Transformation of the form $U(\vec{\alpha}) = e^{-i\vec{\alpha}\vec{k}}$

$\vec{\alpha}$ real $\vec{k}$ set of operators _ Generators of transformation

$U^+U = 1$ · $U = 1 - i\vec{\alpha}\vec{k} + \ldots$ $U^{-1} = U^+$

$U^+U = (1 + i\vec{\alpha}\vec{k}^+ + \ldots)(1 - i\vec{\alpha}\vec{k} + \ldots)$

$u^+u = 1 + i\vec{\alpha}(k^+ - k) + O(\alpha^2) + \ldots$

$K^+ = K$ hermitian, observable

We have seen translations $\vec{k} = \vec{p}/\hbar$

rotations $\vec{k} = \vec{J}/\hbar$

Isospin $\vec{k} = \vec{\sigma}/2$

Good Symmetry: $U^+HU = H$

(1) Transitions $a \to b$ — physics 115c

$A_{ab} = \langle b|\hat{O}|a\rangle$

strength of transition $\propto |A_{ab}|^2$

$\hat{O}$ contains details of interaction, function of H $[u\ \hat{o}] = 0$

consider

$|a'\rangle = U|a\rangle$

$|b'\rangle = u|b\rangle$

$A_{ab} = \langle b|u^+u O u^+u|a\rangle = \langle b'|\underbrace{uOu^+}_{=0}|a'\rangle = A_{a'b'}$

$\Rightarrow$ SAME AMPLITUDE FOR TRASITIONS BETWEEN STATES RELATED BY SYMMETRY OPERATION

(2) $\boxed{[k; H] = 0}$

Simple to show

Consider infinitesimal transformation

$U = 1 - \vec{\alpha}\vec{K} = 1 - i\alpha_i k_i$ ($\vec{\alpha}$ small)

$U^+ H U = H$

$(1 + i\alpha_i k_i^+)\, H\, (1 - i\alpha_i k_i) = H$

To first order in α_i, and using $k_i^+ = k_i$

$H + i\alpha_i \underbrace{(k_i H - H k_i)}_{\text{must be zero}} = H \Rightarrow [k_i\ H] = 0$

(3) $\boxed{\text{Conservation of quantum numbers}}$

$k_i |a\rangle = k_a |a\rangle$

$k_i |b\rangle = k_b |b\rangle$

Since $[k\ O] = 0$

$\langle b | [k\ O] | a\rangle = 0$

$\langle b | kO - Ok | a\rangle = 0$ (remember $k^+ = k$)

$(k_b - k_a)\langle b | O | a\rangle = 0$

either $\langle b|O|a\rangle = 0$ <u>OR</u>, $k_b = k_a$

(4) $\boxed{\text{DEGENERACY}}$

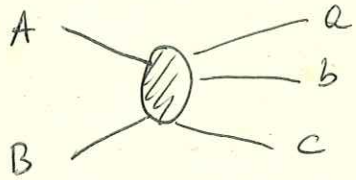
Take $|a\rangle$ to be eigenstate of H

$H|a\rangle = E_a|a\rangle$ $\quad |b\rangle = U|a\rangle$ (Assume $|a\rangle \neq |b\rangle$, otherwise trivial)

$E_a = \langle a|H|a\rangle = \langle a|U^+ H U|a\rangle = \langle b|H|b\rangle = E_b$ $\quad E_A = E_B$

$\boxed{\text{STATES RELATED BY SYMM. TRANSF. ARE DEGENERATE}}$

(5) CONTINOUS SYMMETRIES LEAD TO ADDITIVE CONSERVATION LAWS



example, momentum

$AB \to abc$

$|i\rangle$ = initial state $= |\vec{P}_A\rangle |\vec{P}_B\rangle$

$|f\rangle$ = final state $= |\vec{P}_a\rangle |\vec{P}_b\rangle |\vec{P}_c\rangle$

Translation

$U(\vec{\alpha})|i\rangle = e^{-i\vec{\alpha}\hat{\vec{P}}_A} |\vec{P}_A\rangle \, e^{-i\vec{\alpha}\hat{\vec{P}}_B} |\vec{P}_B\rangle$

$U(\vec{\alpha})|i\rangle = e^{-i\alpha\hat{P}_i} \underbrace{|P_A\rangle |P_B\rangle}$ $\leftarrow = |i\rangle$

where $\hat{\vec{P}}_i = \hat{\vec{P}}_A + \hat{\vec{P}}_B$

Similarly for $U(\vec{\alpha})|f\rangle = e^{-i\alpha\hat{\vec{P}}_f} |f\rangle$

$\langle f|O|i\rangle = \langle f|U^{\dagger} O U|i\rangle = e^{i\alpha(\vec{P}_f - \vec{P}_i)} \langle f|O|i\rangle$

either $\langle f|O|i\rangle = 0$ <u>OR</u> $\vec{P}_f = \vec{P}_i$

(6) DISCRETE SYMMETRIES $\to$ MULTIPLICATIVE CONS. LAW

eg Parity - Not continous, no generators

We already saw, possible eigenvalues ± 1

$P|a\rangle = P_a|a\rangle \qquad P|b\rangle = P_b|b\rangle$

$\langle b|O|a\rangle = \langle b|P^{\dagger} O P|a\rangle = P_b \cdot P_a \langle b|O|a\rangle$

either $P_a P_b = 1$ OR $\langle b|O|a\rangle = 0$

$P_a = 1, \; P_b = 1 \qquad P_a = -1, \; P_b = -1$ $\longleftarrow$ <u>These are the possibilities</u> <u>Conservation law</u>