These slides describe an example Monte Carlo simulation of a physical process

The code is in :

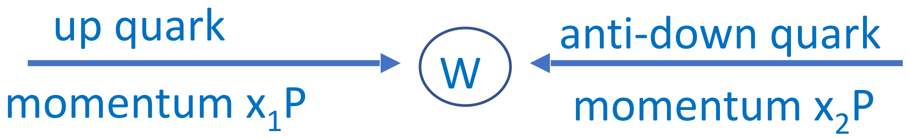
/home/pi/physrpi/campagnari/python/muonSim.py

We will “simulate” the process $pp \to W^+ \to \mu^+\nu$

In particular we are interested in the momentum distribution of the muons.

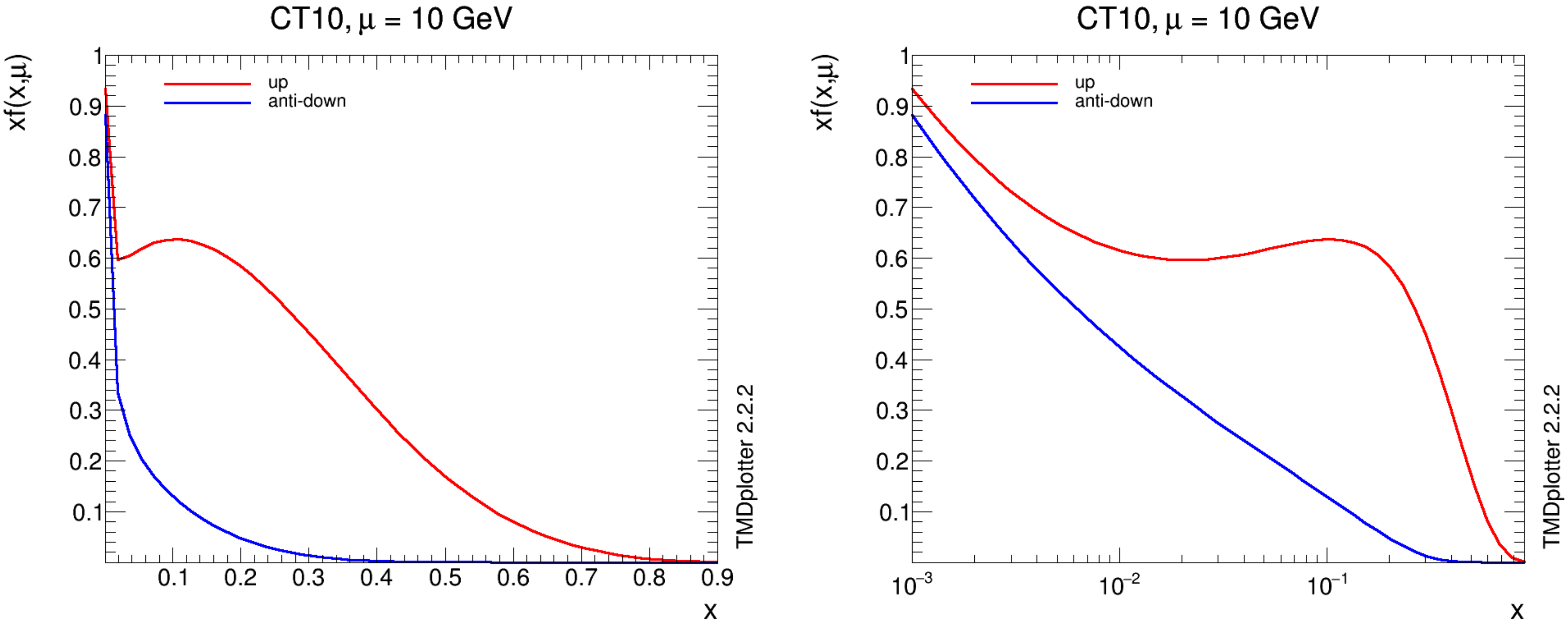
What really happens is $u\bar{d} \to W^+ \to \mu^+\nu$

