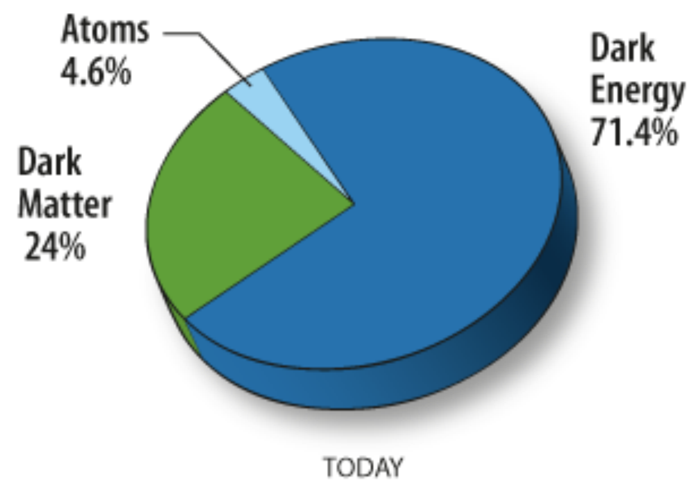
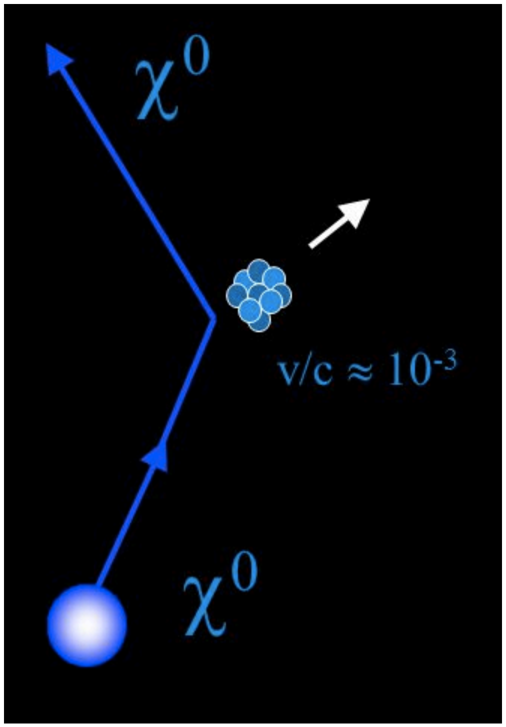
About limits on counting. An example

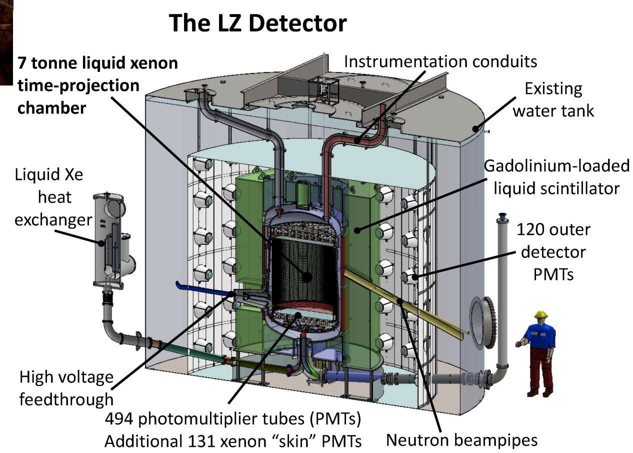
- Want to find dark matter (DM)
  - DM: something that does nor interact with photons
- Astrophysics: 25% of universe is DM



I will look for a DM particle (χ^0) to "hit" a nucleus and make it recoil. I invent a clever way to "detect" the recoil

Collisions with neutrons from cosmic rays/radioactivity look like DM. DM collisions are rare, I go underground to do the expt.





