Solutions – Practice Problems for the Final
Physics 25

Problem 1

(a) The probability of finding the electron at polar angle θ is proportional to

$$P(\theta)\,d\theta \propto |\Theta(\theta)|^2 \sin\theta\,d\theta = C^2 \sin^2\theta\cos^2\theta \cdot \sin\theta\,d\theta.$$

Setting $\frac{d}{d\theta}\big(\sin^3\theta\cos^2\theta\big) = 0$ gives $\sin^2\theta\cos\theta(3\cos^2\theta - 2\sin^2\theta) = 0$, whose non-trivial solution is $\tan^2\theta = 3/2$, i.e. $\theta \approx 50.8$. This lies strictly between the z-axis ($\theta = 0$) and the xy-plane ($\theta = 90$).

Answer: C – Somewhere in between.

(b) The maximum kinetic energy in the photoelectric effect is $K = hf - \phi = hc/\lambda - \phi$, which depends only on the photon *frequency* (wavelength), not on intensity. Since the wavelength is unchanged, K is unchanged.

Answer: C – Stay the same.

(c) The ground-state wavefunction is $\psi(x) = \sqrt{2/L}\sin(\pi x/L)$. The probability in an interval dx is $|\psi(x)|^2 dx \propto \sin^2(\pi x/L)\,dx$.

$$p_1 \propto \sin^2\left(\frac{\pi \cdot 0.5L}{L}\right) = \sin^2\left(\frac{\pi}{2}\right) = 1, \qquad p_2 \propto \sin^2\left(\frac{\pi \cdot 0.05L}{L}\right) = \sin^2(9) \approx 0.0245.$$

Answer: A – $p_1 > p_2$.

(d) The magnitude of angular momentum is $L = \hbar\sqrt{\ell(\ell+1)}$ and its z-component satisfies $|L_z| = \hbar|m| \le \hbar\ell$. For $L \neq 0$ (i.e. $\ell \ge 1$):

$$L = \hbar\sqrt{\ell(\ell+1)} > \hbar\ell \ge \hbar|m| = |L_z|,$$

so $L > L_z$ for every allowed state.

Answer: B – $L > L_z$ always.

(e) Large-angle scattering requires a single, very strong deflecting encounter. Atomic electrons are far too light to deflect a massive α particle appreciably, and multiple small deflections wash out statistically rather than adding coherently. Wave interference (C) is negligible at the relevant energies. The true cause – scattering off a tiny, massive, highly charged nucleus – is not listed among A–C.

Answer: D – None of the above.

Problem 2

The radial wave function is $R(r) = Are^{-r/2a_0}$. The radial probability density is

$$P(r) = r^2|R(r)|^2 = A^2 r^4 e^{-r/a_0}.$$

(a) Most probable distance. Set $dP/dr = 0$:

$$\frac{d}{dr}\left(r^4 e^{-r/a_0}\right) = e^{-r/a_0} r^3 \left(4 - \frac{r}{a_0}\right) = 0 \implies \boxed{r_{\mathrm{mp}} = 4a_0.}$$

(b) Average distance. Using $\int_0^\infty r^n e^{-r/a_0}\,dr = n!\,a_0^{n+1}$:

$$\langle r \rangle = \frac{\displaystyle\int_0^\infty r \cdot r^4 e^{-r/a_0}\,dr}{\displaystyle\int_0^\infty r^4 e^{-r/a_0}\,dr} = \frac{\displaystyle\int_0^\infty r^5 e^{-r/a_0}\,dr}{\displaystyle\int_0^\infty r^4 e^{-r/a_0}\,dr} = \frac{5!\,a_0^6}{4!\,a_0^5} = \frac{120\,a_0}{24} = \boxed{5a_0.}$$

Problem 3

The photoelectric-effect relation is $eV_{\mathrm{stop}} = hc/\lambda - \phi$, where ϕ is the work function and $hc = 1240$ eV nm.

(a) Work function. From the first measurement:

$$\phi = \frac{hc}{\lambda_1} - eV_1 = \frac{1240 \text{ eV nm}}{471 \text{ nm}} - 1.25 \text{ eV} = 2.633 \text{ eV} - 1.25 \text{ eV} = \boxed{1.38 \text{ eV.}}$$

(b) Second wavelength. From the second measurement:

$$\lambda_2 = \frac{hc}{eV_2 + \phi} = \frac{1240 \text{ eV nm}}{1.68 \text{ eV} + 1.38 \text{ eV}} = \frac{1240}{3.06} \text{ nm} \approx \boxed{405 \text{ nm.}}$$

Problem 4

The Rutherford cross section gives a count rate proportional to

$$N \propto \frac{1}{\sin^4(\theta/2)} \cdot \frac{\Delta A}{r^2},$$

where ΔA is the (fixed) detector area and r is the foil-to-detector distance.

(a) $\theta_2 = 60$, $r_2 = 1$ m (same distance).

$$\frac{N_2}{N_1} = \frac{\sin^4(\theta_1/2)}{\sin^4(\theta_2/2)} = \frac{\sin^4(45)}{\sin^4(30)} = \frac{(1/\sqrt{2})^4}{(1/2)^4} = \frac{1/4}{1/16} = 4.$$

$$\boxed{N_2 = 4 \times 10 = 40 \text{ particles/min.}}$$

(b) $\theta_3 = 60$, $r_3 = 3$ m (same angle as in part (a), tripled distance).
The solid angle $d\Omega = \Delta A/r^2$ decreases by a factor of $(r_2/r_3)^2 = (1/3)^2 = 1/9$:

$$N_3 = N_2 \times \left(\frac{r_2}{r_3}\right)^2 = 40 \times \frac{1}{9} \approx \boxed{4.4 \text{ particles/min.}}$$

Problem 5

Given $\Psi(x) = Ae^{-x^2/a^2}$, compute the second derivative:

$$\frac{d\Psi}{dx} = A\left(-\frac{2x}{a^2}\right)e^{-x^2/a^2},$$

$$\frac{d^2\Psi}{dx^2} = A\left(-\frac{2}{a^2} + \frac{4x^2}{a^4}\right)e^{-x^2/a^2} = \left(\frac{4x^2}{a^4} - \frac{2}{a^2}\right)\Psi.$$

Substituting into the time-independent Schrödinger equation $-\frac{\hbar^2}{2m}\Psi'' + V(x)\Psi = E\Psi$ and dividing by Ψ:

$$V(x) = E + \frac{\hbar^2}{2m}\left(\frac{4x^2}{a^4} - \frac{2}{a^2}\right) = E - \frac{\hbar^2}{ma^2} + \frac{2\hbar^2}{ma^4}x^2.$$

Imposing $V(0) = 0$:

$$0 = E - \frac{\hbar^2}{ma^2} \implies \boxed{E = \frac{\hbar^2}{ma^2}.}$$

The potential is then

$$\boxed{V(x) = \frac{2\hbar^2}{ma^4}x^2.}$$

This is a harmonic oscillator potential $V = \frac{1}{2}m\omega^2x^2$ with $\omega = 2\hbar/(ma^2)$, and the energy $E = \frac{1}{2}\hbar\omega$ confirms Ψ is the ground state.