up-quark from one proton and anti-down quark from the other proton



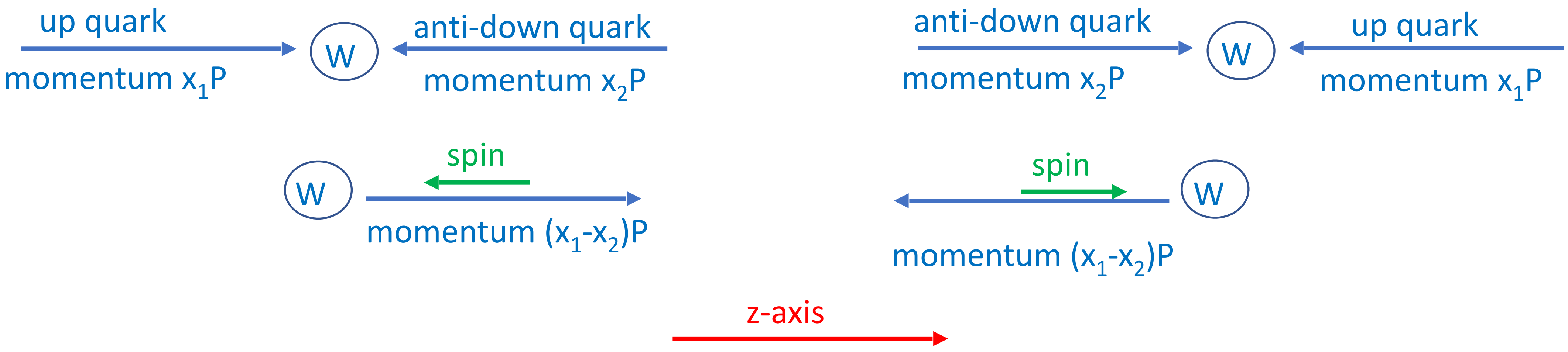
x_1 and x_2 are the fraction of total proton momentum carried by the quarks

The PDFs for x(up-quark) and x(anti-down quark) inside a proton



Note: nobody knows how to calculate these!!!!

They are “measured”



up quark

momentum x_1P

W

anti-down quark

momentum x_2P

anti-down quark

momentum x_2P

W

up quark

momentum x_1P

W

spin

momentum $(x_1-x_2)P$

spin

W

momentum $(x_1-x_2)P$

z-axis

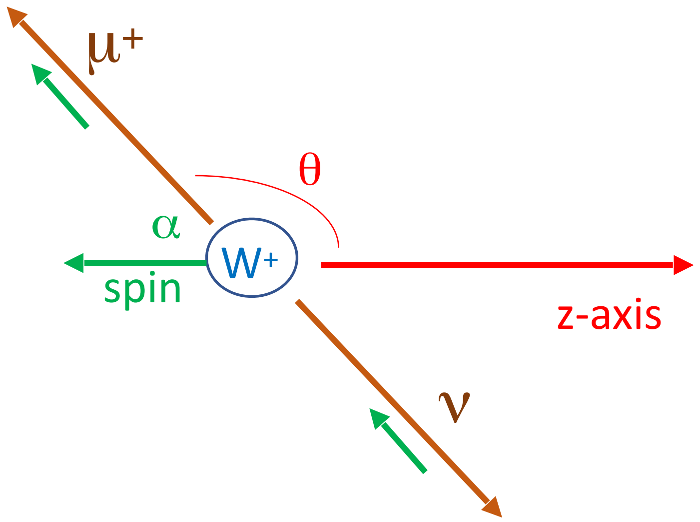
Interesting physics, the W is polarized.

- This is a weak interaction process
- Only left handed quarks and right handed antiquarks contribute
  - Parity violation….the world is left handed!
- As a result the W which is S=1 will have $S_z=\pm1$ and never $S_z=0$

In real life one would have to generate the z component of the W momentum, ie, pick the x’s appropriately. However

1. This is complicated (there are dedicated libraries to do this -- probably not in python)
2. Not important for what we care about (more on this later)

So let’s work in the W rest frame (equivalent to $x_1=x_2$)



The direction of the muon with respect to the spin has PDF:

$P(\cos\alpha)\, d\cos\alpha \sim (1 + \cos\alpha)^2\, d\cos\alpha$

Since the spin can be in the +z or –z direction with equal probability, the pdf for the space angle θ is

$P(\cos\theta)\, d\cos\theta \sim (1 + \cos^2\theta)\, d\cos\theta$

Mass of W:	$M = 80.4$ GeV/c^2
Mass of μ:	$m = 0.105$ GeV/$c^2 \sim 0$
Mass of ν:	$m < 10^{-9}$ GeV/$c^2 \sim 0$

Therefore we should

- **Pick a $\cos\theta$ according to**
- **Give the muon this momentum**

In the massless approximation both muon and neutrino have momentum
$P = \frac{1}{2} Mc = 40.2$ GeV/c

