Physics 25 Final — Solutions

Spring Quarter 2026

Throughout these solutions we set $c = 1$.

Problem 1

(a)

Use the relativistic energy-momentum relation $E^2 = p^2 + m^2$ with $E = 4m$:

$$16m^2 = p^2 + m^2 \implies p^2 = 15m^2$$

$$\boxed{C: \quad p = \sqrt{15}\, m}$$

(b)

$L = \sqrt{l(l+1)}\,\hbar = \sqrt{2}\,\hbar$ implies $l(l+1) = 2$, so $l = 1$. The magnetic quantum number takes values $m_\ell = -1,\ 0,\ +1$, giving $L_z = m_\ell \hbar$.

$$\boxed{D: \quad L_z = -\hbar,\ 0,\ \hbar}$$

(c)

A valid radial wavefunction $R(r)$ must be normalizable ($\int_0^\infty |R|^2 r^2\, dr < \infty$), finite at $r = 0$, and vanish as $r \to \infty$.

- I: Ae^{-br} — finite at $r = 0$, decays exponentially to zero. ✓
- II: $A \sin(br)$ — oscillates forever, not normalizable. ×
- III: A/r — $\int_0^\infty (A/r)^2 r^2\, dr = A^2 \int_0^\infty dr$ diverges. ×

$$\boxed{A: \quad \text{I only}}$$

(d)

The photoelectric effect is explained by Einstein's photon hypothesis: light is absorbed in discrete quanta of energy $E = h\nu$, where ν is the frequency.

$$\boxed{D}$$

Problem 2

At the threshold wavelength λ_0, the photon energy exactly equals the work function ϕ:

$$\phi = \frac{h}{\lambda_0}$$

For incident light of wavelength λ, the maximum kinetic energy of the ejected electron is:

$$KE_{\max} = \frac{h}{\lambda} - \frac{h}{\lambda_0}$$

$$\boxed{KE_{\max} = h\left(\frac{1}{\lambda} - \frac{1}{\lambda_0}\right) = \frac{h(\lambda_0 - \lambda)}{\lambda\,\lambda_0}}$$

Problem 3

(a) Electron momentum in the muon rest frame

In the muon rest frame, conservation of 3-momentum requires the electron and neutrino momenta to be equal in magnitude and opposite in direction: $|\vec{p}_e| = |\vec{p}_\nu| = q$. With $c = 1$ the energies are

$$E_e = \sqrt{q^2 + m^2}, \qquad E_\nu = q \quad \text{(massless neutrino)}.$$

Conservation of energy:

$$M = \sqrt{q^2 + m^2} + q \implies M - q = \sqrt{q^2 + m^2}.$$

Squaring both sides:

$$M^2 - 2Mq + q^2 = q^2 + m^2 \implies 2Mq = M^2 - m^2.$$

$$\boxed{q = \frac{M^2 - m^2}{2M}}$$

(b) Neutrino momentum in the boosted frame

In the muon rest frame S, the electron moves in $+x$ with momentum $+q$, so the neutrino moves in $-x$ with $(E_\nu,\, p_{\nu x}) = (q,\, -q)$.

In the boosted frame S' the muon has momentum p' in $+x$ and energy $E' = \sqrt{p'^2 + M^2}$. The boost parameters are

$$\gamma = \frac{E'}{M}, \qquad \beta = -\frac{p'}{E'}$$

(negative because S' moves in the $-x$ direction relative to S).

Applying the Lorentz transformation:

$$k' = \gamma(p_{\nu x} - \beta\, E_\nu) = \frac{E'}{M}\left(-q - \left(-\frac{p'}{E'}\right)q\right) = \frac{E'}{M}\cdot q\left(-1 + \frac{p'}{E'}\right) = \frac{q(p' - E')}{M}.$$

$$\boxed{k' = -\frac{q\left(\sqrt{p'^2 + M^2} - p'\right)}{M}}$$

An equivalent form is:

$$\boxed{k' = -\frac{qM}{\sqrt{p'^2 + M^2} + p'}}.$$

This can be derived by taking the first boxed solution, multiplying numerator and denominator by $\sqrt{p'^2 + M^2} + p'$ and using the trivial identity $(a+b)(a-b) = a^2 - b^2$.

