# Winter Quarter 2019 – UCSB Physics 129L
# Homework 8 – last one for credit
# Due Friday March 22, 5PM

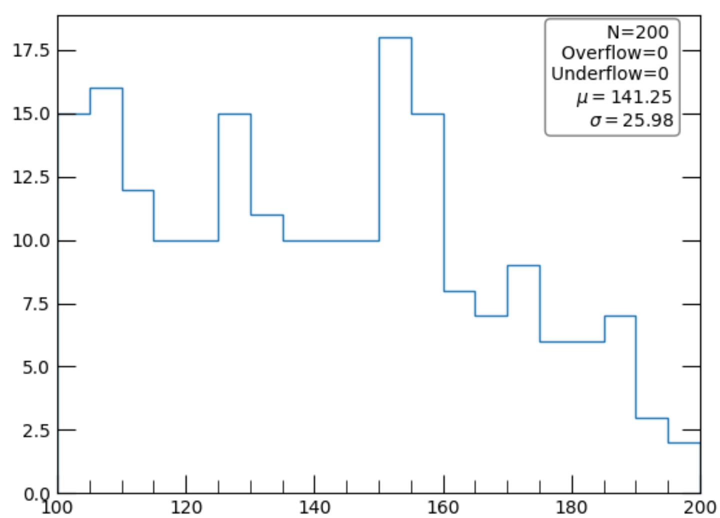
Read carefully the instructions on the website on how to prepare your homework.

**This week you are required to turn in, in addition to the code, also a short report for Exercise 1** (see further instructions below). Include the report in the `tgz` file.

**THE SOLUTIONS TO THIS HOMEWORK SHOULD BE EMAILED TO THE INSTRUCTOR ONLY**

**Exercise 1**

Refer back to Exercise 3 in Homework 5. The mass distribution with some reasonable binning looks like this:




We now want to fit this dataset to the sum of two contributions: a smooth background component and a signal component for particle $A$ at a mass of 155 GeV. As mentioned in Homework 5, the text file that contains these data

is `/home/pi/physrpi/campagnari/python/mass.txt`
Recycle your solution (or use the solution provided for you) to save yourself some efforts in reading these data in. Here are other things to consider.

- We know (well, I am telling you) that the experimental resolution for the mass is 5 GeV. Therefore the pdf for the signal is a Gaussian of mean 155 GeV and sigma 5 GeV.

- The background pdf is unknown, but it must be a smoothly falling distribution. Therefore we will parametrize the background pdf with one (actually, several, see below) "reasonable" functions.

- The number of events in each reasonably-sized bin is too small to perform a $\chi^2$ fit assuming gaussian counting uncertainties (we are in the Poisson regime). Instead, we need a log-likelihood fit to account for Poisson statistics.

- Reasonable bins are comparable in size to the mass resolution. Squeezing a Gaussian into 2-3 bins loses a lot of information. Therefore we perform an **unbinned** log-likelihood fit.

We discussed extended negative log-likelihood (NLL) fits (binned and unbinned) in class.
General notes:
http://hep.ucsb.edu/people/claudio/ph129-w19/MaximumLikelihood.pdf
Slides:
http://hep.ucsb.edu/people/claudio/ph129-w19/ExtendedNLLExample.pdf
Use iminuit (the python port of minuit) to perform unbinned NLL fits for this dataset to a sum of signal ($S$) and background ($B$). As mentioned above, we parametrize the background pdf with a "reasonable" function. Here are some hints:

- An example minuit fit for a binned $\chi^2$ fit was shown in class and can be found as `/home/pi/physrpi/campagnari/python/testFit_v3.py`

- In the slides linked above there is a detailed discussion of how to setup a binned NLL fit with minuit. There is also an explanation of how to modify it for an unbinned fit. The corresponding code can be found as `/home/pi/physrpi/campagnari/python/maxLikFit.py`

- The background pdf has some parameters, e.g., if it was an exponential it would go like $e^{-\alpha m}$, where $m$ is the mass and $\alpha$ is a parameter. It is up to you to choose a reasonable functional form for the pdf, but the parameters of the function should not be fixed. Instead, you will be fitting for them as well. Thus you will be fitting simultaneously for the following quantities:

  - $S$, the number of signal events;
  - $B$, the number of background events;
  - all the parameters of the background pdf, e.g., in the exponential case, the single parameter $\alpha$.

