Physics 115B, Spring 2022, Final Exam

Common notation: Let $|\ell, m\rangle$ represent an eigenstate of angular momentum with eigenvalues of $\hat{L}^2$ and $\hat{L}_z$ given by $\ell(\ell+1)\hbar^2$ and $m\hbar$, respectively. In the case of spin states, simply replace $L \to S$, $\ell \to s$. Similarly $|I, I_3\rangle$ is an eigenstate of isospin with eigenvalues of $\hat{I}^2$ and $\hat{I}_3$ equal to $I(I+1)$ and I_3 respectively.
Scoring: Every problem is worth 10 points. In the case of multiple questions in a given problem, all questions are worth the same number of points.

Please put a "box" around each of your final answers.

1 Problem 1

Answer the following quick questions. No explanation needed.
(a) A particle moves in a potential $V(r) = -k/r^6$ where k is a positive constant. The bound state wavefunctions will be proportional to spherical harmonics. True or False?
(b) Isospin is a **perfect** symmetry of the strong interaction. True or False?
(c) What is the energy in eV of the photon emitted when a hydrogen atom makes a transition from the $n = 2$ to the $n = 1$ energy level?
(d) The spin of an electron is associated with the rotation of the electron around its axis. True or False?
(e) Consider a charged particle moving in three dimensions in the potential of two identical charges fixed on the x-axis at $x = 0$ and $x = a$. The eigenfunctions of the hamiltonian are also eigenfunctions of the parity operator? True or False?
(f) Let $|a\rangle$ and $|b\rangle$ represent two quantum states in Dirac notation. What is $|b\rangle\langle a|$? A complex number? A real number? Another quantum state? An operator? Pick one.
(g) Same question as (f), but now for $\langle b|a\rangle$.

2 Problem 2

The spin state of an electron is $|\psi\rangle = (a^2+b^2)^{-\frac{1}{2}}(a\,|\frac{1}{2},\frac{1}{2}\rangle + b\,|\frac{1}{2},-\frac{1}{2}\rangle)$ where a and b are real numbers.
(a) Find the expectation value of $\hat{S}_x$.
(b) Find the probability that a measurement of S_x yields $\frac{\hbar}{2}$.

3 Problem 3

(a) Consider the operator $\hat{A} = \hat{L}_x\hat{L}_y + \hat{L}_y\hat{L}_x$. Show that $\hat{A}$ is hermitian.
(b) Find the expectation value of $\hat{A}$ in the $|\ell, m\rangle$ state.

4 Problem 4

(a) Write the $\hat{L}_z$ operator in matrix form for states of $\ell = 1$ in the basis with eigenvectors $|u_1\rangle = |1,1\rangle$, $|u_2\rangle = |1,0\rangle$, $|u_3\rangle = |1,-1\rangle$.
(b) Write the $\hat{L}_+$ operator in matrix form for the same basis.

5 Problem 5

Consider a particle of mass m in a 3d infinite "cubical well" of length a to a side, corresponding to the potential

$$V(x,y,z) = \begin{cases} 0 & x, y, z \text{ all between 0 and } a \\ \infty & \text{otherwise} \end{cases}$$

You can think of this as a particle in a box with infinitely thick walls.
Use separation of variables in Cartesian coordinates to find the eigenfunctions and the corresponding energies. To keep things simple, you do not need to normalize the eigenfunctions.

6 Problem 6

Consider the K^*, K, and π states (particles). The K^* and K states have $I = \frac{1}{2}$, the π states have $I = 1$. The $|I, I_3\rangle$ assignments are:

$$\begin{array}{lllllllll} |K^{*+}\rangle & = & |\frac{1}{2}, \frac{1}{2}\rangle & |K^{*0}\rangle & = & |\frac{1}{2}, -\frac{1}{2}\rangle & & & \\ |K^{+}\rangle & = & |\frac{1}{2}, \frac{1}{2}\rangle & |K^{0}\rangle & = & |\frac{1}{2}, -\frac{1}{2}\rangle & & & \\ |\pi^{+}\rangle & = & |1, 1\rangle & |\pi^{0}\rangle & = & |1, 0\rangle & |\pi^{-}\rangle & = & |1, -1\rangle \end{array}$$

What are the relative probabilities for $K^{*+} \to K^0\pi^+$ and $K^{*+} \to K^+\pi^0$.

7 Problem 7

The wave function of a particle moving in a spherical potential is $\psi(\vec{r}) = (x + y)f(r)$ where $f(r)$ is some function of r.
(a) Is $\psi(\vec{r})$ an eigenfunction of the $\hat{L}^2$ and/or $\hat{L}_z$ operators?
(b) What are the probabilities for the particle to be measured in various $|\ell, m\rangle$ states?

8 Problem 8

Consider two particles. Particle 1 has spin $s_1 = 1/2$, while particle 2 has spin $s_2 = 1$. The particles are at rest and subject to an interaction that depends on their relative spin orientations, corresponding to the Hamiltonian

$$H = \epsilon \vec{S}_1 \cdot \vec{S}_2$$

Here $\vec{S}_1$ is the spin operator for particle 1 and $\vec{S}_2$ is the spin operator for particle 2. What are the energy eigenvalues of this Hamiltonian?