Physics aside: the ang. distribution is from the QM rotation matrices. Has to do with changing the axis of quantization for angular momentum

A complication

Heisenberg uncertainty principle ΔEΔt ~ h/2π

- Energy is mass and mass is energy
- W bosons live for a very short Δt
- Therefore the "mass" of a W boson in a given collision is not always 80.4 GeV
- It is "broadened" according to the relativistic Breit Wigner function

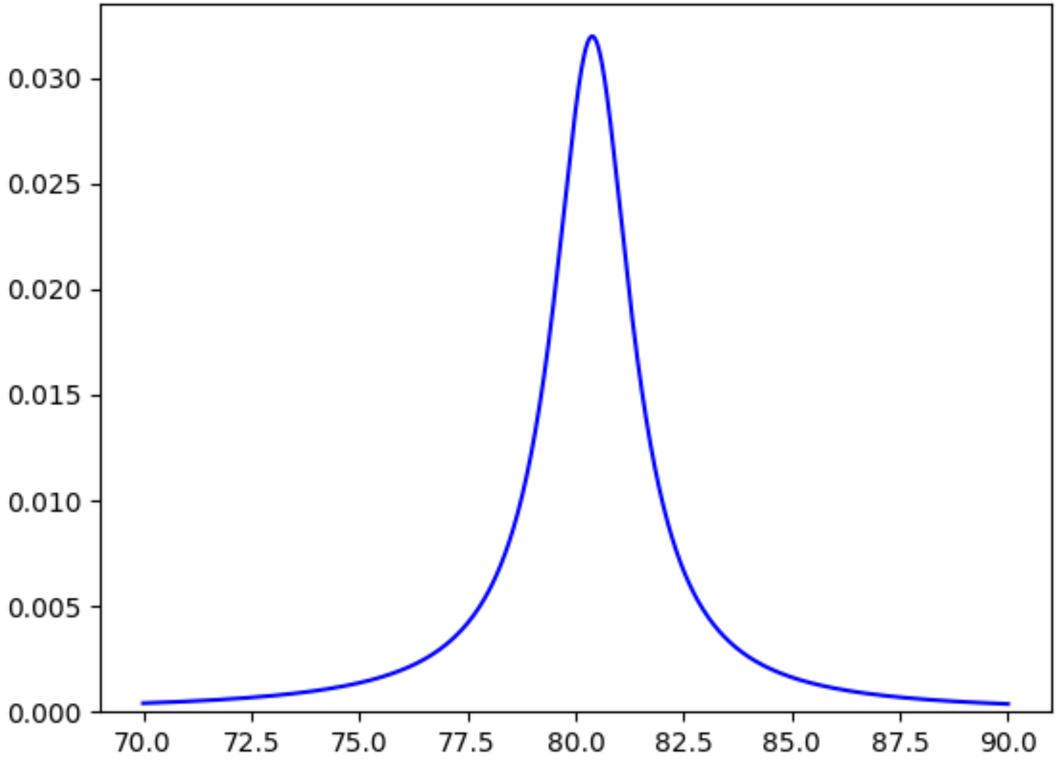
$$f(M) = \frac{C}{(M^2 - M_0^2)^2 + M^2\Gamma^2}$$