  Since the number of events in the dataset is limited, you should keep the background pdf simple, e.g., 2–4 parameters maximum.

- Remember that pdfs have to be normalized. In our case the background pdf must integrate to one between $m_1 = 100$ and $m_2 = 200$ GeV, since all the data to be consideed has $m$ in this interval. This means that you need to explicitly write the normalization of the pdf in terms of its parameters. For example, in the case mentioned above the pdf would be $f(m)dm = \alpha e^{-\alpha m} dm/(e^{-\alpha m_1} - e^{-\alpha m_2})$.

- For a log-likelihood fit the error analysis in minuit requires setting `errordef=0.5` at the very beginning, as explained in the slides.

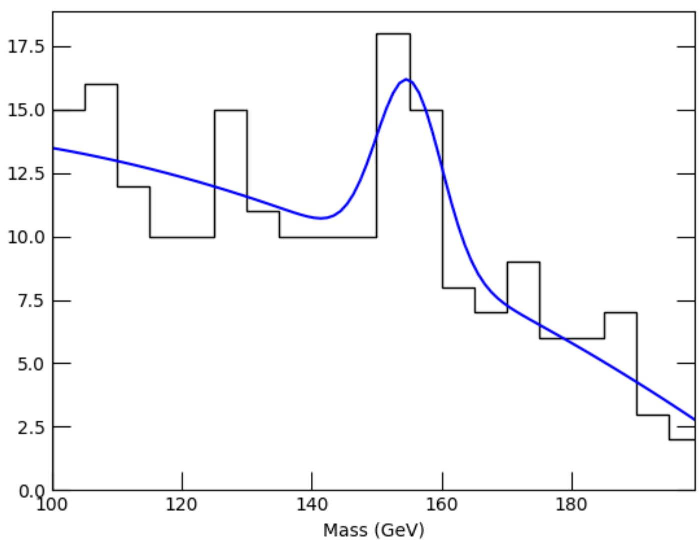
Repeat this procedure for a few (3?) reasonable choice of background pdfs with different functional forms. Write a short (2 pages?) report where you describe what you did:

- What background pdfs did you try?

- What result did you get for $S$ (fitted value and uncertainty) for each background pdf?

- For each fit, show the results of the sum of signal + background pdfs on top of the mass histogram. Look at the figure below to see what I mean. This was done for one particular example of background pdf. Note: even if the fit is unbinned, you can still bin the data after the fit and superimpose the results of the fit to the binned data.

- Use your judgement to decide whether any given fit is reasonable.
- Quote a final answer as $S$ = XX ± YY ± ZZ, where XX ± YY is the result of one of the fits which you think is most representative and ZZ is your estimate of the systematic uncertainty. Base your estimate for ZZ on how you found the results to vary as you changed the background pdf. Do not stress too much about getting ZZ "right"... it is a judgment call.




**Exercise 2**

In this exercise you should make use of the `LVector` class from Homework 6, Exercise 1. You can use the solution that was provided. This solution also includes member functions to rotate vectors in 3-dimensional space. These may be of use to you. **Work in "Natural Units" where $c$=1.**[1]. Then all masses, momenta, and energies are measured in GeV.
Consider the following decay chain

[1] https://telescoper.wordpress.com/2010/03/05/the-joy-of-natural-units/

$$B^+ \to D^{*0}\pi^+$$
$$D^{*0} \to D^0\pi^0$$
$$D^0 \to K^-\pi^+ \qquad \pi^0 \to \gamma\gamma$$

With the exception of the photons ($\gamma$'s), all the particles in this decay chain are *mesons*, i.e., bound states of quark-antiquark pairs, sort of like the hydrogen atom is a bound state of a proton and an electron. The masses and spins are listed in the Table below. The Particle Data Group (PDG) maintains a listing of the properties of these mesons (and much more) at http://pdg.lbl.gov/ if you want to learn more.

