MARKOV CHAINS (MC)

A sequence $\vec{x}_1 \ldots\ldots \vec{x}_n \ldots\ldots$
where prob of $\vec{x}_{n+1}$ only depends on $\vec{x}_n$
A random walk is an example

MARKOV CHAIN MONTE CARLO (MCMC)

We saw the utility of generating $\vec{x}$ according to pdf $f(\vec{x})$
We can use MC to do this.
Will not show how this works mathematically
Will instead give "recipe" - 1D for simplicity -

1. Pick arbitrary x_1
2. Decide on x_2 based on a <u>proposal</u>

Proposal could be : x_2 random btw $x_1 + \Delta x$, $x_1 - \Delta x$

This is a symmetric proposal

$$\text{prob}\,(x_1 \xrightarrow{\text{propose}} x_2) = p\,(x_2 \xrightarrow{\text{propose}} x_1)$$

(Metropolis condition)

The "proposed" value is x_2^{prop}

(3) I have a "proposal". Should I accept it?

Calculate $a = f(x_2^{prop}) / f(x_1)$

$a = \min(1, a) \leftarrow a \in [0\ 1]$

Accept the proposal with probability a

$\Rightarrow$ if accepted $x_2 = x_2^{proposal}$

if not $x_2 = x_1$

(4) Repeat

After many trials the sequence is a good sampling of $f(x)$

Arbitrariness about starting point - Throw away the first few members of the chain ("burn in")

Some things to notice

- Normalization of $f(x)$ not needed, since we are only taking ratios
- The initial value must be such that $f(x_1) > 0$
- The "proposal" function can be any function as long as it is non-zero in the domain of validity of $f(x)$