7.33) At any given time, the flux is \[ \Phi = TA^2B. \]

If \( B \) changes, then \( \Phi \) changes, therefore there is an E.M.F. In magnitude \[ E_{\text{m.f.}} = \frac{d\Phi}{dt} \]

If the ring was a conductor, we would get current flowing. But the ring is an insulator, so current does not flow. However, there is an \( E \) field associated with the E.M.F. inside the ring. Since the ring is charged, this \( E \) field will cause a force, and this force will cause a torque. Why a torque?

Because the forces on each charge element are tangential:

\[ \tau = F a = qEa. \]

Now we need to find \( E \):

\[ E_{\text{m.f.}} = \frac{d\phi}{dt} \text{ but } E_{\text{m.f.}} = \int E \cdot d\ell = 2\pi aE \]

\[ 2\pi aE(t) = TA^2 \frac{dB}{dt} \]

\[ E(t) = \frac{1}{2} \frac{q^2 a}{L} \frac{dB}{dt} \]

Let \( L \) = angular momentum, \( L = Iw \), where \( I \) is the moment of inertia.
We have \( \gamma = \frac{dL}{dt} = I \frac{dw}{dt} \)
\[
\frac{1}{2} qa^2 \frac{dB}{dt} = I \frac{dw}{dt} = ma^2 \frac{dw}{dt}
\]
\((I = ma^2)\)

\[
\frac{dw}{dt} = \frac{q dB}{2M dt}
\]
Integrate \( w \) from 0 \( \rightarrow \) \( w \)
Integrate \( B \) from \( B_0 \rightarrow 0 \)
Up to a sign, I then get \( \boxed{w = \frac{q B_0}{2M}} \)

\[7.36\]
(a) If \( I_2 \) increases, there will be extra flux through the top circuit pointing up. Therefore, the induced emf in the top circuit must make an extra downward flux. The induced current must then be negative: \( \boxed{\varepsilon_1 = -L_1 \frac{dI_1}{dt} - M \frac{dI_2}{dt}} \) (1)

Exact same argument leads to \( \boxed{\varepsilon_2 = -L_2 \frac{dI_2}{dt} - M \frac{dI_1}{dt}} \) (2)

If we were to switch the convention for the \( I_2 \) and the \( \varepsilon_2 \), we would get the opposite sign, i.e., \( +M \frac{dI_1}{dt} \)

(b) Key idea here is that in this case \( I_1 = I_2 = I \)
Also the total emf \( \varepsilon \) is \( \varepsilon_1 + \varepsilon_2 \)

Then adding equations (1) and (2) above, I get \( \boxed{\varepsilon = -(L_1 + L_2 + 2M) \frac{dI}{dt}} \)
\( \Rightarrow I = L_1 + L_2 + 2M \)
If instead we connect the coils like in figure (c) \( I_1 = -I_2 \). Write \( I = I_1 \).

Then
\[
\mathcal{E} = \mathcal{E}_1 - \mathcal{E}_2 = -L_1 \frac{dI}{dt} - M \frac{d(-I)}{dt} + L_2 \frac{d(-I)}{dt} + M \frac{dI}{dt}
\]

\[
\mathcal{E} = -(L_1 + L_2 - 2M) \frac{dI}{dt}
\]

\[
L'' = L_1 + L_2 - 2M
\]

(c) Since \( L'' \geq 0 \), must have \( M \leq \frac{L_1 + L_2}{2} \).

\[\text{7.41}\]

\[\mathcal{E} = 10 \text{V} \quad R_1 = 150 \Omega \quad R_2 = 50 \Omega \quad L = 0.1 \text{H}\]

When the switch has been closed for a long time, all transients have died out.

Thus
\[
V_A = \mathcal{E} = 10 \text{V} \quad V_B = V_A = 10 \text{V}
\]

\[
I_1 = \frac{V_A}{R_1} = 0.067 \text{A}
\]

\[
I_2 = \frac{V_B}{R_2} = \frac{V_A}{R_2} = 0.20 \text{A}
\]
When we open the switch, the circuit now looks like this:

\[ V_A - V_B = L \frac{dI}{dt} \quad (1) \]

\[ V_B - V_C = V_B = I R_2 \quad (2) \]

\[ V_C - V_A = -V_A = I R_1 \quad (3) \]

\[ L \frac{dI}{dt} + I(R_1+R_2) = 0 \]

Assuming that the switch is closed at \( t=0 \), the solution is \( I = I_0 e^{-t/\tau} \)

with \( \tau = L / (R_1+R_2) = 0.1 / 200 \) sec = 0.5 msec

\( I_0 \) is given by the value from the previous page i.e. \( I_0 = 0.2 \) A. Then from equations 2 & 3 we can get \( V_A \) and \( V_B \) as a function of time:

\[ V_B = I R_2 = 10 V e^{-t/\tau} \]

\[ V_A = -I R_1 = -30 V e^{-t/\tau} \]

Note that \( V_A \) is discontinuous!!
\[ I = I_0 \left(1 - e^{-t/t_0}\right) \quad t_0 = \frac{L}{R} \quad I_0 = \frac{E}{R} \]

\[ e^{-t/t_0} = 0.1 \quad \frac{t}{t_0} = \log_{10} 0.1 \]

\[ t = \frac{\log 10 \cdot L}{R} = 0.115 \text{ sec} \]

\[ E = \frac{1}{2} L I^2 = \frac{1}{2} L \left(\frac{9}{10} I_0\right)^2 = \frac{81}{200} L \frac{E^2}{R^2} \]

\[ E = \frac{81}{200} \times 5 \times 10^{-4} \times \frac{12^2}{10^{-4}} \quad J = 292 \text{ J} \]

