1.21 \text{ emf } E = - \frac{d\Phi}{dt}

\Phi = NAB_0 \cos(\omega t + \phi)

E = N(\pi R^2)B_0 2\pi f \sin(\omega t + \phi)

E_{\text{max}} = 2N\pi^2 R^2 B_0 = 2.410^3 \pi^2 (12 \times 10^{-2})^2 30.5 \times 10^{-5} V

E_{\text{max}} = 1.7 V

7.22 \text{ Consider circle of radius } R = 3 \text{ cm centered on } \text{axis} \text{. }

\Phi = TTR^2 B

B = B_0 \cos(\omega t + \phi) \quad B_0 = 4 G = 4 \times 10^{-4} T

w = 2\pi f \quad f = 2.5 \times 10^6

\text{emf: } E = - \frac{d\Phi}{dt} = wTT^2 B_0 \sin(\omega t + \phi)

\text{Amplitude of emf} = E_0

E = \int E \, dl

\text{Amplitude of } E = E_0 = \frac{E_0}{2\pi R} = \frac{\omega R B_0}{2}

E_0 = \frac{2\pi - 2.5 \times 10^6 \times 3 \times 10^{-2} \times 4 \times 10^{-4}}{2} \frac{V}{m}

E_0 = 94 \frac{V}{m}

\text{GO TO PAGE 8 FOR A JUSTIFICATION THAT } E \text{ IS PURELY TANGENTIAL}
The field at the edge of the loop is \( B = \frac{\mu_0 I}{2\pi r} \). Only the \( z \)-component of this field contributes to \( \Phi \), \( |B_z| = B \sin \theta \)

But \( \frac{b}{2} = r \sin \theta \Rightarrow |B_z| = \frac{\mu_0 I b}{4\pi r^2} \)

But \( r^2 = h^2 + \frac{b^2}{4} \Rightarrow B_z = \frac{\mu_0 I b}{4\pi \left( h^2 + \frac{b^2}{4} \right)} \)

We want \( \frac{d\Phi}{dt} \). In \( dt \) the loop will have moved an amount \( \mathbf{v} \) \( dt \) to the right. Since \( B_z \) has opposite sign on the two sides of the loop, in magnitude

\[ |d\Phi| = 2 |B_z| \mathbf{v} dt b \]

Area swept out by moving loop
\[
\left| \Phi \right| = 2 \frac{\mu_0 I b}{4\pi} \frac{b \sigma dt}{h^2 + \frac{1}{4} R^2}
\]

\[\Rightarrow \left| \Delta \Phi \right| = \frac{\mu_0 G b^2}{2\pi} \frac{V}{h^2 + \frac{1}{4} R^2} \]

\[\text{(1.26)}\]

\[
\frac{d\Phi}{dt} = V b B = E, \quad I = \frac{E}{R} = \frac{B b V}{R}
\]

Because the bar carries current \( I \), there will be a force on the bar \( F = I B b \)

\[F = \frac{B^2 b^2 V}{R}\]

Now we have to decide whether the force is going to be against the motion or in the direction of motion. Clearly physics tells us that it has to be against the motion, i.e., in the direction of slowing down the rod. Otherwise we have just invented a perpetual motion machine (!!)
You can check the direction using right hand rules, etc.

So we have \( \frac{mdv}{dt} = -\frac{B^2v^2}{R} \)

The solution is \( v(t) = v_0 e^{-t/\tau} \)

with \( \tau = \frac{mR}{B^2v_0} \)

(a) The rod slows down but never stops moving \((v \to 0 \text{ as } t \to \infty)\)

(b) \( v = \frac{dx}{dt} \implies x(t) = C - v_0 \tau e^{-t/\tau} \)

Where \( C \) is a constant. Now choose \( C \) such that \( x(t=0) = x_0 \)

This gives \( x_0 = C - v_0 \tau \) or

\[ C = x_0 + v_0 \tau \]

\[ \implies x(t) = x_0 + v_0 \tau (1 - e^{-t/\tau}) \]

