Compact Notation

The Levi-Civita symbol in three dimension, ϵ_{ijk}, where i, j, and k are integers between 1 and 3, is defined as:

- $\epsilon_{ijk} = 0$ if any of i, j, and k are the same. For example $\epsilon_{122} = 0$.
- $\epsilon_{ijk} = +1$ if ijk is an **even** permutation of 123. So $\epsilon_{123} = \epsilon_{312} = \epsilon_{231} = +1$.
- $\epsilon_{ijk} = -1$ if ijk is an **odd** permutation of 123. So $\epsilon_{132} = \epsilon_{321} = \epsilon_{132} = -1$.

Another convention that leads to a more compact notation is that **sum over repeated indeces is implied**.
For example, imagine that we have vectors $\vec{a}$ and $\vec{b}$ in 3D with components $(a_x, a_y, a_z) = (a_1, a_2, a_3)$ and $(b_x, b_y, b_z) = (b_1, b_2, b_3)$. Then we could write
$|\vec{a}|^2 = \sum_i a_i a_i = a_i a_i$
$\vec{a} \cdot \vec{b} = \sum_i a_i b_i = a_i b_i$
$(\vec{a} \times \vec{b})_k = \sum_{i,j} \epsilon_{ijk} a_i b_j = \epsilon_{ijk} a_i b_j$.

The Levi-Civita symbol allows you to write cross-products in a very compact way. If you do not believe me, write out explicitely the components of $\vec{a} \times \vec{b}$ and check for yourself.
Furthermore, the commutation relations of the angular momentum operator can be written in compact form as
$[L_i, L_j] = i\epsilon_{ijk}\hbar L_k$
As you can see, the ϵ_{ijk} factor insures that the commutator of any component of angular momentum with itself ($i = j$) is zero, as it should; further, $[L_i, L_j] = -[L_j, L_i]$ since $\epsilon_{ijk} = -\epsilon_{jik}$; finally, the only non-zero term in the implied sum over k on the right side is the one with $k \neq i$ and $k \neq j$, meaning that the commutator of two different components of $\vec{L}$ gives $\pm\hbar$ times the third component, with the sign determined by the order of the two elements of the commutator.