- I do the experiment and count how many nuclear recoils I see (N)
- Despite my best efforts, some neutron recoils sneak into the sample.
- I work very hard to
  1. Reduce them as much as possible
  2. Estimate how many I should see on average (B)
- What I have seen is in principle the sum of neutron background and DM signal (S) → N=S+B

- What do I expect S to be? I don't really know
- Astro observation tell me what the DM density ρ (mass/unit volume) should be
- The number of DM particles crossing my detector will go like ~ ρ/M
- I do not know the strength of the interaction between DM and a nucleus. I can quantify it by an (unknown) interaction cross section σ
- My detector is not perfect. There is an efficiency to actually detect a DM-nucleus collision, which in general will depend on mass $\varepsilon(M)$
  - Good news: since I built the detector I (should) know $\varepsilon(M)$

Bottom line: $S \propto \frac{\rho \cdot \sigma \cdot \epsilon(M)}{M}$

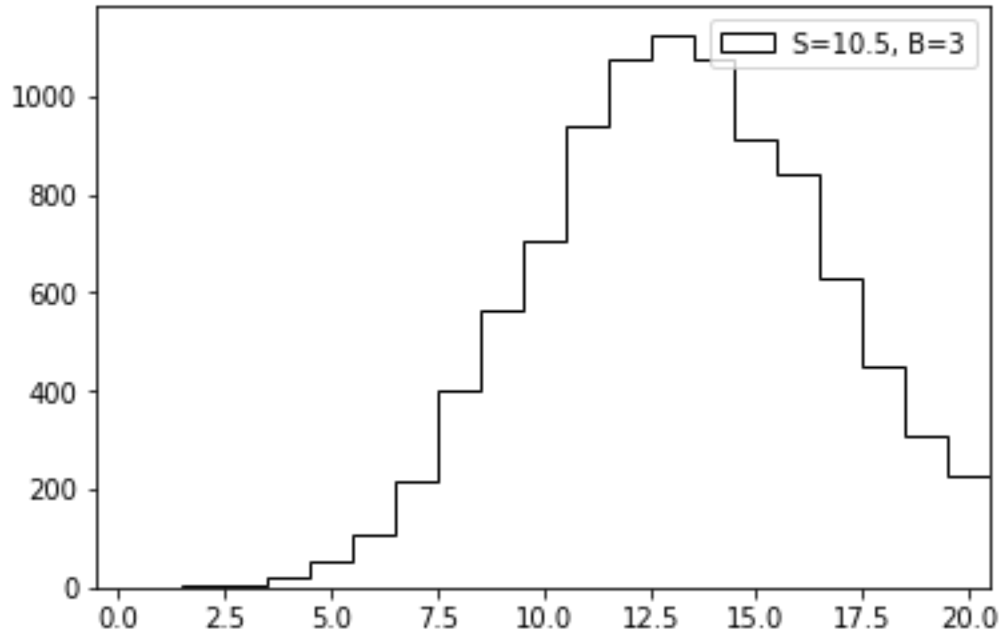
This is the average expectation.
Based on two unknown parameters: σ and M.

In my one and only experiment I have seen N and N=S+B
Both S and B are subject to fluctuations.
If N>>B, I have seen DM, I book a trip to Stockholm.
If not, S is too small for me to discover DM, but I still want statement about DM, in particular about σ and M.

What is the largest possible value of S compatible with N and B?

N=5 B=3. For given S, prob. of seeing N is Poisson of mean S+3
What is the prob. of seeing ≤ 5 as a function of S?

