Spring Quarter 2026 – UCSB Physics 25
Final

Put a box around the final answer for each problem/question

Problem 1
Quick conceptual questions. Pick an answer, no need to explain. No penalty for mistakes. Note: these are straight from Physics GRE exams.

(a) An electron (rest mass = m) has total energy equal to four times its rest energy. The momentum of the electron is:
A. mc
B. $\sqrt{2}mc$
C. $\sqrt{15}mc$
D. $4mc$
E. $2\sqrt{15}mc$

(b) Consider a single electron atom with orbital angular momentum $L = \sqrt{2}\hbar$. Which of the following gives all possible values of L_z, the z-component of angular momentum:
A. 0
B. $0,\ \hbar$
C. $0,\ \hbar,\ 2\hbar$
D. $-\hbar,\ 0,\ \hbar$
E. $-2\hbar,\ -\hbar,\ 0,\ \hbar,\ 2\hbar$

(c) Which of the following functions could possibly represent the radial wavefunction for an electron in an atom? (r is the distance of the electron from the nucleus. A and b are constants).
I: Ae^{-br}
II: $A\sin br$
III: A/r

A. I only
B. II only
C. I and III only
D. I, II, and III

(d) The photoelectric effect can be explained assuming that
A. electrons are restreicted to orbits of angular momentum $n\hbar$, where n is an integer
B. electrons are associated with waves of wavelength $\lambda = h/p$, where p is the momentum
C. light is emitted ony when electrons jump between orbits
D. light is absorbed in quanta of energt $E = h\nu$, where ν is the frequency of the light.
E. light behaves like a wave.

Problem 2
The threshold wavelength (i.e.: the longest wavelength) for emission of electrons from a given metal surface is λ_0. What is the maximum kinetic energy for an ejected electron when the wavelength is changed from λ_0 to λ.

Problem 3

A muon (rest mass M) decays into an electron (rest mass m) and a neutrino (rest mass=0).
(a) Find the magnitude of the momentum of the electron in the muon rest frame. Call this momentum q.
(b) Now imagine that in the rest frame the electron was emitted in the positive X-direction. Boost the system into a frame where the muon is moving with momentum p' in the positive X-direction. In this frame what is the momentum k' of the neutrino? Leave your answer expressed in terms of q. If you do not know how to do part (a), attempt part (b) taking q as a given.

Problem 4

A positron (e^+) is the antimatter partner of the electron, i.e., a particle identical to the electron (e^-) but with positive charge.
A positron of energy E collides with an electron which is at rest. The electron and the positron annhilate into two photons, $e^+e^- \to \gamma\gamma$. The two photons are measured to emerge from the collision with equal energies E_γ.
(a) What is E_γ?
(b) What is the angle made by the momentum vector of one of the two photons and the momentum vector of the incoming positron.
(Take the mass of one electron or one positron to be m).

Problem 5

According to Bob, an observer on the Earth, a rocket carrying Martha from Earth to the planet Zorg travels at a speed of $0.8c$ and takes 30 years to reach Zorg. (Zorg is at rest with respect to the earth). Give answers in light-years (for distances) and years (for time intervals).
(a) What is the distance between Earth and Zorg, according to Bob.
(b) How long does Martha think that the trip takes.
(c) What is the distance between Earth and Zorg, according to Martha?
(d) A third observer, Ralph, thinks that the two events of Martha passing the earth and Martha reaching Zorg occur 25 years apart. In Ralph's reference frame what is the distance between these two events? Ralph's velocity relative to the earth does not change between these two events.

Problem 6

For an electron in any Bohr orbit of the Hydrogen atom, calculate the the ratio of **magnitude** of the potential energy to that of the kinetic enerhy, $|U/K|$ where U is the potential energy and K is the kinetic energy.

Problem 7

The wavefunction of the first excited state of a one-dimensional harmonic oscillator is given by $\psi(x) = Axe^{-\alpha x^2}$ where $A = (\frac{32\alpha^3}{\pi})^{\frac{1}{4}}$. What is the average value of x.