| Particle | Spin | Mass |
|---|---|---|
| $B^+$ | 0 | 5.28 GeV/c$^2$ |
| $D^{*0}$ | 1 | 2.01 GeV/c$^2$ |
| $D^0$ | 0 | 1.86 GeV/c$^2$ |
| $K^-$ | 0 | 0.494 GeV/c$^2$ |
| $\pi^+$ | 0 | 0.1396 GeV/c$^2$ |
| $\pi^0$ | 0 | 0.1350 GeV/c$^2$ |
| $\gamma$ | 1 | 0 GeV/c$^2$ |

The exercise consists of generating (via Monte Carlo) 1,000 such decay chains, starting from a $B^+$ at rest. The rest frame of the $B^+$ is the LAB frame. Some things to keep in mind:

- You are starting from a $B^+$ decay at rest. The $B^+$ is spinless, so there is no preferred direction in space.
- In spherical coordinates $(r, \theta, \phi)$[2] a "random" direction means $\phi$ random (equally likely) between 0 and $2\pi$ and $\cos\theta$ random between -1 and 1. If you are not convinced about this, remember that the solid angle in spherical coordinates is $d\Omega = d\phi \, d\cos\theta$.
- For a two-body decay $A \to BC$ in the rest frame of $A$ the 3-momentum of $B$ or $C$ is given in Section 48.4.2 of http://pdg.lbl.gov/2017/reviews/rpp2017-rev-kinematics.pdf. (You **really** should be able to calculate this for yourself!).

[2] I define spherical coordinates such that $\theta$ is the angle between $\vec{r}$ and the $z$-axis, and $\phi$ is the angle between the projection of $\vec{r}$ onto the $(x, y)$ plane and the $x$-axis. This is the usual definition in Physics. I believe in some of the math classes that you have taken at UCSB the definitions of $\theta$ and $\phi$ are switched.

- For the decay $A \to BC$ the "decay angle" $\theta_D$ is defined in the rest frame of $A$ as the angle between the direction of $B$ and the direction of motion of $A$ in the LAB frame[3]. If $A$ is spinless (S=0) the $\cos\theta_D$ distribution is uniform between -1 and 1. Almost all of the decaying particles in this decay chain are spinless.

- The $D^{*0}$ decay is more complicated (S=1). Conservation of angular momentum in the $B^+$ rest frame implies that in its rest frame the $D^{*0}$ has $S_3 = 0$ where the axis of quantization is defined as the direction of the $D^{*0}$ in the $B^+$ rest frame. Then, the pdf for $\cos\theta_D$ in $D^{*0} \to K^-\pi^+$ is proportional to the Wigner d-matrix $|d^1_{00}|^2 = \cos^2\theta_D$ [4][5].

- To generate $A \to BC$ starting in the frame where $A$ is moving, you should consider rotating the $xyz$ axes so that the $z$-axis points along the direction of motion of $A$. Then, in the rest frame of $A$ you have $\theta_D$ $= \theta$, where $\theta$ is the angle in spherical coordinates defined above. After generating the decay in the rest frame you will need to rotate the axes back and boost into the LAB frame appropriately.
  Think carefully how you should rotate the $xyz$ axes such that after rotation the $z$-axis points in the direction of a vector $\vec{v}$ whose direction was defined by spherical coordinates $\alpha$ and $\theta$ before the rotation of the axes. By what angle should you be rotating the axes, and about which axis should the rotation be performed? The member functions for rotations in `LVector.py` should help you. Play around with them separately to test your understanding.

At the end of the generation you should have a list with 1000 entries, and each entry will be itself a list with five `LVectors` for the five final state particles $(\pi^+, K^-, \pi^+, \gamma, \gamma)$. The vectors should all be in the LAB frame. You will then save these events and write them out in a "pickle" file as follows (read about pickle at https://docs.python.org/3/library/pickle.html):

```
import LVector as lv
import pickle
```

[3] The decay angle is also called "helicity angle". If the explanation is not clear to you, try this: https://www.quantumdiaries.org/tag/helicity/

[4] http://charm.physics.ucsb.edu/people/richman/ExperimentersGuideToTheHelicityFormalism.pdf, equation B.1.4.

