PHYSICS 115 B
FINAL EXAM

① (a) $\boxed{\text{True}}$ becouse spherically symmetric

(b) $\boxed{\text{False}}$ becouse $m_p \neq m_n$

(c) $E = R\left(\frac{1}{1^2} - \frac{1}{2^2}\right)$ $\boxed{R = \frac{3}{4}R = \frac{3}{4}\, 13.6\,eV = 10.2\,eV}$

(d) $\boxed{\text{False}}$

(e) $\boxed{\text{False}}$ becouse $[\pi\ H] \neq 0$

(g) $\boxed{\text{Operator}}$

(h) $\boxed{\text{Complex numb.}}$

② (a) $\psi = \frac{1}{\sqrt{a^2+b^2}}\begin{pmatrix} a \\ b \end{pmatrix}$ $S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ $a^* = a$, $b^* = b$

$$\langle S_x \rangle = \frac{\hbar}{2(a^2+b^2)}(a^*\ b^*)\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \end{pmatrix} = \frac{\hbar}{2(a^2+b^2)}(a\,b)\begin{pmatrix} b \\ a \end{pmatrix}$$

$$\boxed{\langle S_x \rangle = \frac{ab}{a^2+b^2}\hbar}$$

(b) Let p = prob of measuring $+\frac{\hbar}{2}$

Then, $1-p$ = probability of measuring $-\hbar/2$

$$\langle S_x \rangle = p\frac{\hbar}{2} + (1-p)\left(-\frac{\hbar}{2}\right) = p\hbar - \frac{\hbar}{2}$$

$$\frac{ab}{a^2+b^2}\hbar = p\hbar - \frac{\hbar}{2}$$

$$p = \frac{ab}{a^2+b^2} + \frac{1}{2} = \frac{a^2+b^2+2ab}{2(a^2+b^2)}$$

$$\boxed{p = \frac{1}{2}\frac{(a+b)^2}{a^2+b^2}}$$

3 (a)

$$\boxed{\begin{aligned} A &= L_x L_y + L_y L_x \\ A^\dagger &= L_y^\dagger L_x^\dagger + L_x^\dagger L_y^\dagger = L_y L_x + L_x L_y = A \end{aligned}}$$

since $L_i^\dagger = L_i$

(b)

$$\langle A \rangle = \langle l m | L_x L_y + L_y L_x | l m \rangle$$

$$L_x = \frac{1}{2}(L_+ + L_-) \qquad L_y = \frac{1}{2i}(L_+ - L_-)$$

$$L_x L_y = \frac{1}{4i}\left(L_+^2 - L_-^2 - L_+ L_- + L_- L_+\right)$$

$$L_y L_x = \frac{1}{4i}\left(L_+^2 - L_-^2 + L_+ L_- - L_- L_+\right)$$

$$A = \frac{1}{4i}\left(L_+^2 - L_-^2\right)$$

$$\boxed{\langle A \rangle = 0}$$

Because eg $L_-^2 |l m\rangle = C |l\, m-2\rangle$

and $\langle l m | l\, m-2 \rangle = 0$

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(a)

$$\boxed{L_z = \hbar \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}}$$

(b)

$$L_+ = \begin{pmatrix} \langle 1 1 | L_+ | 1 1 \rangle & \langle 1 1 | L_+ | 1 0 \rangle & \langle 1 1 | L_+ | 1 -1 \rangle \\ \langle 1 0 | L_+ | 1 1 \rangle & \langle 1 0 | L_+ | 1 0 \rangle & \langle 1 0 | L_+ | 1 -1 \rangle \\ \langle 1 -1 | L_+ | 1 1 \rangle & \langle 1 -1 | L_+ | 1 0 \rangle & \langle 1 -1 | L_+ | 1 -1 \rangle \end{pmatrix}$$

$$L_+ = \begin{pmatrix} 0 & \langle 1 1 | L_+ | 1 0 \rangle & 0 \\ 0 & 0 & \langle 1 0 | L_+ | 1 -1 \rangle \\ 0 & 0 & 0 \end{pmatrix}$$

$$L_+ |1 0\rangle = \hbar \sqrt{1(1+1) - 0 \cdot 1}\, |1 1\rangle = \sqrt{2}\hbar |1 1\rangle$$

$$L_+ |1 -1\rangle = \hbar \sqrt{1(1+1) + 1 \cdot 0}\, |1 0\rangle = \sqrt{2}\hbar |1 0\rangle$$

$$\boxed{L_+ = \hbar \begin{pmatrix} 0 & \sqrt{2} & 0 \\ 0 & 0 & \sqrt{2} \\ 0 & 0 & 0 \end{pmatrix}}$$

5 SAME AS HOMEWORK 1, PROBLEM 1

Inside the box $-\frac{\hbar^2}{2m}\nabla^2\psi = E\psi$

$$-\frac{\hbar^2}{2m}\left[\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}+\frac{\partial^2\psi}{\partial z^2}\right]=E\psi \quad (1)$$

Separation of variables $\psi(xyz)=X(x)Y(y)Z(z)$

Substitute into (1), divide by ψ, multiply by $-2m/\hbar$

$$\underbrace{\frac{1}{X}\frac{d^2X}{dx^2}}_{f(x)}+\underbrace{\frac{1}{Y}\frac{d^2Y}{dy^2}}_{f(y)}+\underbrace{\frac{1}{Z}\frac{d^2Z}{dz^2}}_{f(z)}=-\frac{2m}{\hbar}E$$