Some Commonly Used Constants and Conversion Factors

(see Appendix A for a more complete list)

Speed of light	$c = 2.998 \times 10^8$ m/s
Electronic charge	$e = 1.602 \times 10^{-19}$ C
Boltzmann constant	$k = 1.381 \times 10^{-23}$ J/K $= 8.617 \times 10^{-5}$ eV/K
Planck's constant	$h = 6.626 \times 10^{-34}$ J·s $= 4.136 \times 10^{-15}$ eV·s
Avogadro's constant	$N_A = 6.022 \times 10^{23}$ mole $^{-1}$
Electron mass	$m_e = 5.49 \times 10^{-4}$ u $= 0.511$ MeV$/c^2$
Proton mass	$m_p = 1.007276$ u $= 938.3$ MeV$/c^2$
Neutron mass	$m_n = 1.008665$ u $= 939.6$ MeV$/c^2$
Bohr radius	$a_0 = 0.0529$ nm
Hydrogen ionization energy	13.6 eV
Thermal energy	$kT = 0.02525$ eV $\cong \frac{1}{40}$ eV ($T = 293$ K)
$hc = 1240$ eV·nm (MeV·fm)	$\hbar c = 197$ eV·nm (MeV·fm)
$\dfrac{e^2}{4\pi\varepsilon_0} = 1.440$ eV·nm (MeV·fm)	1 u $= 931.5$ MeV$/c^2$ 1 eV $= 1.602 \times 10^{-19}$ J

$\Gamma(n+1) = n!$ if n is a positive integer

DEFINITE INTEGRALS INVOLVING EXPONENTIAL FUNCTIONS

Some integrals contain Euler's constant $\gamma = 0.5772156\ldots$ [see 1.20, page 3].

18.68 $$\int_0^\infty e^{-ax}\cos bx\,dx = \frac{a}{a^2+b^2}$$

18.69 $$\int_0^\infty e^{-ax}\sin bx\,dx = \frac{b}{a^2+b^2}$$

18.70 $$\int_0^\infty \frac{e^{-ax}\sin bx}{x}\,dx = \tan^{-1}\frac{b}{a}$$

18.71 $$\int_0^\infty \frac{e^{-ax}-e^{-bx}}{x}\,dx = \ln\frac{b}{a}$$

18.72 $$\int_0^\infty e^{-ax^2}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}$$

18.73 $$\int_0^\infty e^{-ax^2}\cos bx\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}\,e^{-b^2/4a}$$

18.74 $$\int_0^\infty e^{-(ax^2+bx+c)}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}\,e^{(b^2-4ac)/4a}\,\operatorname{erfc}\frac{b}{2\sqrt{a}}$$

where $$\operatorname{erfc}(p) = \frac{2}{\sqrt{\pi}}\int_p^\infty e^{-x^2}\,dx$$

18.75 $$\int_{-\infty}^\infty e^{-(ax^2+bx+c)}\,dx = \sqrt{\frac{\pi}{a}}\,e^{(b^2-4ac)/4a}$$

18.76 $$\int_0^\infty x^n e^{-ax}\,dx = \frac{\Gamma(n+1)}{a^{n+1}}$$

18.77 $$\int_0^\infty x^m e^{-ax^2}\,dx = \frac{\Gamma[(m+1)/2]}{2a^{(m+1)/2}}$$

18.78 $$\int_0^\infty e^{-(ax^2+b/x^2)}\,dx = \frac{1}{2}\sqrt{\frac{\pi}{a}}\,e^{-2\sqrt{ab}}$$

18.79 $$\int_0^\infty \frac{x\,dx}{e^x-1} = \frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\cdots = \frac{\pi^2}{6}$$

18.80 $$\int_0^\infty \frac{x^{n-1}}{e^x-1}\,dx = \Gamma(n)\left(\frac{1}{1^n}+\frac{1}{2^n}+\frac{1}{3^n}+\cdots\right)$$

For even n this can be summed in terms of Bernoulli numbers [see pages 139 to 140].