = string mass/unit length = M/L
- Ignore weight (gravity), ignore stretching of string
- Look at forces on length Δx of string

No horizontal motion, so x-components are equal and opposite





Slopes:

$$\left(\frac{\partial y}{\partial x}\right)_{x+\Delta x} = \frac{F_{2y}}{F}$$

$$\left(\frac{\partial y}{\partial x}\right)_{x} = -\frac{F_{1y}}{F}$$

Net vertical force:

$$F_y = F_{1y} + F_{2y} = F\left[\left(\frac{\partial y}{\partial x}\right)_{x+\Delta x} - \left(\frac{\partial y}{\partial x}\right)_{x}\right]$$

Newton's 2nd Law:

$$F_y = (\mu \Delta x)\frac{\partial^2 y}{\partial t^2}$$

$$F\left[\left(\frac{\partial y}{\partial x}\right)_{x+\Delta x} - \left(\frac{\partial y}{\partial x}\right)_{x}\right] = (\mu\Delta x)\frac{\partial^2 y}{\partial t^2}$$

$$\frac{\left(\frac{\partial y}{\partial x}\right)_{x+\Delta x} - \left(\frac{\partial y}{\partial x}\right)_{x}}{\Delta x} = \left(\frac{\mu}{F}\right)\frac{\partial^2 y}{\partial t^2}$$

Take limit $\Delta x \rightarrow 0$:

$$\frac{\partial^2 y}{\partial x^2} = \left(\frac{\mu}{F}\right)\frac{\partial^2 y}{\partial t^2}$$

Remember wave equation:

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y(x,t)}{\partial t^2}$$

$$v = \sqrt{\frac{F}{\mu}}$$

Wave equation for string

- Derived with no assumption on shape
- $V^2 = F/\mu$
- Does it make intuitive sense?
  - V increases with F (restoring force)
  - V decreases with mass (inertia)