In order for this equation to be satisfied for all (xyz) each individual term must be equal constant

Write $\frac{1}{X}\frac{d^2X}{dx^2}=-k_x^2 \quad \frac{1}{Y}\frac{d^2Y}{dy^2}=-k_y^2 \quad \frac{1}{Z}\frac{d^2Z}{dz^2}=-k_z^2$

$$k_x^2+k_y^2+k_z^2=\frac{2mE}{\hbar} \Rightarrow E=\frac{\hbar^2}{2m}(k_x^2+k_y^2+k_z^2)$$

Solution of the X equation

$X(x)=A_x\sin k_x x + B_x\cos k_x x$

Boundary condition $X(0)=X(\ell)=0$

this gives $B_x = 0$ $k_x = n_x \pi / a$

And similarly for y and z. Therefore

$$\boxed{\psi(xyz) = A \sin \frac{n_x \pi}{a} x \ \sin \frac{n_y \pi}{a} y \ \sin \frac{n_z \pi z}{a}}$$

and $$\boxed{E = \frac{\hbar^2 \pi^2}{2ma^2}(n_x^2 + n_y^2 + n_z^2)}$$

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$K^{*+} \left| \tfrac{1}{2} \, \tfrac{1}{2} \right\rangle$ $K^{*0} \left| \tfrac{1}{2} \, -\tfrac{1}{2} \right\rangle$

$K^{*+} \to K^0 \pi^+$ or $K^+ \pi^0$

$K^+ \pi^0 = \left| \tfrac{1}{2} \, \tfrac{1}{2} \right\rangle \left| 1 \, 0 \right\rangle = \sqrt{\tfrac{2}{3}} \left| \tfrac{3}{2} \, \tfrac{1}{2} \right\rangle + \sqrt{\tfrac{1}{3}} \left| \tfrac{1}{2} \, \tfrac{1}{2} \right\rangle$

$K^0 \pi^+ = \left| \tfrac{1}{2} \, -\tfrac{1}{2} \right\rangle \left| 1 \, 1 \right\rangle = \sqrt{\tfrac{1}{3}} \left| \tfrac{3}{2} \, \tfrac{1}{2} \right\rangle + \sqrt{\tfrac{2}{3}} \left| \tfrac{1}{2} \, \tfrac{1}{2} \right\rangle$

$$\boxed{\frac{\Gamma(K^{*+} \to K^0 \pi^+)}{\Gamma(K^{*+} \to K^+ \pi^0)} = \left(\frac{\sqrt{1/3}}{\sqrt{2/3}} \right)^2 = \frac{1}{2}}$$

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(7) (a) $\psi(\vec{r}) = (x+y)\, f(r)$

$x = r\sin\theta\cos\phi$

$y = r\sin\theta\sin\phi$

Note $Y_1^1 = -C\sin\theta\, e^{i\phi} = C(-\sin\theta\cos\phi - i\sin\theta\sin\phi)$

$Y_1^{-1} = +C\sin\theta\, e^{-i\phi} = C(\sin\theta\cos\phi - i\sin\theta\sin\phi)$

$\Rightarrow Y_1^1 - Y_1^{-1} = -2C\sin\theta\cos\phi = \dfrac{-2Cx}{r}$ $\qquad C = \sqrt{\dfrac{3}{8\pi}}$

$Y_1^1 + Y_1^{-1} = -2iC\sin\theta\sin\phi = \dfrac{-2iCy}{r}$

$\Rightarrow x = \dfrac{-r}{2C}\left(Y_1^1 - Y_1^{-1}\right)$

$y = -\dfrac{r}{2iC}\left(Y_1^1 - Y_1^{-1}\right) = \dfrac{ir}{2C}\left(Y_1^1 - Y_1^{-1}\right)$

$\psi = \dfrac{1-i}{2C}\left(Y_1^{-1} - Y_1^1\right) r f(r)$

$\Rightarrow$ $\boxed{\text{eigenstate of } L^2\text{, eigenvalue } \ell(\ell+1)\hbar^2}$

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$\boxed{\text{Not eigenstate of } J_z}$

(b)

$$\boxed{\begin{array}{ll} 50\% \text{ prob} & |1\ -1\rangle \\ 50\% \text{ prob} & |1\ \ 1\rangle \end{array}}$$

8 SAME AS HOMEWORK 5 PROBLEM 3

$H = \varepsilon \vec{S}_1 \cdot \vec{S}_2$

$S^2 = S_1^2 + S_2^2 + 2\vec{S}_1 \cdot \vec{S}_2$

$H = \frac{1}{2}\varepsilon\left[S^2 - S_1^2 - S_2^2\right]$

$\frac{1}{2} \otimes 1 = \frac{3}{2} \oplus \frac{1}{2}$

When $S = \frac{3}{2}$ eigenvalue is

$$E = \frac{\hbar^2}{2}\varepsilon\left[\frac{3}{2}\left(\frac{3}{2}+1\right) - \frac{1}{2}\left(\frac{1}{2}+1\right) - 1(1+1)\right]$$

$$\boxed{E = \frac{1}{2}\varepsilon\hbar^2}$$

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When $S = 1/2$ eigenvalue is

$$E = \frac{\hbar^2}{2}\varepsilon\left[\frac{1}{2}\left(\frac{1}{2}+1\right) - \frac{1}{2}\left(\frac{1}{2}+1\right) - 1(1+1)\right]$$

$$\boxed{E = -\varepsilon\hbar^2}$$