Where

M_0 = 80.4 GeV/c²

Γ = 2.2 GeV/c²

C is a normalization constant



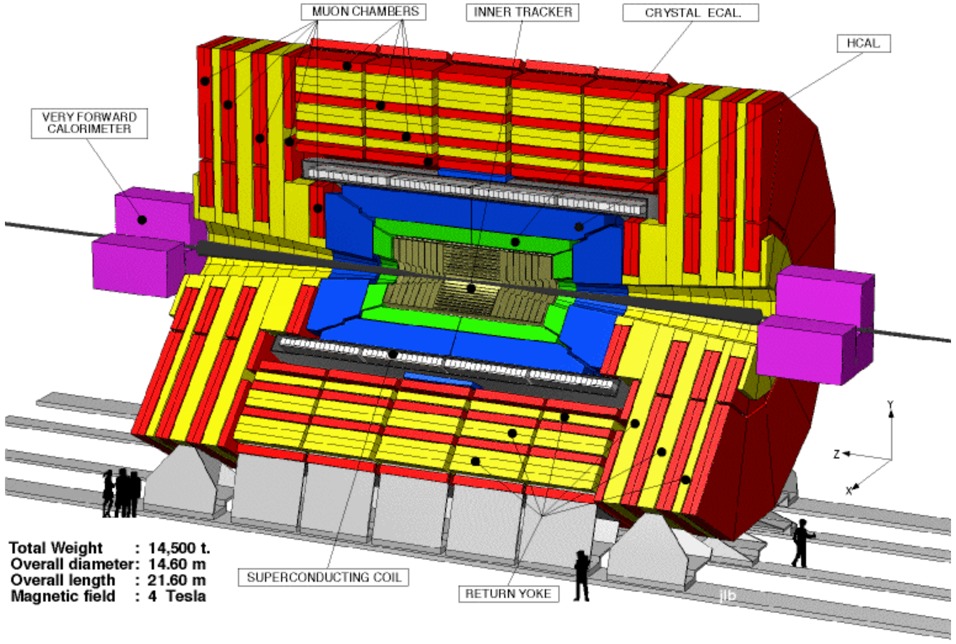
Another ingredient

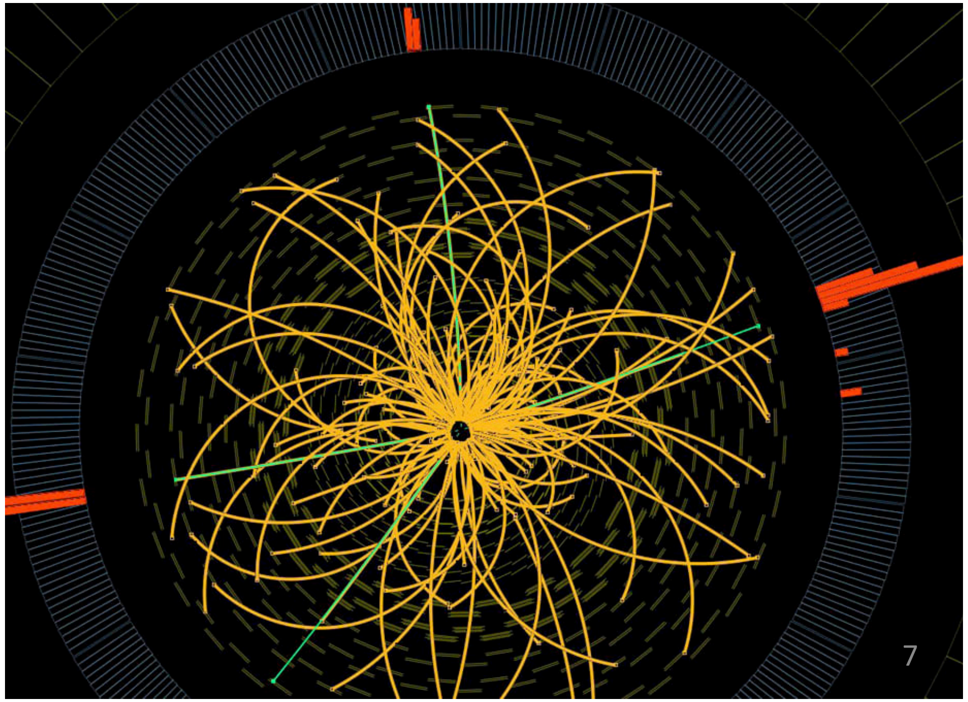
We are “simulating” pp collisions at CERN
The two big experiments Atlas and CMS are embedded in solenoids with constant B-field in the z direction.

Charged particles bend in the magnetic field.
Their trajectories are measured.
Knowing the B-field and the curvature, can measure “transverse momentum”

