Practice problems for the final
Physics 25

About one-half of the problems on the final will be on Special Relativity (SR), and the other half on ther rest of the Modern Physics material.
As far as SR is concerned, you already have problems from the practice midterm and the midterm itself as examples of the type of problems that are likely to appear on the final.
Here I have collected sample non-SR Modern Physics problems from old Physics 25 finals that I gave many many years ago.

Problem 1
Quick conceptual questions. Pick an answer, no need to explain. No penalty for mistakes.

(a) For a certain electronic state in hydrogen, the angular part of the wavefunction is $\Theta(\theta) = C \sin\theta \cos\theta$, where θ is the usual polar angle. Is the electron described by this wavefunction most likely to be found
A. Close to the z-axis.
B. Close to the xy-plane.
C. Somewhere in between.

(b) When a certain light source illuminates a metal surface, electrons are emitted from the metal with kinetic energies up to the value K. The light source is replaced with one that has the same wavelength but less intensity. With this new light source, does the maximum electron kinetic energy
A. Increase.
B. Decrease.
C. Stay the same.

(c) A particle is trapped inside an infinite potential well $0 < x < L$. The particle is in its ground state. The probability to locate the particle in a small interval of width dx near $x = 0.5L$ is p_1. The probability to locate the particle in a small interval of the same width dx near $x = 0.05L$ is p_2.
A. $p_1 > p_2$.
B. $p_1 < p_2$.
C. $p_1 = p_2$.

(d) Let L be the magnitude of angular momentum, and L_z be its z-component. Excluding cases in which $L = 0$, for an electron in a hydrogen atom:
A. $L = L_z$ always.
B. $L > L_z$ always.
C. $L = L_z$ in some cases, but not always.

(e) In Rutherford scattering of alpha particles by a gold foil, large deflections can occasionally be observed because
A. a single alpha particle can be scattered many times in encounters with many atoms.
B. the alpha particle can be deflected by the many atomic electrons of the gold atoms.
C. the wave nature of the alpha particles causes interference effects.
D. none of the above.

Problem 2
The radial part of the $2p$ hydrogen wave function is $R(r) = Are^{-r/2a_0}$ where A is a constant.
(a) Find the most probable distance of the electron from the nucleus.
(b) Find the average of this distance.

Problem 3
The stopping potential for a certain surface is $V_1 = 1.25$ V when it is illuminated with light of wavelength $\lambda_1 = 471$ nm. When the wavelength is changed to a new value λ_2, the stopping potential becomes $V_2 = 1.68$ V
(a) What is the work function of this surface (in electronVolts)?
(b) What is λ_2?

Problem 4
In a Rutherford scattering experiment the scattered α particles are detected by a small, movable, square particle detector. The detector is initially placed at a distance of 1 m from the thin gold foil and at an angle of 90^o from the direction of the incoming α particles. The detector is hit (on average) by 10 α particles per minute.
(a) Next, the detector is moved. It is placed at an angle of 60^o from the direction of the incoming α particles, but keeping the distance between the detector and the gold foil fixed at 1 m. How many α particles per minute

will hit the detector in this configuration (on average)?
(b) Finally, the detector is moved again. In this case, the angle is kept to be 60^o, but the distance is increased from 1 m to 3 m. How many α-particles per minute will hit the detector in this configuration (on average)?
You should work in the approximation that the detector dimensions are much much smaller than the distance between the foil and the detector.

Problem 5
One of the solutions of the one-dimensional time independent Schrödinger equation for a particular potential V(x) is $\Psi(x) = Ae^{-\frac{x^2}{a^2}}$. Find V(x) taking the arbitrary constant in the definition of potential energy to be such that V(x=0)=0. Also, find the energy corresponding to this solution.