S	p	Excluded with 95% confidence?
10.5	0.8%	YES



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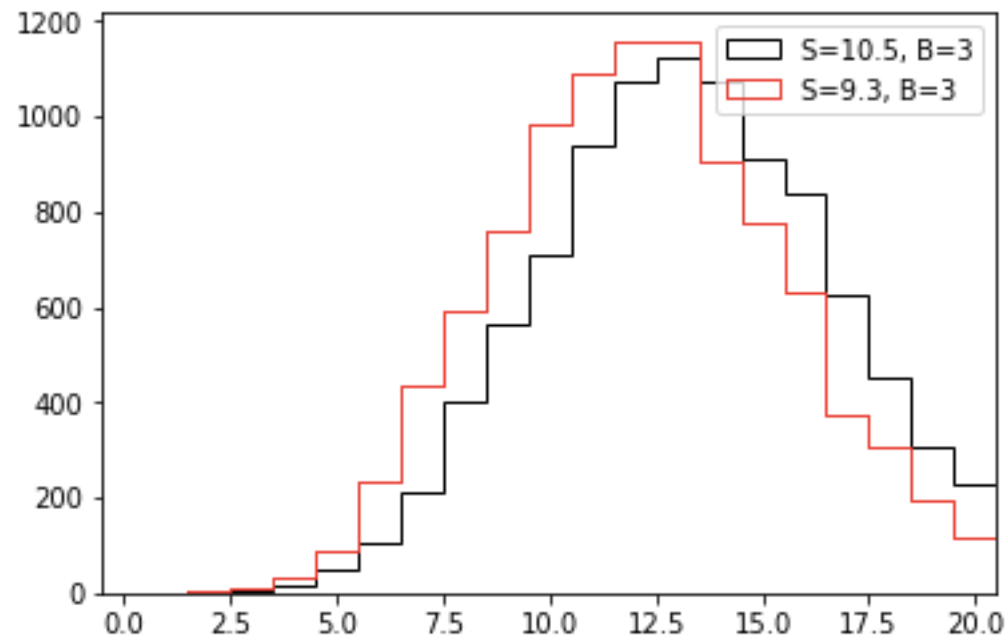
9.3	1.4%	YES



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S	p	Excluded with 95% confidence?
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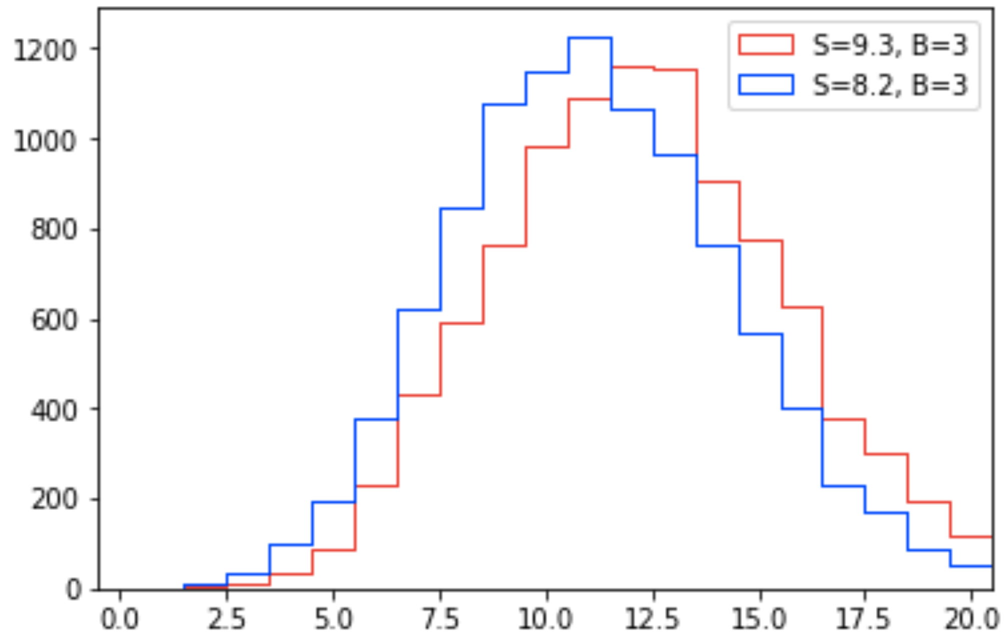
9.3	1.4%	YES
8.2	3.4%	YES