$$P_T = \sqrt{P_x^2 + P_y^2}$$

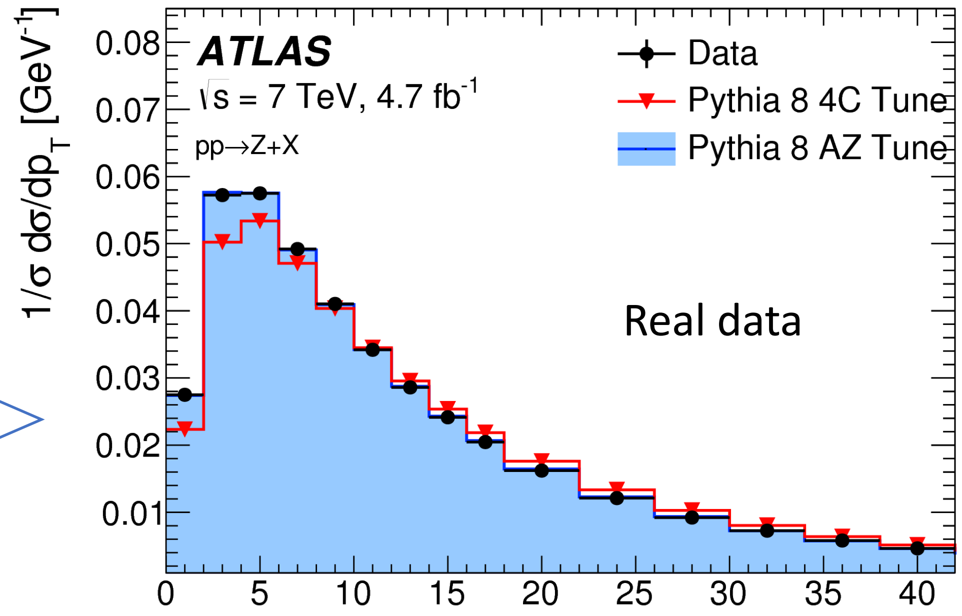
P_T is measured with finite resolution.
About 1.5%
(It is more complicated than that, but OK)



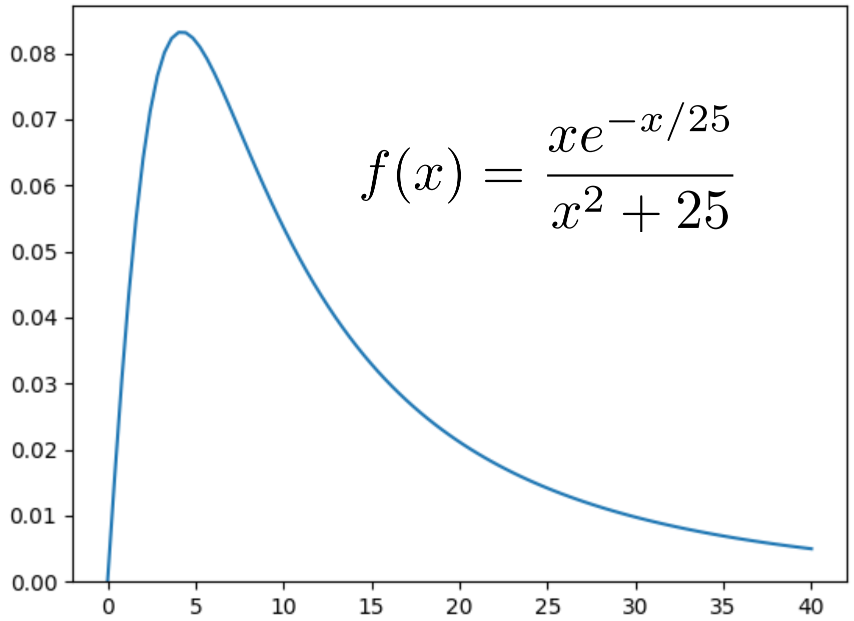


Another complication

The W actually has a some P_T as well



This made-up function kind of look like it. So we will use it



Plan of work to obtain the P_T distribution of muons

1. Pick a W mass M from a Breit Wigner
2. Pick a $\cos\theta$ for the muon in the W rest frame
3. $P = ½\,Mc$ and $P_T = P\sin\theta = ½\,Mc\sin\theta$
4. Pick a transverse momentum for the W from the distribution of the previous page. Call that Q_T. The x-axis will be the direction of Q_T
   - Choosing Q_T along x-axis (or y-axis) is arbitrary. Makes algebra in (6) a tiny bit easier
5. Pick a random azimuthal angle ϕ for the muon in the W rest frame
   - $P_x = P_T \sin\phi$ $P_y = P_T \cos\phi$
6. Boost the muon momentum from the rest frame to the LAB frame
   - The boost is in the x-direction. $\beta\gamma = cQ_T/M$
7. Calculate the P_T of the muon in the LAB after the boost
8. "Smear" it by its resolution
9. Plot it

What about the motion of the W in z?

- We wanted to look at the transverse momentum of the muon
- Can go from a W with P_z=0 to a W with finite P_z by performing a boost of the system in the z-direction.
- But a boost in the z direction leaves all x and y momentum components unchanged
- Therefore: it does not matter that we worked with P_z=0
  - At least for the one simple plot we want to make

CDF experiment, Fermilab, 2007