At \( t=\infty \), \( x(t) = x_0 + v_0 \tau \), therefore the rod will travel \( \Delta x = v_0 \tau \)

\[ \Delta x = \frac{v_0 mR}{B^2v_0} \]
(c) The kinetic energy lost by the rod, \( \frac{1}{2} m v_0^2 \) must have been dissipated in the resistor. Let's check that. Power in resistor: \( P = \frac{dE}{dt} = I^2 R \)

\[
\frac{dE}{dt} = \left( \frac{B b v}{R} \right)^2 R = \frac{B^2 b^2 v^2}{R} = \frac{B^2 b^2 v_0^2 e^{-2t/r}}{R}
\]

Total energy dissipated is

\[
E = \int_0^\infty \frac{B^2 b^2 v_0^2}{R} e^{-2t/r} dt = \frac{\pi}{2} \frac{B^2 b^2 v_0^2}{R} = m R \frac{B^2 b^2 v_0^2}{2 B^2 b^2} = \frac{1}{2} m v_0^2 \sqrt{2} \text{ as expected}.
\]

7.27 (a) \( B(t) = \mu_0 n I(t) = \mu_0 n I_0 \cos wt \)

\( \Phi(t) = B(t) \pi r^2 = \mu_0 n I_0 \pi r^2 \cos wt \)

\[
\varepsilon = -\frac{d\Phi}{dt} = \mu_0 n I_0 \omega \pi r^2 \sin wt
\]

\[
I = \frac{\varepsilon}{R} = \frac{\mu_0 n I_0 \omega \pi r^2 \sin wt}{R}
\]
(6) \[ \mathbf{F} = I \text{(loop)} \ d\mathbf{e} \times \mathbf{B} \]

where \(d\mathbf{e}\) is along the ring.

The force is always radial.

Note that \(d\mathbf{e}\) is always \(\perp\) to \(\mathbf{B}\).

So in magnitude

\[ F = I \text{(loop)} \ d\mathbf{e} \cdot \mathbf{B} = F = I \text{(loop)} \ d\mathbf{e} \mu_0 n I_0 \cos \omega t \]

\[ F = \frac{\pi r^2 \mu_0 n^2 I_0^2 \omega d\mathbf{e}}{R} \sin \omega t \cos \omega t \]

But \(\sin \omega t \cos \omega t = \frac{1}{2} \sin 2\omega t\)

\[ F = \frac{\pi r^2 \mu_0 n^2 I_0^2 \omega d\mathbf{e}}{2R} \sin 2\omega t \]

Force is maximum at

\[ 2\omega t = \pm \frac{\pi}{2} + m\pi \]

where \(m\) is an integer.
Note - The force on opposite sides of the ring is in opposite directions, e.g. either

\[ \text{de} \quad \text{F} \quad \text{de} \quad \text{F} \]

or

\[ \text{de} \quad \text{F} \quad \text{de} \quad \text{F} \]

because \( \text{de} \) points in different directions.

(C) The force tends to shrink or stretch the ring (it alternates between the two)
Addendum to 7.22 - (By CC and Dan P.) - We assumed that the field was totally tangential. Now we want to show it. We are going to work in cylindrical coordinates \( \vec{E} = E_r \hat{r} + E_\theta \hat{\theta} + E_z \hat{z} \).

Consider a Gaussian surface which is a cylinder, on axis, length \( l \).

\[
\text{Gauss' law } \oint \vec{E} \cdot d\vec{A} = 0 = E_r(r) 2\pi r \cdot l
\]

\[-\int E_z(z = -\frac{R}{2}) dA + \int E_z(z = \frac{R}{2}) dA = 0 \]

Since the solenoid is infinitely long, \( E_2 \) must be independent of \( z \), therefore the last two integrals cancel. So we are left with \( E_r(r) 2\pi rl = 0 \).

\[\Rightarrow E_r = 0 \quad \text{NO RADIAL COMPONENT} \]
Now let's look at $E_z$ - it cannot depend on $\theta$ by obvious symmetry. It cannot depend on $z$ by the argument of the previous page (infinitely long solenoid). Now consider square path across solenoid.

The flux is 0 through path. All fields must be zero outside solenoid. The radial $E$ is 0 $\Rightarrow \oint E \cdot dl$ only has contribution from the segment inside solenoid parallel to $z$-axis. $\oint E \cdot dl = E_z l = 0$ $\Rightarrow E_z = 0$