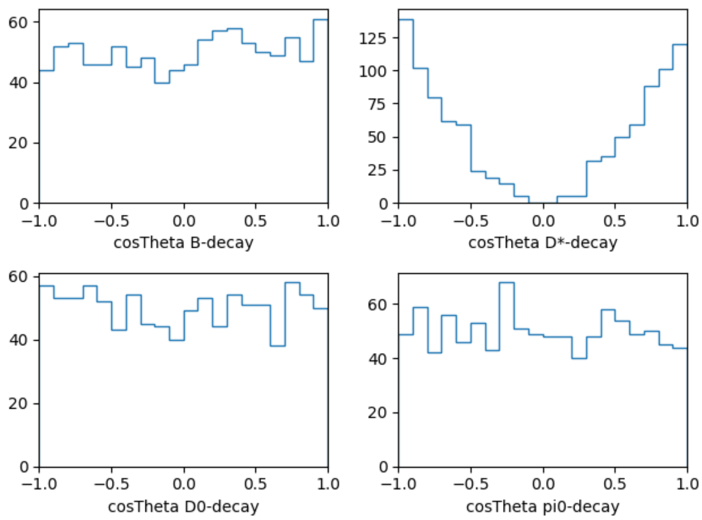
[5] http://home.fnal.gov/ kutschke/Angdist/angdist.ps, Section 14.

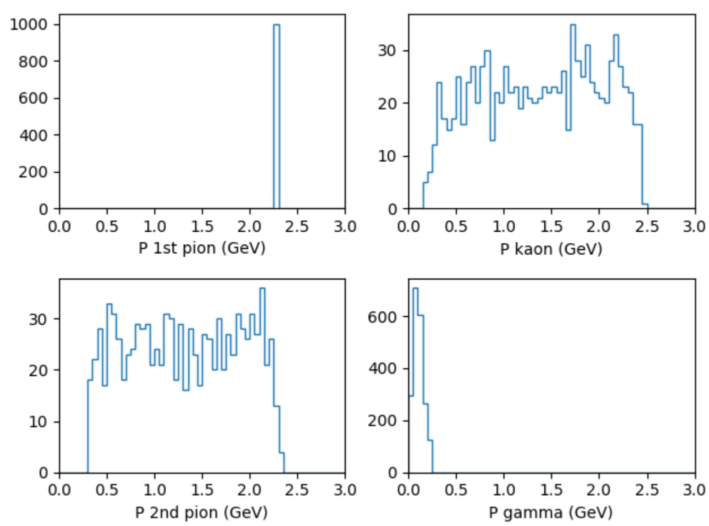
```
out = []
for i in range(1000):

    # here goes the code to generate the decay chain

    # Ppi1 = LVector of pi+ from B+ decay
    # Ppi2 = LVector of pi+ from D0 decay
    # PK   = LVector for K-
    # Pg1  = LVector for one of the gammas from pi0 decay
    # Pg2  = LVector for the other gamma from pi0 decay
    # (the order of the gammas should be random)
    # put final state particles in list
    out.append([Ppi1, PK, Ppi2, Pg1, Pg2])

#-----------------------------------
# Write the list to a pickle file
#-----------------------------------
with open ('data.pik', 'wb') as f:
    pickle.dump(out, f)
```

Test that your generation makes sense by running the program
`/home/pi/physrpi/campagnari/python/checkDecayChain.py`
This program does the following:

- Reads the `data.pik` file.
- Checks for energy momentum conservation throughout the decay chain.
- Checks that in all decays $A \to BC$, the $B$ and $C$ particles are back-to-back in the rest frame of $A$.
- Plots the decay angles for all decays.
- Plots the momenta of the final state particles.

If everything is well, you should see no error messages and the plots will look like what is shown below (not **exactly** because of different random seeds). The TAs will run this program to see whether you have generated things correctly.


cosTheta B-decay
cosTheta D*-decay
cosTheta D0-decay
cosTheta pi0-decay


P 1st pion (GeV)
P kaon (GeV)
P 2nd pion (GeV)
P gamma (GeV)