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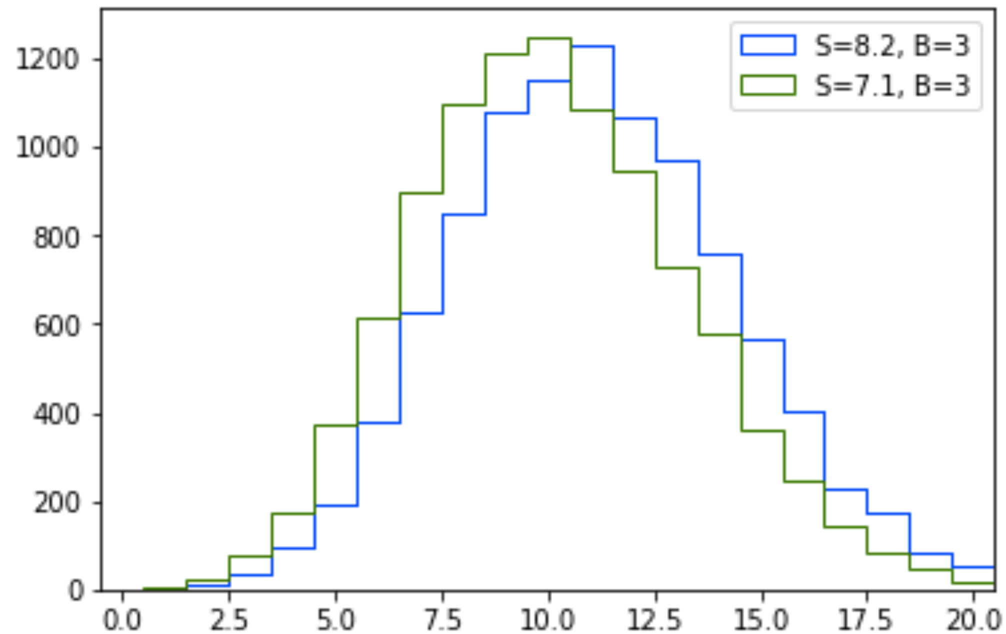
8.2	3.4%	YES
7.1	6.5%	NO



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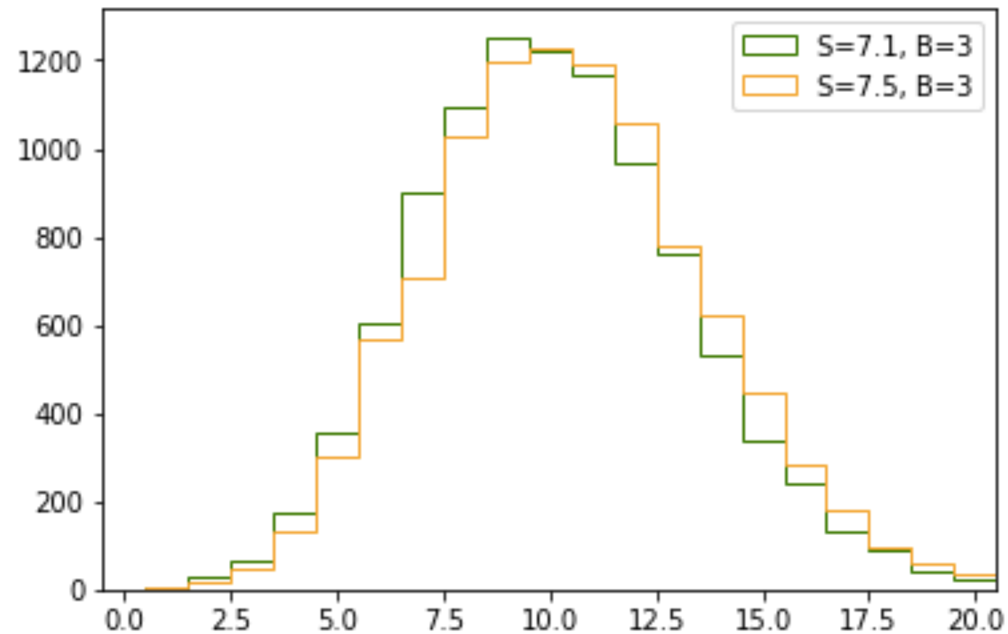
8.2	3.4%	YES
7.1	6.5%	NO
7.5	5.0%	YES (just)



Frequentist

The average value of S must be ≥ 7.5 with 95% confidence

$$S \propto \frac{\rho \cdot \sigma \cdot \epsilon(M)}{M}$$

Recast the exclusion on S in term of exclusion in a plot σ vs. M