\[ \vec{B} = B_y \hat{y} + B_z \hat{z} \]

Line element: \[ d\vec{s} = dx \hat{x} + dy \hat{y} \]

\[ d\vec{F} = I d\vec{s} \times \vec{B} = I \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ dx & dy & 0 \\ 0 & B_y & B_z \end{vmatrix} \]

\[ = I dy B_z \hat{x} - I dx B_z \hat{y} + I dx B_y \hat{z} \]

Torque: \[ d\vec{N} = \vec{r} \times d\vec{F} \]

\[ d\vec{N} = I \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ x & y & 0 \\ dy B_z & -dx B_z & dx B_y \end{vmatrix} \]

Remember: \( z = 0 \) because we are in the \( xy \) plane.

The \( x \)-component is: \[ dN_x = I B_y y \ dx \]

When I integrate \( y \ dx \) over \( x \) for a closed loop, I get the area of the loop: \[ N_x = I B_y A \]

The \( y \)-component is: \[ dN_y = -I B_y x \ dx \]

Integrating \( x \ dx \) over the closed
loop will give 0. Similarly, the $z$-component will have integrals $\int x \, dx$ and $\int y \, dy$ which are zero over the loop. So $\mathbf{N} = I A B_y \mathbf{\hat{x}}$.

Write $\mathbf{N} = -I A \mathbf{\hat{z}}$ as suggested.

Then

$$\begin{vmatrix}
\mathbf{M} \times \mathbf{B} & = & \begin{vmatrix}
x & y & z \\
0 & 0 & -I A \\
B_x & B_y & 0 \\
\end{vmatrix} \\
\end{vmatrix}$$

$$\mathbf{M} \times \mathbf{B} = I A B_y \mathbf{\hat{x}} = \mathbf{N} \mathbf{\circ}$$

(6.37) This is equivalent to superposition of a solid rod with current into the page and a smaller rod with equal and opposite current out of the page. The solid rod contributes no field at its center. So $\mathbf{B} = \frac{M_0 I}{2\pi \gamma}$ where $\gamma = 2 \text{ cm}$. Actually current density.
We now need to figure out \( I \). As mentioned before, the current densities must be the same in the rod and the fictitious rod - so

If \( I_o = 900 \, \text{A} \quad r_o = 4 \, \text{cm} \quad l = 2 \, \text{cm} \)

then \( \frac{I}{r^2} = \frac{I_o}{r_o^2 - r^2} \quad I = I_o \frac{r^2}{r_o^2 - r^2} = 1200 \, \text{A} \)

Then \( B = \frac{\mu_0 I}{2 \pi r} \rightarrow B = 3 \times 10^{-3} \, \text{T} \)

\[ 6.42 \]
\[ \nabla \times \vec{A} = \left| \begin{array}{ccc}
\hat{x} & \hat{y} & \hat{z} \\
\frac{\partial A_y}{\partial z} & \frac{\partial A_z}{\partial x} & \frac{\partial A_x}{\partial y} \\
\frac{\partial A_z}{\partial y} & \frac{\partial A_x}{\partial z} & \frac{\partial A_y}{\partial x}
\end{array} \right| = 0 \quad 0 \quad 0 = B_o \]

Here is an easy one:

\( A_x = 0 \quad A_y = B_o \times A_z = 0 \)
\[ B = \frac{\mu_0 I r}{2 \pi \Gamma_0^2} \]

\[ \mathbf{A}' = A_0 (x^2 + y^2) \hat{z} = A_0 r^2 \hat{z} \]

Work in cylindrical coordinates \((r, \theta, z)\)

\(\mathbf{B}\) is in \(\hat{\theta}\) direction

\[ \mathbf{B} = \frac{\mu_0 I r}{2 \pi \Gamma_0^2} \hat{\theta} \]

Use the equation for the curl of a vector in cylindrical coordinates.

The equation is in the back of the book.

In the case of \(A_\theta = A_r = 0\) we have

\[ \nabla \times \mathbf{A} = \frac{1}{r} \frac{\partial A_z}{\partial \theta} \hat{r} - \frac{\partial A_r}{\partial \theta} \hat{\theta} \]

For \(A_z = A_0 \rho^2 \hat{\rho}\) this reduces to

\[ \nabla \times \mathbf{A} = -2A_0 \rho \hat{\theta} \]

And \(\mathbf{B} = \nabla \times \mathbf{A}\) if \(A_0 = -\frac{\mu_0 I}{4 \pi \Gamma_0^2} \)

You can also do this in cartesian coordinates but it is much more painful!
\[ dL = \frac{rd\theta}{\sin \theta} \]

\[ d\vec{B} = \frac{\mu_0 I}{4\pi r^2} dL \times \hat{r} \quad d\vec{B} \text{ is } I \text{ to paper} \]

In magnitude \[ |d\vec{L} \times \hat{r}| = dL \sin \theta \]

\[ \Rightarrow d\vec{B} = \frac{\mu_0 I}{4\pi r^2} dL \sin \theta = \frac{\mu_0 I}{4\pi r^2} \frac{r \, d\theta}{\sin \theta} \sin \theta \]

\[ dB = \frac{\mu_0 I \, d\theta}{4\pi r} \quad \text{But } \theta = r \sin \theta \]

\[ d\vec{B} = \frac{\mu_0 I \sin \theta \, d\theta}{4\pi b} \]

Now integrate \( A \) from \( 0 \to \pi \)

\[ B = \frac{\mu_0 I}{4\pi b} \int_0^\pi \sin \theta \, d\theta = \frac{\mu_0 I}{4\pi b} \left[ -\cos \theta \right]_0^\pi \]

\[ B = \frac{\mu_0 I}{2\pi b} \]
Explanation for first problem

Consider \( \int x \, dx \) over closed loop \( Y \).

Break it into two pieces:

\[
\int_{x_1}^{x_2} x \, dx + \int_{x_1}^{x_2} x \, dx = 0
\]

Same for \( S \) dy etc.