

Winter Quarter 2019 – UCSB Physics 129L

Homework 6

Due Friday, March 1, 5 pm

Read carefully the instructions on the website on how to prepare your homework for turning it in to the TAs.

If your last name starts with A through N, send the homework to Jenny. Otherwise send it to Francesco.

The emails of the TAs are on the website.

Put the instructor in cc to the email.

Make sure to have your rpi updated to pick up the latest/greatest example programs from class.

Exercise 1

In lecture we went over the creation of a class to represent a triangle object. The associated code is

```
/home/pi/physrpi/campagnari/python/myTriangle.py
```

Write the necessary code to represent a 4-vector (or Lorentz vector) class `LVector.py`. I have “specified” what methods are required (**including their names**) in the file

```
/home/pi/physrpi/campagnari/python/LVector_specification.py
```

Please use the same naming convention. This class would be a fairly complete 4-vector class. It would be missing rotations in real 3D space, boosts through hyperbolic functions, and Lorentz transformations implemented by specifying the Λ^ν_μ tensor. Add those methods if you are so inclined! (space rotations around an arbitrary axis in 3D are a bit tricky).

Exercise 2

In Homework 4 Exercise 5 you calculated an integral that (almost) gives you the Bayesian posterior pdf with a uniform prior for a signal ($S = x$) in a counting experiment with N counts in the presence of a background ($B = y$) estimated to be (on average) $B = \mu$ with a Gaussian uncertainty σ . The “almost” caveat is related to the fact that the pdf needs to be limited to $S > 0$, and then of course needs to be normalized properly.

Take your solution from Homework 4 (or alternatively the solution provided on the class webpage), modify it to accept keyboard inputs for N , μ , and σ , and use the posterior pdf to calculate the 95% Bayesian Confidence Level (CL) lower limit on S . That is to say: what is the value of S for which you can say that all true values of S can be excluded with at least 95% confidence? Make a note of your results for $N = 5$ and $N = 1$ when $\mu = 3$ and $\sigma = 0.5$. We will compare them with the results from the following exercise.

Exercise 3

An alternative way of establishing the 95% CL limit for the situation of Exercise 2 is through a frequentist analysis. You could do the following:

- Scan over reasonable values of S .
- For each value of S , pick a bunch of number of counts n from a Poissonian of mean $S + B$, where B is picked from a truncated gaussian of mean μ and standard deviation σ . Make sure that B is not the same from one pick of n to the next.
- For each value of S keep track of the fraction of picks that result in $n < N$.
- Values of S for which these fraction are < 0.05 are excluded at 95% CL.
- The 95% CL limit that you would quote is the smallest excluded value of S .

You have already written very similar code (Exercise 2, Homework 4). You can recycle it, or even start from the solutions that are provided on the web. However, make sure to add the ability to input N , μ , and σ from the command line, just as you did in the previous exercise.

Try it out for $N = 5$ and $N = 1$ when $\mu = 3$ and $\sigma = 0.5$ and compare with the results from Exercise 2. Do not be surprised if the answers are not the same.

Take home message: do not overinterpret limits. What you get in detail depends on what method you use. People have been accused of picking a method to make their results look more sensitive. (Makes for interesting foodfights at conferences).

Also, while the method sketched here is frequentist, it is not **the** frequentist method. It has a problem (feature?) that when N fluctuates low, with less than 5% probability even for background-only (i.e. with $S = 0$), the limit on S does not exist (or it is zero). A possible way (but not **the only** way) around this, as well as other issues, is discussed in <https://arxiv.org/abs/physics/9711021> (this paper has almost 3000 citations to date!).

Finally, note that while frequentists do not like to talk about the “probability of the true value”, this method uses the pdf for B which is fundamentally a Bayesian concept (!).

Exercise 4

In Homework 3 Exercise 4 you drew a color map of the square of the wavefunction for a particle in a 2D box for a particular eignestate. Modify your code for that exercise (or the solution that is provided on the web) to instead show the square of the wavefunction as a contour plot. Don't worry too much about the normalization, you can take the maximum of $|\Psi|^2$ to be equal to one. There are many different ways to show contours, a couple of examples are shown below. In any case, figure out on the web how to do contours, and then decide what to do to make it look *good* (unleash you inner artist).