Useful Information

Rutherford scattering: For a single α-nucleus scatter, the probability of scattering into a solid angle $d\Omega = \sin\theta \, d\theta \, d\phi$ is

$$p(\Omega)d\Omega = A\frac{Z^2}{K}\frac{d\Omega}{\sin^4\frac{\theta}{2}}$$

where A is a proportionality constant, K is the kinetic energy of the α-particle, and Z is the charge of the nucleus in units of e.

Some Commonly Used Constants and Conversion Factors

(see Appendix A for a more complete list)

Speed of light	$c = 2.998 \times 10^8$ m/s
Electronic charge	$e = 1.602 \times 10^{-19}$ C
Boltzmann constant	$k = 1.381 \times 10^{-23}$ J/K $= 8.617 \times 10^{-5}$ eV/K
Planck's constant	$h = 6.626 \times 10^{-34}$ J·s $= 4.136 \times 10^{-15}$ eV·s
Avogadro's constant	$N_A = 6.022 \times 10^{23}$ mole $^{-1}$
Electron mass	$m_e = 5.49 \times 10^{-4}$ u $= 0.511$ MeV$/c^2$
Proton mass	$m_p = 1.007276$ u $= 938.3$ MeV$/c^2$
Neutron mass	$m_n = 1.008665$ u $= 939.6$ MeV$/c^2$
Bohr radius	$a_0 = 0.0529$ nm
Hydrogen ionization energy	13.6 eV
Thermal energy	$kT = 0.02525$ eV $\cong \frac{1}{40}$ eV ($T = 293$ K)
$hc = 1240$ eV·nm (MeV·fm)	$\hbar c = 197$ eV·nm (MeV·fm)
$\dfrac{e^2}{4\pi\varepsilon_0} = 1.440$ eV·nm (MeV·fm)	1 u $= 931.5$ MeV$/c^2$
	1 eV $= 1.602 \times 10^{-19}$ J

Some integrals contain Euler's constant $\gamma = 0.5772156\ldots$ [see 1.20, page 3].

18.68 $$\int_0^\infty e^{-ax}\cos bx\,dx = \frac{a}{a^2+b^2}$$

18.69 $$\int_0^\infty e^{-ax}\sin bx\,dx = \frac{b}{a^2+b^2}$$

18.70 $$\int_0^\infty \frac{e^{-ax}\sin bx}{x}\,dx = \tan^{-1}\frac{b}{a}$$

18.71 $$\int_0^\infty \frac{e^{-ax}-e^{-bx}}{x}\,dx = \ln\frac{b}{a}$$

18.72 $$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

18.73 $$\int_0^\infty e^{-ax^2}\cos bx\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}\,e^{-b^2/4a}$$

18.74 $$\int_0^\infty e^{-(ax^2+bx+c)}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}\,e^{(b^2-4ac)/4a}\operatorname{erfc}\frac{b}{2\sqrt{a}}$$

where $$\operatorname{erfc}(p) = \frac{2}{\sqrt{\pi}}\int_p^\infty e^{-x^2}\,dx$$

18.75 $$\int_{-\infty}^\infty e^{-(ax^2+bx+c)}\,dx = \sqrt{\frac{\pi}{a}}\,e^{(b^2-4ac)/4a}$$

18.76 $$\int_0^\infty x^n e^{-ax}\,dx = \frac{\Gamma(n+1)}{a^{n+1}}$$

18.77 $$\int_0^\infty x^m e^{-ax^2}\,dx = \frac{\Gamma[(m+1)/2]}{2a^{(m+1)/2}}$$

18.78 $$\int_0^\infty e^{-(ax^2+b/x^2)}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}\,e^{-2\sqrt{ab}}$$

18.79 $$\int_0^\infty \frac{x\,dx}{e^x-1} = \frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots = \frac{\pi^2}{6}$$

18.80 $$\int_0^\infty \frac{x^{n-1}}{e^x-1}\,dx = \Gamma(n)\left(\frac{1}{1^n}+\frac{1}{2^n}+\frac{1}{3^n}+\cdots\right)$$

For even n this can be summed in terms of Bernoulli numbers [see pages 139 to 140].