For the energy delivered by the battery, we cannot simply answer "292 J" because there have been IR losses in the resistor. (Remember: \( I^2R \) is power, i.e., \( \frac{dE}{dt} \). We can do this two ways: (a) integrate \( I^2R \) and add it to 292 J; or (b) integrate \( EI \), which is the power instantaneously delivered by the battery. Let's do it both ways.
\[ E = \int_0^t E_0 I_0 \, dt \quad I(t) = \frac{\log 10}{R} t \]

\[ E = E_0 I_0 \left[ t - \frac{9}{10} t \right] = \frac{E_0^2}{R^2} \left[ \log 10 - \frac{9}{10} \log 10 \right] \]

\[ E = \frac{12.5 \times 10^{-4}}{10^{-4}} \left[ \log 10 - \frac{9}{10} \log 10 \right] = 1010 J \]

This means that \(1010 J - 292 J = 718 J\) must have been dissipated in the resistor.

Let's verify that

\[ E_{\text{resistor}} = \int I^2 R \, dt = I_0^2 R \int_0^t (1 - e^{-t/\tau})^2 \, dt \]

\[ = \frac{E_0^2}{R} \left[ t - 2e^{-t/\tau} + e^{-2t/\tau} \right] dt \]

\[ = \frac{E_0^2}{R} \left[ t - \frac{9}{10} \log 10 + \frac{1}{2} \frac{99}{100} \log 10 \right] = \frac{E_0^2}{R^2} \left[ \log 10 - \frac{18}{100} \log 10 \right] \]

\[ = \frac{12.5 \times 10^{-4}}{10^{-4}} \left[ 0.997 \right] = 718 J \]

\(\checkmark\) Yes!!
\[ V_A - V_B = V_0 \cos \omega t \quad \text{(voltage across C)} \]
\[ V_B - V_C = -V_0 \cos \omega t \quad \text{(voltage across L)} \]

Current \[ L \frac{dI}{dt} = -V_0 \cos \omega t \Rightarrow I = -\frac{V_0}{\omega L} \sin \omega t \]

or \[ I = V_0 \sqrt{\frac{C}{L}} \sin \omega t \]

Energy in the inductor \[ U_L = \frac{1}{2} L I^2 = \frac{1}{2} \frac{V_0^2}{C} \sin^2 \omega t \]

Energy in the capacitor \[ U_C = \frac{1}{2} C (V_A - V_B)^2 = \frac{1}{2} \frac{V_0^2}{C} \cos^2 \omega t \]

Note: \[ U_L + U_C = \frac{1}{2} CV_0^2 \text{ is constant} \]

At \[ t = 0 \]
\[ U_L = 0 \quad U_C = \frac{1}{2} CV_0^2 \]

At \[ t = \frac{T}{2} \]
\[ \frac{\pi}{2 \omega} \quad U_L = \frac{1}{2} CV_0^2 \quad U_C = 0 \]
\[
IR + \frac{Q}{C} = E_0 \cos \omega t
\]

\[
R \frac{dQ}{dt} + \frac{Q}{C} = E_0 \cos \omega t = E_0 e^{i\omega t}
\]

Write \(Q = \tilde{Q}_0 e^{i\omega t}\) \(\Rightarrow \frac{dQ}{dt} = i\omega \tilde{Q}_0 e^{i\omega t}\)

\[
\tilde{Q}_0 = \frac{C E_0}{1 + i\omega RC}
\]

But we wanted current, so \(\tilde{I} = \tilde{I}_0 e^{i\omega t}\)

\[
\tilde{I}_0 = i\omega \tilde{Q}_0 = \frac{i\omega CE_0}{1 + i\omega RC} = \frac{i\omega CE_0 (1 - i\omega RC)}{1 + \omega^2 R^2 C^2}
\]

\[
\tilde{I}_0 = \frac{E_0 \omega^2 RC^2 + i E_0 \omega C}{1 + \omega^2 R^2 C^2}
\]

Now write \(\tilde{I}_0 = I_0 e^{i\phi}\)

\[
\tan \phi = \frac{E_0 \omega C}{E_0 \omega^2 RC^2 - \omega RC}
\]

\[
I_0 = \frac{E_0 \sqrt{\omega^2 RC^4 + \omega^2 C^2}}{1 + \omega^2 R^2 C^2} = \frac{E_0 \omega C \sqrt{1 + \omega^2 RC^2}}{1 + \omega^2 R^2 C^2}
\]
\[ I_0 = \frac{\omega C E_0}{\sqrt{1 + \omega^2 R^2 C^2}} = \frac{E_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \]

Now take the real part of \( I e^{i\omega t} = I_0 e^{i(\omega t + \phi)} \)

\[ I(t) = \frac{E_0}{\sqrt{R^2 + \frac{1}{\omega^2 C^2}}} \cos(\omega t + \phi) \]

with \( \phi = \tan^{-1} \frac{1}{R \omega C} \)

For large \( \omega \), amplitude \( \Rightarrow \frac{E_0}{R} \)

phase \( \Rightarrow 0 \)

This makes sense, because at high \( \omega \) the capacitor is 'like a short'.

For small \( \omega \), the amplitude goes to zero. This makes sense because for DC currents, the capacitor does not conduct.