The neutrino continues to move in the $-x$ direction in the boosted frame.

Problem 4

(a) Energy of each photon

The initial total energy is $E + m$ (positron energy plus electron rest mass). Since the two photons have equal energies, conservation of energy gives:

$$E + m = 2E_\gamma$$

$$\boxed{E_\gamma = \frac{E + m}{2}}$$

(b) Angle of photon momenta with the incoming positron

Let the two photons be emitted at angles $\pm\theta$ relative to the positron direction. (Equal energies and conservation of transverse momentum force the angles to be equal and opposite.)

Conservation of x-momentum (with $p_\gamma = E_\gamma$ for massless photons):

$$p_{\text{positron}} = 2E_\gamma \cos\theta.$$

The positron's momentum satisfies $p_{\text{positron}} = \sqrt{E^2 - m^2}$ and $2E_\gamma = E + m$, so:

$$\sqrt{E^2 - m^2} = (E + m)\cos\theta.$$

Solving:

$$\cos\theta = \frac{\sqrt{(E - m)(E + m)}}{E + m}$$

$$\boxed{\cos\theta = \sqrt{\frac{E - m}{E + m}}}$$

Problem 5

We work in units where $c = 1$, measuring distances in light-years and times in years. Bob's data: $v = 0.8$, $\Delta t_B = 30$ yr, giving $\gamma = 1/\sqrt{1 - 0.64} = 5/3$.

(a) Distance Earth–Zorg (Bob's frame)

$$d = v\,\Delta t_B = 0.8 \times 30 \text{ yr}$$
$$\boxed{d = 24 \text{ light-years}}$$

(b) Martha's travel time (proper time)

Martha travels with the rocket, so she measures the proper time:

$$\Delta\tau = \frac{\Delta t_B}{\gamma} = 30 \times \frac{3}{5}$$
$$\boxed{\Delta\tau = 18 \text{ years}}$$

(c) Distance Earth–Zorg (Martha's frame)

In Martha's frame the distance is length-contracted:

$$d' = \frac{d}{\gamma} = 24 \times \frac{3}{5}$$
$$\boxed{d' = 14.4 \text{ light-years}}$$

(d) Distance between events in Ralph's frame

The spacetime interval $s^2 = \Delta t^2 - \Delta x^2$ is Lorentz-invariant. Using Bob's frame:

$$s^2 = 30^2 - 24^2 = 900 - 576 = 324 \text{ ly}^2.$$

In Ralph's frame $\Delta t_R = 25$ yr:

$$(\Delta x_R)^2 = \Delta t_R^2 - s^2 = 625 - 324 = 301 \text{ ly}^2.$$
$$\boxed{\Delta x_R = \sqrt{301} \approx 17.35 \text{ light-years}}$$

Problem 6

In a circular Bohr orbit the Coulomb force provides the centripetal acceleration:

$$\frac{ke^2}{r^2} = \frac{mv^2}{r} \implies ke^2 = mv^2 r.$$

The kinetic and potential energies are

$$K = \tfrac{1}{2}mv^2, \qquad U = -\frac{ke^2}{r}.$$

From the force balance, $ke^2/r = mv^2 = 2K$, so $|U| = 2K$.

$$\boxed{\left|\frac{U}{K}\right| = 2}$$

This is a direct consequence of the virial theorem for an inverse-square force.

Problem 7

The expectation value of x in the state $\psi(x) = Axe^{-\alpha x^2}$ is:

$$\langle x \rangle = \int_{-\infty}^{\infty} \psi^*(x)\, x\, \psi(x)\, dx = A^2 \int_{-\infty}^{\infty} x^3\, e^{-2\alpha x^2}\, dx.$$

The integrand $x^3 e^{-2\alpha x^2}$ is an *odd* function (odd $\times$ even), so its integral over the symmetric domain $(-\infty, \infty)$ vanishes:

$$\boxed{\langle x \rangle = 0}$$