Matrix representation of operators

Complete set of orthonormal basis states $|u_i\rangle$

Usually eigenstates of hermitian operator

$$\langle u_i | u_j \rangle = \delta_{ij}$$

Any state $|\psi\rangle$ can be written as $|\psi\rangle = \psi_i |u_i\rangle$ where the sum over repeated indeces is implied and ψ_i's are just numbers. Let O be some operator

$$O|\psi\rangle = \psi_j \, O|u_j\rangle \qquad (1)$$

But $O|\psi\rangle = \phi_i \, |u_i\rangle$ (2)

Acting with $\langle u_i$ on $O|\psi\rangle$ we find

$$\phi_i = \langle u_i | O | \psi\rangle = \psi_j \langle u_i | O | u_j\rangle = O_{ij}\psi_j$$

with $O_{ij} = \langle u_i | O | u_j \rangle$

This is a matrix equation

$$\begin{pmatrix} \phi_1 \\ \phi_2 \\ \vdots \end{pmatrix} = \begin{pmatrix} O_{11} & O_{12} & \cdots \\ O_{21} & O_{22} & \cdots \\ \cdots & \cdots & \cdots \end{pmatrix} \begin{pmatrix} \psi_1 \\ \psi_2 \\ \vdots \end{pmatrix}$$

Therefore the operator O can be <u>represented</u> by a matrix $O_{ij} = \langle u_i | O | u_j \rangle$